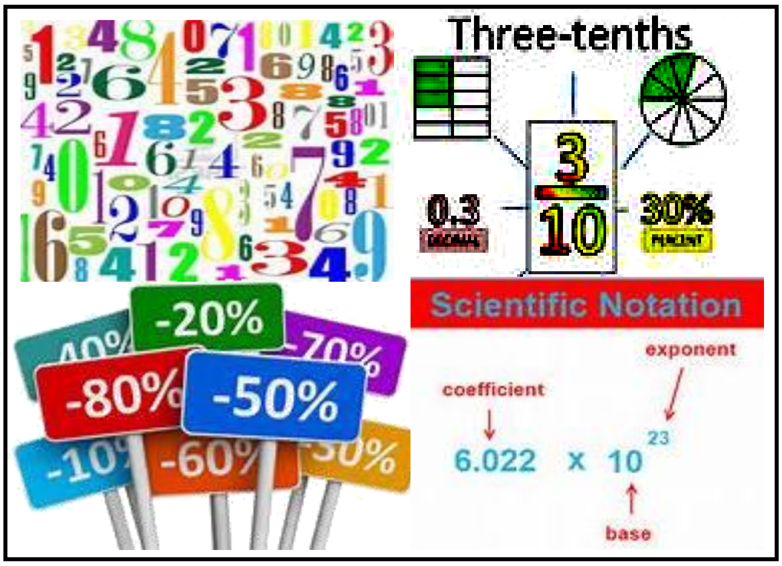
DEPARTMENT OF EDUCATION

GRADE 10

MATHEMATICS

UNIT 1



BASIC NUMERACY SKILLS

Published by:

FLEXIBLE OPEN AND DISTANCE EDUCATION
PRIVATE MAIL BAG, P.O. WAIGANI, NCD
FOR DEPARTMENT OF EDUCATION
PAPUA NEW GUINEA
2017

GRADE 10

MATHEMATICS

UNIT 1

BASIC NUMERACY SKILLS

TOPIC 1:	**NUMBERS**
TOPIC 2:	**ESTIMATION**
TOPIC 3:	**DIRECTED NUMBERS**
TOPIC 4:	**NUMBER SEQUENCES AND PATTERNS**

ACKNOWLEDGEMENT

We acknowledge the contributions of all Secondary Teachers who in one way or another have helped to develop this Course.

Our profound gratitude goes to the former Principal of FODE, Mr. Demas Tongogo for leading FODE team towards this great achievement. Special thanks to the Staff of the English Department of FODE who played an active role in coordinating writing workshops, outsourcing lesson writing and editing processes, involving selected teachers of Central Province and NCD.

We also acknowledge the professional guidance provided by Curriculum and Development Assessment Division throughout the processes of writing, and the services given by member of the English Review and Academic Committees.

The development of this book was Co-funded by GoPNG and World Bank.

DIANA TEIT AKIS
PRINCIPAL

Flexible Open and Distance Education
Papua New Guinea

Published in 2017

ISBN: 978-9980-89-701-5
National Library Services of Papua New Guinea

Compiled and finalized by: Mathematics Department-FODE

Printed by the Flexible, Open and Distance Education

CONTENTS

SECRETARY'S MESSAGE

Achieving a better future by individuals students, their families, communities or the nation as a whole, depends on the curriculum and the way it is delivered.

This course is part and parcel of the new reformed curriculum – the Outcome Base Education (OBE). Its learning outcomes are student centred and written in terms that allow them to be demonstrated, assessed and measured.

It maintains the rationale, goals, aims and principles of the National OBE Curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision of Flexible, Open and Distance Education as an alternative pathway of formal education.

The Course promotes Papua New Guinea values and beliefs which are found in our constitution, Government policies and reports. It is developed in line with the National Education Plan (2005 – 2014) and addresses an increase in the number of school leavers which has been coupled with a limited access to secondary and higher educational institutions.

Flexible, Open and Distance Education is guided by the Department of Education's Mission which is fivefold;

- to facilitate and promote integral development of every individual
- to develop and encourage an education system which satisfies the requirements of Papua New Guinea and its people
- to establish, preserve, and improve standards of education throughout Papua New Guinea
- to make the benefits of such education available as widely as possible to all of the people
- to make education accessible to the physically, mentally and socially handicapped as well as to those who are educationally disadvantaged

The College is enhanced to provide alternative and comparable path ways for students and adults to complete their education, through one system, many path ways and same learning outcomes.

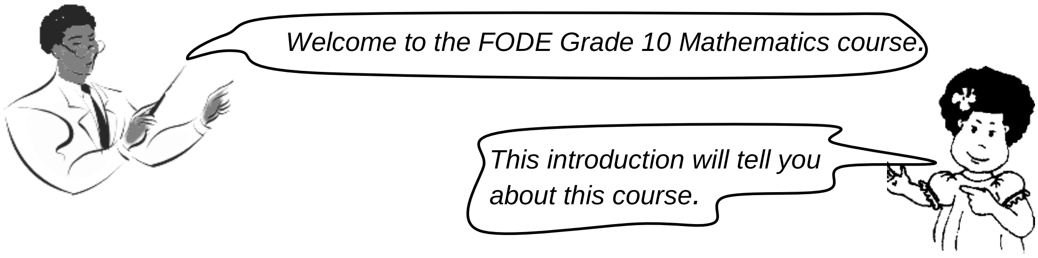
It is our vision that Papua New Guineans harness all appropriate and affordable technologies to pursue this program.

I commend all those teachers, curriculum writers and instructional designers, who have contributed so much in developing this course.

....

DR. UKE KOMBRA, PhD
Acting Secretary for Education

COURSE INTRODUCTION



HOW TO STUDY YOUR GRADE 10 MATHEMATICS COURSE?

1. YOUR LESSONS

In Grade 10 Mathematics there are 6 books for you to study. Each book corresponds to each of the six strands of the course.

Unit 1: Basic Numeracy Skills
Unit 2: Managing your Money
Unit 3: Basic Algebra
Unit 4: Functions and Graphs
Unit 5: Trigonometry
Unit 6: Measurement

Each Unit is divided into 4 Topics and each Topic consists of a minimum of 5 to 7 lessons.

Here is a list of the Units in this Grade 9 course and the topics you will study:

UNITS	TOPICS	TITLE
1 BASIC NUMERACY SKILLS	1	NUMBERS
	2	ESTIMATION
	3	DIRECTED NUMBERS
	4	NUMBER SEQUENCES AND PATTERNS
2 MANAGING YOUR MONEY	1	PERCENTAGES AND MONEY
	2	RATIO AND PROPORTION
	3	RATES
	4	LOANS(BORROWING MONEY)
3 BASIC ALGEBRA	1	EXPANDING ALGEBRAIC EXPRESSIONS
	2	FACTORIZING ALGEBRAIC EXPRESSIONS
	3	POWERS/INDICES
	4	EQUATIONS
4 FUNCTIONS AND GRAPHS	1	STATISTICAL GRAPHS
	2	LINEAR OR STRAIGHT LINE GRAPHS
	3	QUADRATIC FUNCTIONS AND PARABOLA
	4	SYSTEMS OF EQUATIONS

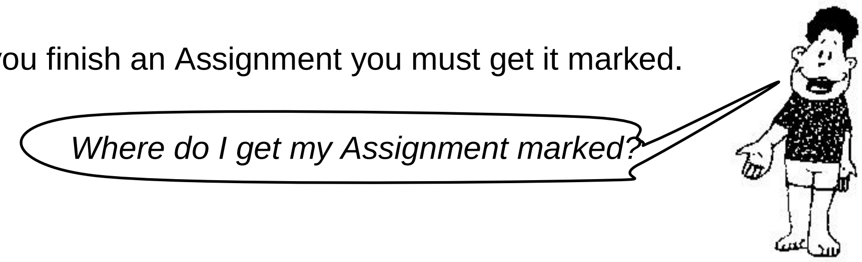
5 TRIGONOMETRY	1	ANGLES AND TRIANGLES
	2	PYTHAGORAS’ THEOREM
	3	POLYGONS AND CIRCLES
	4	INTRODUCTION TO TRIGONOMETRIC FUNCTIONS
6 MEASUREMENTS	1	INDIRECT MEASUREMENTS
	2	SCALE AND SIMILARITY
	3	AREA
	4	VOLUME

2. YOUR ASSIGNMENTS

In this course you will also do six ASSIGNMENTS.

You will study Unit 1 and do Assignment 1 at the same time. Then you will study Unit 2 and do Assignment 2 at the same time, and so on up to Assignment 6.

When you finish an Assignment you must get it marked.

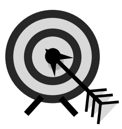


1. **Students in a Registered Study Centre** must give their finished Assignments to their Supervisor for marking.
2. **Students who study at Home by themselves but who live in the Provinces** must send their finished Assignment books to their Provincial Centres for marking.

3. LESSON ICONS

Below are the icons used by FODE in its courses.

Lesson Introduction



Aims

Summary

Activities/Practice Exercises

UNIT 1: BASIC NUMERACY SKILLS- REVISION

Introduction

Dear Student,

This is the **first Unit** of the **Grade 10 Mathematics Course**. You will study it by following the steps suggested in the **Study Guide** on the next page. This Unit is based on the NDOE Lower Secondary Mathematics Syllabus. You will study at Home what students in High Schools study at school.

The four Topics in this Unit are:

Topic 1: Numbers
Topic 2: Estimation
Topic 3: Directed Numbers
Topic 4: Number Sequences and Patterns

In Topic 1- **Numbers -** You will consider revising basic facts whole numbers. Perform the fundamental operations (addition, subtraction, multiplication and division) with whole numbers, fractions and decimals with more speed and accuracy. Our emphasis will be on how the operations are related to each other, as well as their applications to real world situations.

In Topic 2- **Estimation**- You will learn to estimate and round off numbers. You will also introduce with significant figures their importance in estimation. You will learn the basic rules for significant figures in adding, subtracting, multiplying and dividing numbers.

In Topic 3- **Directed Numbers**- You will revise the meaning of integers. You will also revise addition, subtraction, multiplication and division of integers. And lastly perform the order of operation rules in using integers.

In Topic 4- **Numbers Sequences and Patterns** – You will learn the meaning of sequences and the different types of sequences. You will also learn the about the most common number patterns and how they are made. .And lastly, you will

All lessons in this unit are written in simple language and many worked examples to help you.

STUDY GUIDE

Follow the steps given below as you work through the Unit.

Step 1: Start with Topic 1 Lesson 1 and work through it.

Step 2: When you complete Lesson 1, do Practice Exercise 1.

Step 3: After you have completed Practice Exercise 1, check your work. The answers are given at the end of TOPIC 1.

Step 4: Then, revise Lesson 1 and correct your mistakes, if any.

Step 5: When you have completed all these steps, tick the Lesson check-box on the Contents Page like this:

$\boxed{\surd}$ Lesson 1: Whole Numbers-Revision

Then go on to the next Lesson. Repeat the process until you complete all of the lessons in Topic 1.

> As you complete each lesson, tick the check-box for that lesson, on the Contents Page, like this $\boxed{\surd}$ .This helps you to check on your progress.

Step 6: Revise the Topic using Topic 1 Summary, then, do Topic test 1 in Assignment 1.

Then go on to the next Topic. Repeat the process until you complete all of the four Topics in Unit 1.

<u>Assignment:</u> (Four Topic Tests and a Unit Test)
When you have revised each Topic using the Topic Summary, do the Topic Test in your Assignment Book. The Course Book tells you when to do each Topic Test.

When you have completed the four Topic Tests, revise well and do the Topic test. The Assignment Book tells you when to do the Unit Test.

> The Topic Tests and the Unit test in the Assignment will be marked by your Distance Teacher. The marks you score in each Assignment Book will count towards your final mark. If you score less than 50%, you will repeat that Assignment Book.

Remember, if you score less than 50% in three Assignments, you will not be allowed to continue. So, work carefully and make sure that you pass all of the Assignments.

TOPIC 1

NUMBERS

TOPIC 1: NUMBERS

Introduction

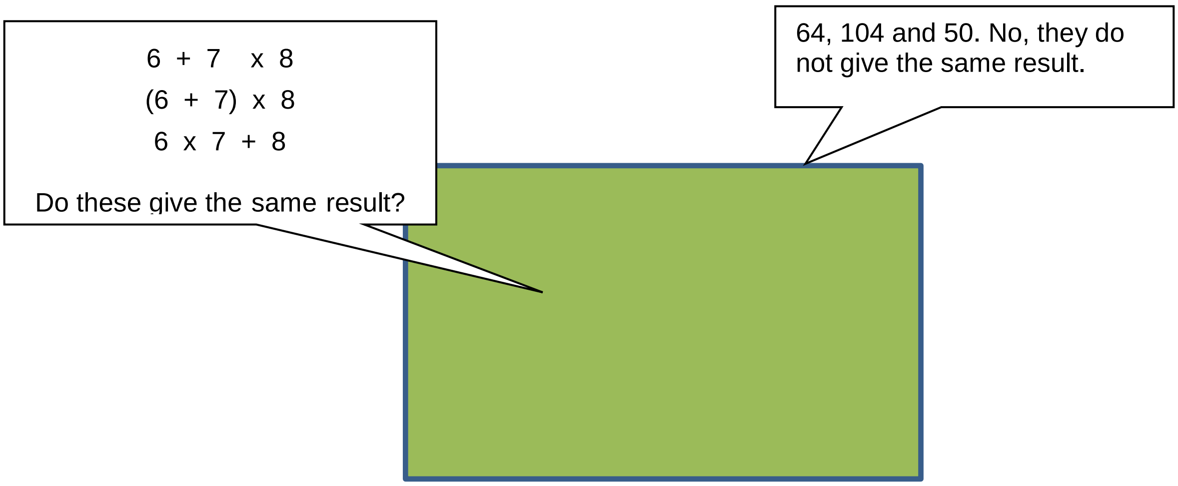
You have done problems involving whole numbers, fractions and decimals. You also know that the decimal point separates the whole number part from the fractional part of a number.

For example, here are two problems.

> "9670 kg of goods are packed into 100 equal loads. How many kilos will be in each load?"
>
> "Tony's fortnightly pay is K316.09. He spends three-quarters of it and saves the balance. How much does he save in a year?"

Ability to do such problems is useful in daily life. It is also necessary for further studies. It is essential that you do the operations with confidence.

Whenever you come across two or more operations, you have to use the correct order in performing the operations.



Is there any special order in which you should do the addition and multiplication?

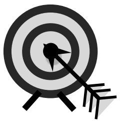
At this level, you should know the correct order to do the operations. You have covered this in Grade 9.

Number skills are gained and developed by practice. Repeated practice helps you gain confidence and speed.

In this topic, you will have another chance to revise and practice operations with whole numbers, fractions, decimals, and percentages as well as their application in real life problem solving.

Lesson 1: Whole Numbers: Revision

You have learned about the properties of whole numbers and work out further problems by using these properties.



In this lesson, you will

- revise the properties of whole numbers and work out further problems by using these properties.
- define and identify prime and composite numbers
- determines the prime factors of a given number
- determine the divisibility of whole numbers.

The Decimal System

Our counting system is based on the decimal system or base 10 - system. We write numbers using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 in different places. The place-value of a digit fixes its value.

The place value table below will help you to read and write numbers correctly.

The Place-Value table

1 000 000's	**100 000's**	**10 000's**	**1000's**	**100's**	**10's**	**1's**

Spacing in Threes

Spacing the digits in threes from the right makes reading of numbers easy. Such spacing is necessary only for numbers with **five or more digits**.

For example:

millions

7 203 567

thousands

7 233 567 is 'Seven million, two hundred and three thousand, five hundred and sixty-seven. Zero is an important 'place holder'. We put zeros in places where there are no other digits.

For example:

8 000 006

The five zeros between 8 and 6 tell us that the place value of 8 is a 'million'. You should place the zeros carefully. In multiplications and divisions, you have to put in zeros yourself if necessary to fill vacant places.

Prime and Composite Numbers

2, 3, 5, 7, 11 are some prime numbers.

A **prime number** has only **1** and **itself** as its factors.

For example:

11 = 1 x 11 ....11 has no factors other than 1and 11.

4, 6, 8, 9, 10, 12 are composite numbers.

A **composite number** has more than two numbers as factors.

For example:

12 = 1 x 12 = 2 x 6 = 3 x 4 .... 12 has factors 2 and 3 and 4 and 6

Common Factors and Greatest Common Factor (G.C.F)

Two or more numbers may have factors in common. Any two prime numbers have no common factor other than 1.

Say we have worked out the factors of two numbers

Example 1

Factors of 12 and 30

Factors of 12 are 1, 2, 3, 4, 6 and 12.

Factors of 30 are 1, 2, 3, 5, 6, 10, 15 and 30.

Then the **common factors** are those that are found in both lists.

- Notice that **1, 2, 3** and **6** appear in both lists?
- So, the **common factors** of 12 and 30 are: **1, 2, 3** and **6**

A number is a **common factor** when it is a factor of two or more numbers.

Example 2

Find the common factors of 15, 30 and 105.

Solution:

Factors of 15 are **1, 3, 5,** and **15**
Factors of 30 are **1, 2, 3, 5, 6, 10, 15** and **30**
Factors of 105 are **1, 3, 5, 7, 15, 21, 35** and **105**

The factors that are common to all three numbers are **1, 3, 5** and **15.** In other words, the **common factors** of 15, 30 and 105 are **1, 3, 5** and **15.**

When we find all the factors of two or more numbers, and some factors are the same ("common"), then the largest of those common factors is the **Greatest Common Factor**. It is abbreviated "**GCF**" and also called "**Highest Common Factor**".

In our previous example, the largest of the common factors is 15, so the **Greatest Common Factor** of 15, 30 and 105 is **15.**

> The **Greatest Common Factor** is the largest of the common factors of two or more numbers. It is the highest number that divides exactly into two or more numbers.

We can find the GCF of two or more numbers by writing them as product of their prime factors.

Example

We can write the following numbers 12 and 16 as

12 = 2 x 2 x 3

18 = 2 x 3 x 3

2 is common to both 12 and 18. 3 is also common to them. So, the highest factor common to both of them is 2 x 3 = 6.

We say: The **GCF** of 12 and 18 is **6.**

Common Multiples and Lowest Common Multiples (LCM)

We get a **multiple** of a number when we **multiply it by another number**, such as multiplying by 1, 2, 3, 4, 5, and so on, but not zero. Just like the multiplication table.

Here are some examples:

The multiples of **4** are: 4,8,12,16,20,24,28,32,36,40,44,...

The multiples of **5** are: 5,10,15,20,25,30,35,40,45,50,...

What is a "Common Multiple"?

Say we have listed the first few multiples of **4** and **5**: the **common multiples** are those that are found in both lists:

The multiples of 4 are: 4,8,12,16,**20**,24,28,32,36,**40**,44,...

The multiples of 5 are: 5,10,15,**20**,25,30,35,**40**,45,50,...

Notice that **20** and **40** appear in both lists?

So, the common multiples of 4 and 5 are: **20, 40,** (and **60**, **80**, etc ..., too).

What is the "Least Common Multiple"?

It is simply the smallest of the common multiples.

In our previous example, the smallest of the common multiples is **20**. Hence, the Least Common Multiple of 4 and 5 is **20**.

To find the Least Common Multiple of two or more numbers, list the multiples of the numbers until we get our first match.

Example 1

Find the least common multiple of 6 and 10:

Solution:

The multiples of 6 are: 6, 12, 18, 24, 30, 36, …
The multiples of 10 are: 10, 20, 30, 40, 50, …

The first match is 30.

So the least common multiple of 4 and 10 is **30.**

Example 2

Find the least common multiple of 4 and 6.

Solution:

The multiples of 4: 4, 8, **12,** 16, 20, **24**, 28, …

The multiples of 6: 6, **12**, 18, **24,** 30, 36, …

12 and 24 are common multiples of 4 and 6.

We can still write more, like 36, 48, 60, 72,… as common multiples of 4 and 6. The least (smallest) of the common multiples is 12. We say 12 is the L.C.M. of 4 and 6.

> A **common multiple** is a number that is a **multiple** of two or more numbers.
>
> The least **common multiple** (LCM) of two numbers is the smallest number (not zero) that is a **multiple** of both.

Now, look at the following examples:

1. Here is a statement.

   "In 2013, the population of P.N.G. was seven million, three hundred and twenty one thousand, and eight hundred and sixty seven".

   Write the population using numbers, spacing them correctly.

   Answer: 7 321 867

2. Work out:

(a) 520 x 40

$$\begin{array}{r} 520 \\ \times\ 40 \\ \hline 20\ 800 \end{array}$$

Answer: 20 800

(b) 5544 ÷ 18

$$\begin{array}{r} 308 \\ 18\overline{)5544} \\ 54 \\ \hline 14 \\ -00 \\ \hline 144 \\ -144 \\ \hline 000 \end{array}$$

Answer: 308

(c) The difference between 80 000 and 9 023.

$$\begin{array}{r} 80000 \\ -\ 9023 \\ \hline 70977 \\ \hline \end{array}$$

Answer: 70 977

3. Find the G.C.F. of 48, 60 and 84.

In order to find the H. C. F. of 48, 60, 84, we write the common factors of these.

Factors of 48 = 2 x 2 x 2 x 2 x 3
Factors of 60 = 2 x 2 x 3 x 5
Factors of 84 = 2 x 2 x 3 x 7

Clearly from the above, we can see that 2 x 2 is common to all three numbers 48, 60 and 84. 3 is also common to them.

So, G.C.F. of all these three numbers = 2 x 2 x 3 = 12

Answer: 12

4. Find the L.C.M. of 14, 21 and 28

In order to find the L. C. M. of 14, 21, 28, we write the common multiples of these.

Multiples of 14:	14	28	42	56	70	<u>84</u>	98
Multiples of 21:	21	42	63	<u>84</u>	105	126	147
Multiples of 28:	28	56	<u>84</u>	112	140	168	196

84 is the common multiple of 14, 21 and 28. We can still write more multiples like 84 x 2 = 168, 84 x 3 = 252 and so on.

The smallest of the common multiples is 84.

So, the L.C.M. of 14, 21, 28 = 84

Answer: 84

5. What is the smallest number divisible by 8, 10 and 12?

All common multiples of 8,10 and 12 are divisible by each number.
The smallest common multiple is the L.C.M. of 8, 10 and 12.

Multiples of 8: 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96,104,112, <u>**120,**</u> 128

Multiples of 10:10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110, <u>**120**</u>, 130, 140, 150

Multiples of 12:12, 24, 36, 48, 60, 72, 84, 96,108, <u>**120**</u>,132,144, 156, 168, 180

120 is common to all the three numbers 8, 10, 12.

We may have more multiples like 240, 360 and so on but the least common multiple is **120.**

Therefore, the L.C.M. of 8, 10 and 12 is **120**.

Answer: 120

NOW DO PRACTICE EXERCISE 1

Practice Exercise 1

1. Find the H. C. F. of:

   (a) 12 and 18

   Answer: __________

   (b) 15, 30 and 60

   Answer: __________

2. Find the smallest number divisible by:

   (a) 6 and 8

   Answer: __________

   (b) 2, 7 and 8

   Answer: __________

3. Work out:

   (a) 2772 ÷ 18

   Answer: ________________

   (b) 630 x 40

   Answer: __________________

   (c) 90 000 – 17 023

   Answer: __________________

4. (a) Add the following amounts of money:

   K17 130 337 + K477 421 + K46 789

   Answer: __________________

   (b) Round the total to the nearest million Kina.

   Answer: __________________

5. Tony has 9 two Kina notes, 11 twenty toea coins and 4 two toea coins. How much money does he have?

   Answer: __________________

6. Express “ two million, sixty three thousand and seven hundred and twenty five “ in figures.

Answer: ___________________

---

7. Write 27 046 in words.

Answer: ____

---

8. Write 642 as a product of its prime factors

Answer: ___________________

---

9. List multiples of 3, 8 and 15 and find their least common multiple (LCM).

3 = ____

8 = ____

15 = ____

Answer: ___________________

---

10. List factors of 28 and 40. What is the highest common factor (HCF)?

28 = ____

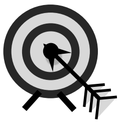
40 = ____

Answer: ___________________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 2: Fractions: Revision

You have revised whole numbers and their properties in the first lesson.



In this lesson, you will;

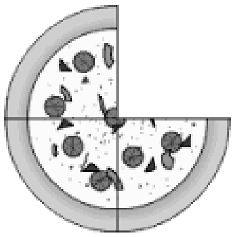
- recall the different types of fractions and their properties
- revise the rules in adding, subtracting, multiplying and dividing fractions and use them in solving word problems.

First let us revise all the new terms and rules about fractions.

A fraction is a part or portion of a whole.

A fraction is made up of 2 numbers. The top number is called the **numerator** and the bottom number is called the **denominator**.

In the fraction below the 3 is the numerator and the 4 is the denominator.



$$\mathbf{\frac{3}{4}}$$

Denominator is number showing how many equal 'pieces' something has been divided into.

Numerator is the number showing how many equal parts has been taken.

Different types of fractions

1) **Unit fractions** are fractions where the numerator is always one (1).

Examples: $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$, etc.

2) **Proper fractions** are fractions whose *numerators are less than the denominators.* These fractions indicate values less than one (1).

Examples: $\frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{7}{8}$, etc.

3) **Improper fractions** are fractions whose *numerators are greater than or equal to the denominators.* These fractions indicate values equal to or greater than one (1).

Examples: $\frac{3}{2}, \frac{7}{3}, \frac{11}{4}, \frac{12}{5}$, etc.

4) **Fractions Written in Mixed Form or Mixed Numbers** are fractions written as a sum of a whole number and a fraction. An improper fraction can also be written as a mixed number or vice versa.

Examples: $1\frac{1}{2}, 2\frac{1}{3}, 2\frac{3}{4}, 2\frac{2}{5}$, etc

5) **Similar Fractions** are two or more fractions whose denominators are the same.

Examples: $\frac{3}{12}, \frac{5}{12}$ and $\frac{7}{12}$, $\frac{2}{5}$ and $\frac{3}{5}$ etc

6) **Dissimilar fractions** are two or more fractions whose numerators and denominators are different.

Examples: $\frac{2}{8}$ and $\frac{3}{7}$; $\frac{1}{2}, \frac{2}{3}$ and $\frac{3}{8}$

Equivalent Fractions

Some fractions may look different, but are really the same, for example:

These are equivalent fractions.

$$\mathbf{\frac{4}{8}} \quad = \quad \mathbf{\frac{2}{4}} \quad = \quad \mathbf{\frac{1}{2}}$$

(Four-Eighths) Two-Quarters) (One-Half)

It is usually best to show an answer using the fraction in in its simplest form ($\frac{1}{2}$ in this case). That is called **Simplifying**, or **Reducing** the fraction.

To simplify a fraction, divide the top and bottom by the **highest number** that can divide into both numbers exactly.

These are fractions in Simplest Form: $\frac{4}{12} \mathbf{=} \frac{1}{3}$; $\frac{3}{9} \mathbf{=} \frac{1}{3}$

Cancel factors common to the numerator and denominator to get the simplest form of that fraction. Sometimes we say it is in its **lowest terms**.

A fraction is said to be in its **simplest form** if the numerator and the denominator cannot be divided by the same factor except 1.

Addition and Subtraction of fractions

Only similar fractions can be added or subtracted. So to add or subtract fractions, we follow the following steps.

Step 1: Make sure the bottom numbers (the denominators) are the same.

Step 2: Add or subtract the top numbers (the numerators), put the answer over the denominator.

Step 3: Simplify the fraction (if needed).

Examples

1. Add: a) $\frac{3}{5} + \frac{1}{5} = \frac{4}{5}$

b) $\frac{2}{3} + \frac{3}{4} = \frac{6}{12} + \frac{9}{12}$ make the denominators the same

$= \frac{15}{12}$ add the top numbers

$= 1\frac{3}{12}$ or $1\frac{1}{4}$ simplify

2. Subtract: a) $\frac{3}{5} - \frac{1}{5} = \frac{2}{5}$

b) $\frac{2}{5} - \frac{1}{4} = \frac{8}{20} - \frac{5}{20}$ make the denominators the same

$= \frac{3}{20}$ subtract the top numbers

Multiplication of Fractions

To multiply fractions, we follow the following steps:

1. Multiply the top numbers (the numerators).
2. Multiply the bottom numbers (the denominators).
3. Simplify the fraction if needed.

Examples

Multiply: a) $\frac{1}{3} \text{ x } \frac{3}{5} = \frac{3}{15}$ ⇒ Multiply 1 and 3 ⇒ Multiply 3 and 5

The GCF of 3 and 15 is 3.

$= \frac{1}{5}$ ⇒ Simplest form (divide both 3 and 15 by 3(GCF)

b) $\frac{1}{3} \text{ x } \frac{9}{16} = \frac{9}{48}$ ⇒ Multiply 1 and 9 ⇒ Multiply 3 and 16

The GCF of 9 and 48 is 3.

$= \frac{3}{16}$ ⇒ Simplest form (divide both 3 and 15 by 3

c) $\frac{2}{3} \text{ x } 6 = \frac{2}{3} \text{ x } \frac{6}{1}$ ⇒ make 6 as fraction by putting over 1.

$= \frac{12}{3}$ ⇒ Multiply 2 and 6 ⇒ Multiply 3 and 1

$= 4$ ⇒ Simplify (divide 12 by 3)

Reciprocal or Multiplicative Inverse

A number multiplied by its reciprocal equals 1.

$\frac{4}{7} \times \frac{7}{4} = 1,$ $\frac{7}{4}$ is the reciprocal of $\frac{4}{7}$

$\frac{4}{7}$ is the reciprocal of $\frac{7}{4}$

$5 \times \frac{1}{5} = 1,$ $\frac{1}{5}$ is the reciprocal of 5

5 is the reciprocal of $\frac{1}{5}$

A number is said to be **a reciprocal** or **multiplicative inverse** of a given number if when multiplied to the given number the result is equal to 1.

Division of Fractions

There are 3 simple steps to divide fractions:

Step 1 Turn the second fraction (the one you want to divide by) upside down. This is now a reciprocal. (Also known as the **multiplicative inverse**)
Step 2 Multiply the first fraction by that reciprocal.
Step 3 Simplify the fraction (if needed)

Examples

Divide: a) $\frac{2}{3} \div 4 = \frac{2}{3} \times \frac{1}{4}$
$= \frac{2}{12}$
$= \frac{1}{6}$

b) $3 \div \frac{3}{4} = 3 \times \frac{4}{3}$
$= \frac{12}{3}$
$= 4$

c) $\frac{1}{12} \div \frac{1}{4} = \frac{1}{12} \times \frac{4}{1}$
$= \frac{4}{12}$
$= \frac{1}{3}$

Now let us look at the following worked out examples.

1. Fill in the missing numbers.

(a) $\frac{2}{3} = \frac{4}{\square}$

Answer: $\frac{2}{3} = \frac{4}{6}$

(b) $\frac{5}{7} = \frac{\square}{42}$

Answer: $\frac{5}{7} = \frac{30}{42}$

2. Write in the simplest form: $\frac{18}{42}$

$\frac{18}{42}$ The GCF of 18 and 42 is 6.

$\frac{3}{7}$ Divide 18 and 42 by the GCF

Answer: $\frac{18}{42} = \frac{3}{7}$

3. Write $\frac{48}{9}$ as a mixed number.

Answer: $\frac{48}{9} = 5\frac{1}{3}$

4. Write $5\frac{6}{7}$ as an improper fraction.

$$5\frac{6}{7} = 5 + \frac{6}{7}$$
$$= \frac{35 + 6}{7}$$
$$= \frac{41}{7}$$

Answer: $5\frac{6}{7} = \frac{41}{7}$

5. Arrange in descending order: $\frac{2}{3}$, $\frac{7}{12}$, $\frac{5}{6}$

$\frac{2}{3}$, $\frac{7}{12}$, $\frac{5}{6}$ ..... L.C.M. of 3,12 and6 =12

$\frac{8}{12}$, $\frac{7}{12}$, $\frac{10}{12}$ …. Equivalent fractions with common denominator 12

Descending Order: $\frac{10}{12}$, $\frac{8}{12}$, $\frac{7}{12}$

Answer: $\frac{5}{6}$, $\frac{2}{3}$, $\frac{7}{12}$

6. Which is larger, a quarter of three quarters or two thirds of a quarter?

a quarter of three quarters $= \frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$

two thirds of a quarter $= \frac{2}{3} \times \frac{1}{4} = \frac{1}{6}$

$\frac{3}{16}$ *or* $\frac{1}{6}$ .... *L.C.M. of 16 and 6 = 48*

$\frac{9}{48}$ *or* $\frac{8}{48}$ .... *Write the fractions with common denominator.*

The larger fraction is $\frac{9}{48} = \frac{3}{16}$

Answer: A quarter of three quarters

7. Paul used $\frac{3}{8}$ of his pay for food and $\frac{2}{5}$ for other expenses and saved the rest.

(a) What fraction did he spend in all?

Paul spent $\frac{3}{8}$ of his pay on food and $\frac{2}{5}$ for other expenses.

Fraction spent $= \frac{3}{8} + \frac{2}{5}$ .... L.C.M. of 8 and 5 = 40

$= \frac{15}{40} + \frac{16}{40}$

$= \frac{15 + 16}{40}$

$= \frac{31}{40}$

Answer: $\mathbf{\frac{31}{40}}$

(b) What fraction did he save?

Fraction saved $= 1 - \frac{31}{40}$ .... from his whole pay, subtract

$= \frac{40}{40} - \frac{31}{40}$ the amount spent.

$= \frac{40 - 31}{40}$

$= \frac{9}{40}$

Answer: $\mathbf{\frac{9}{40}}$

(c) Paul's pay is K160, how much money did he save?

Savings $= \frac{9}{40}$ of 160

$= \frac{9}{40} \times \frac{160}{1}$

$= 9 \times 4$

= K36

Answer: K36

8. A painter needs $1\frac{2}{3}$ litres of paint to paint a room.
How many rooms of the same size could he paint with 15 litres?

Painter requires $1\frac{2}{3}$ litres for 1 room.
So, the number of rooms the painter can paint with 15 litres can be obtained by dividing 15 with $1\frac{2}{3}$,

$$15 \div 1\frac{2}{3} = \frac{15}{1} \div \frac{5}{3}$$
$$= \frac{\cancel{15}\,3}{1} \times \frac{3}{\cancel{5}\,1}$$
$$= 9$$

Answer: 9 rooms

9. Rolu spends half his pay and deposits $\frac{2}{3}$ of the balance in the bank.
   (a) What fraction of his pay did he deposit in the bank?
   (b) If his pay was K207, how much did he deposit?

(a) Spending $\frac{1}{2}$:

So, balance $1 - \frac{1}{2} = \frac{1}{2}$ .... *from whole pay subtract his spending*

Deposit: $\frac{2}{3}$ of the balance $= \frac{2}{3}$ of $\frac{1}{2}$

$\frac{2}{3}$ of $\frac{1}{2} = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$

So, he deposited fractional part $\frac{1}{3}$ of his pay in the bank.

Answer: $\frac{1}{3}$ of his pay

(b) Amount deposited is $\frac{1}{3}$ of his pay which was K207

So, $\frac{1}{3}$ of K207

$= \frac{1}{3} \times 207$
$= 69$

Answer: K69

NOW DO PRACTICE EXERCISE 2

Practice Exercise 2

1. Fill in the missing numbers in the given boxes.

(a) $\frac{5}{6} = \frac{\square}{18}$

Answer:___________________

(b) $\frac{2}{7} = \frac{10}{\square}$

Answer:___________________

(c) $\frac{\square}{5} = \frac{32}{40}$

Answer:___________________

2. Write in their simplest form:

(a) $\frac{4}{16}$

Answer: ___________________

(b) $\frac{9}{27}$

Answer: ___________________

(c) $\frac{12}{20}$

Answer: ___________________

3. Write as mixed numbers:

(a) $\frac{17}{5}$

Answer: ___________________

(b) $\frac{29}{7}$

Answer:___________________

(c) $\frac{25}{6}$

Answer:___________________

4. Write as improper fractions:

(a) $2\frac{3}{5}$

Answer:___________________

(b) $12\frac{1}{3}$

Answer:___________________

(c) $9\frac{3}{11}$

Answer:___________________

---

5. Arrange the following fractions in **descending** order (largest to smallest):

$$\frac{3}{5}\ ,\ \frac{5}{6}\ ,\ \frac{2}{3}\ ,\ \frac{11}{15}$$

Answer:___________________

---

6. Write in **ascending** order (smallest to largest):

$$\frac{3}{4}\ ,\ \frac{5}{6}\ ,\ \frac{2}{3}\ ,\ \frac{3}{5}$$

Answer:___________________

---

7. Find the value of:

(a) $\frac{2}{3}$ of K8.01

Answer: ___________________

(b) 0.32 times K0.50

Answer: ___________________

8. David spent $\frac{4}{9}$ of his pay and then had K120 left.

(a) What fraction of his pay does he still have?

Answer: ____________________

(b) How much was his pay?

Answer: ____________________

---

9. Which is the larger amount of money $\frac{5}{8}$ of K400 or 0.6 of K400?

Answer: ____________________

---

10. Johnnie spends $\frac{1}{2}$ his pay on his family, $\frac{1}{6}$ on his personal expenses, $\frac{1}{9}$ on church activity and he saves the rest.

(a) What fraction of his pay does he save?

Answer: ____________________

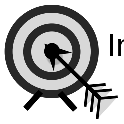
(b) How much money does he save from his pay K180?

Answer: ____________________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 3: Decimals: Revision

You have revised fractions and their properties in the first lesson.



In this lesson, you will:

- revise decimals
- revise the operations on decimals and their application to word problems.

The word "Decimal" really means "based on 10" (from Latin **decima: a tenth part**).

We sometimes say "decimal" when we mean anything to do with our numbering system, but a "Decimal Number" usually means there is a Decimal Point.

The Place-Value table

The decimal point (•) separates the whole number part from the fractional part of a number.

Whole numbers **Fractions**

<.............> <..>

10's	**1's**	**•**	$\frac{1}{10}$ **'s**	$\frac{1}{100}$ **'s**	$\frac{1}{1000}$ **'s**	$\frac{1}{10000}$ **'s**	$\frac{1}{1000000}$ **'s**	$\frac{1}{1000000}$ **'s**

Decimal point (decimal)

- 0.34 is read as "zero point three four" or "34 hundredths". $0.34 = \frac{34}{100}$
- 12.835 to the nearest tenth is 12.8 and to the nearest hundredth is 12.84.
- When writing in columns, the decimal points come one below the other.
- We add zeros to avoid confusion.

For example:
$$\begin{array}{r} 5.70 \\ +\ \underline{0.89} \\ \underline{6.59} \end{array}$$

- Multiplying by, 10s, 100s, 1000s

For example: 0.499 x 100 = 49.9

- Dividing by 10, 100, 1000

For example: 18.5 ÷ 1000 = 0. 0185

- Number of decimal places in multiplications

0.9 x 0.04 = 0.036 $\Rightarrow$ *(one + two = three places)*

Recurring or Repeating Decimal

A decimal number that has digits that repeats continuously (forever).

The part that repeats is usually shown by placing a line over the repeating pattern.

Examples:

$\frac{1}{3}$ = 0.3333... = $\mathbf{0.\bar{3}}$ ⇒ (the 3 repeats forever)

$\frac{77}{600}$ = 0.128333... = $0.128\mathbf{\bar{3}}$ ⇒ (the 3 repeats forever)

$\frac{1}{7}$ = 0.142857142857... = $0.\mathbf{\overline{142857}}$ ⇒ (the "142857" repeats forever)

- To divide by a decimal; change **Divisor** to a whole number, compensate the **Dividend** and then divide.

For example: $\frac{6.48}{1.6} = \frac{64.8}{16}$ *changing denominator to a whole number*

$= 4.05$

Now look at the following worked examples.

1. Place 0**.**06, 0.6, 0.66 and 0.006 in place value table and then write the decimals in words.

Number	**1's**	**•**	$\mathbf{\frac{1}{10}}$ **'s**	$\mathbf{\frac{1}{100}}$ **'s**	$\mathbf{\frac{1}{1000}}$ **'s**	$\mathbf{\frac{1}{10000}}$ **'s**	$\mathbf{\frac{1}{100000}}$ **'s**	$\mathbf{\frac{1}{1000000}}$ **'s**
0.08	**0**	•	**0**	**8**				
0.6	**0**	•	**8**					
0.66	**0**	•	**8**	**8**				
0.006	**0**	•	**0**	**0**	**8**			

In words:

0.08 ⇒ zero point zero eight or eight hundredth

0.8 ⇒ zero point eight or eight tenth

0.88 ⇒ zero point eight eight or eighty eight hundredth

0.008 ⇒ zero point zero zero eight or eight thousandth

2. Write each of the following as a fraction and then as a decimal.

(a) Five - tenths.

Answers: $\mathbf{\frac{5}{10}}$**, 0.5**

(b) Seventeen hundredths.

Answers: $\mathbf{\frac{17}{100}}$ **, 0.17**

3. Here are three decimals: 0.06, 0.3, 0.009, 0.15

(a) Which is the largest of the three?

Answer: 0.2

(b) Which is the smallest?

Answer: 0.009

(c) What is the sum of the smallest and the largest?

Answer: 0.209

4. Round 5.635:

(a) to the nearest whole number.

Answer: 6

(b) to the nearest hundredth.

Answer: 5.64

(c) correct to one decimal place.

Answer: 5.6

5. Yasi buys two pairs of shoes costing K11.99 and K14.00.
How much change does she receive from two twenty Kina notes?

Total cost = K11.99 + K14.00

$$\begin{array}{r} 11.99 \\ +\ 14.00 \\ \hline 25.99 \\ \hline \end{array}$$

Change due = K40.00 – K25.99

$$\begin{array}{r} 40.00 \\ -25.99 \\ \hline 14.01 \\ \hline \end{array}$$

Answer: K14.01

6. A kilogram of beef costs K5.32. What is the cost of 12.5 kg?

Cost = K5.32 x 12.5

$$\begin{array}{r} 5.32 \\ \times\ 12.50 \\ \hline 0\ 00 \\ 26\ 60 \\ 106\ 40 \\ 532\ 00 \\ \hline 66.5\ 00 \\ \hline \end{array}$$

No. of decimal places = Two places + one place = 3 places.

Answer: K66.50

7. Find approximate answers to the following in whole numbers.

(a) 17.4×11.33

Approximate: $\mathbf{20 \times 10 = 200}$

Answer: 200

(b) $40.88 \div 3.8$

Approximate: $\mathbf{40 \div 4 = 10}$

Answer: 10

8. $0.72 \div 0.08 = ?$

We first convert the given expression (in decimals) in terms of fractions.

$$0.72 \div 0.08$$
$$= \frac{72}{100} \div \frac{8}{100} = 72 \div 8 = 9$$

So, $\mathbf{0.72 \div 0.08 = 9}$ **Answer: 9**

9. Change these fractions to decimals.

(a) $\frac{4}{5}$

$$\begin{array}{r} 0.8 \\ 5\,\overline{)\,4.0} \\ -\ \underline{4.0} \\ \underline{0\ 0} \end{array}$$

So, $\mathbf{\frac{4}{5} = 0.8}$ **Answer: 0.8**

(b) $\frac{5}{8}$

$$\begin{array}{r} 0.625 \\ 8\,\overline{)\,5.000} \\ -\ \underline{4.8} \\ 20 \\ \underline{-16} \\ 40 \\ \underline{-40} \\ \underline{00} \end{array}$$

So, $\mathbf{\frac{5}{8} = 0.625}$ **Answer: 0.625**

10. What is the cost of 1000 articles at K0.69 each?

Cost = 0.69 x 1000
= 690 **Answer: K690**

11. Change to decimals and give as recurring decimals.

(a) $\frac{9}{11}$

$\mathbf{\frac{9}{11} = 0.818181}$ **Answer**: $\mathbf{0.\overline{81}}$

(b) $\frac{7}{12}$

$\mathbf{\frac{7}{12} = 0.58333}$ **Answer:** $\mathbf{0.58\overline{3}}$

More examples.

Solve the following word problems.

12. Julia cut a string 8.46 m long into 6 equal pieces. What is the length of each piece of string?

1.41 m

13. The mass of a jar of sweets is 1.4 kg. What is the total mass of 7 such jars of sweets?

9.8 kg

14. The watermelon bought by Peter is 3 times as heavy as the papaya bought by Paul. If the watermelon bought by Peter has a mass of 4.2 kg, what is the mass of the papaya?

1.4 kg

15. There is 0.625 kg of powdered milk in each tin. If a carton contains 12 tins, find the total mass of powdered milk in the carton.

7.5 kg

16. Marcus bought 8.6 kg of sugar. He poured the sugar equally into 5 bottles. There was 0.35 kg of sugar left over. What was the mass of sugar in 1 bottle?

1.65kg

NOW DO PRACTICE EXERCISE 3

Practice Exercise 3

1. Write in decimal notation:

   (a) $\frac{2}{10}$

   Answer: ____________

   (b) $\frac{2}{10} + \frac{3}{100}$

   Answer: ____________

   (c) $4 + \frac{4}{100}$

   Answer: ____________

2. Round the following:

   (a) 14.804 to the nearest whole number.

   Answer: ____________

   (b) 14.804 correct to two decimal places.

   Answer: ____________

   (c) 0.934 correct to the first decimal place.

   Answer: ____________

   (d) 1.505 to the nearest hundredth.

   Answer: ____________

3. Write in columns and then work out:

   (a) 4.06 + 203.7 + 0.0107 + 5

   Answer: _____________

   (b) From the sum of 7.924 and 3.4 take away 1.089

   Answer: _____________

4. Take away K4.99 from K10.00

   Answer: _____________

5. Calculate:

(a) 17.002×100

Answer: ____________

(b) 0.0706×1000

Answer: ____________

(c) 0.004×10

Answer: ____________

(d) $4 \div 100$

Answer: ____________

(e) $0.04 \div 1000$

Answer: ____________

---

6. Find approximate answers to the following:

(a) 18.33×4.7

Answer: ____________

(b) $18.33 \div 4.7$

Answer: ____________

---

7. Work out:

(a) 18.33×4.7

Answer: ____________________

(b) $18.33 \div 4.7$

Answer: ____________________

(c) $35 \div 0.08$

Answer: ____________________

(d) $0.5 \div 0.8$

Answer: ____________________

8. Tony wants to divide K5700 amongst his 6 children equally.
How much does he give to each child?

Answer: ________________

---

9. A factory has 30 technical workers. Each is paid K212.80 per fortnight, what is the total wages of the 30 workers?

Answer: ________________

---

10. On a certain day, 1000 Vita drinks were sold in a store at K0.39 each.
How much were total sales?

Answer: ___________________

---

11. Write down the answers to the following:

(a) The cost of 10 000 kg of bread at K0.67 per kg

Answer: ___________________

(b) The total weight of 4000 chickens each weighing 2.5 kg

Answer: ___________

(c) 0.099 x .... = 990

Answer: ___________

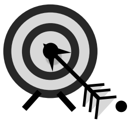
(d) $\frac{1}{8}$ of 1 kg = ?

Answer: ___________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 4: Percentages: Revision

You have revised decimals numbers and their properties in the previous lesson.



In this lesson, you will;

- Revise percentages
- Find the percentage of a quantity
- Find one number as a percentage of another
- Calculate the number when the percentage is known.

Percentages

The word percentage comes from the word percent. Percentage is the result obtained by multiplying a quantity by a percent. Say 10 **percent** of 50 bananas is 5 bananas: 5 bananas is the **percentage**.

But in practice people use either word the same way.

Percentage of a Quantity

Percentage of a quantity means we find a certain percent of a specified quantity. A quantity can be a whole number, distance, mass, time and etc.

Example 1 What is 30 % of 20 kg of rice?

Solution:

$$\begin{aligned}\frac{30}{100}\text{ x } 20\text{ kg} &= \frac{3}{10}\text{ x } 20\text{ kg}\\ &= \frac{60}{10}\text{ kg}\\ &= 6\text{ kg}\end{aligned}$$

30% of K20 kg is 6 kg.

A Number as Percentage of Another

A number can be expressed as a percent of another second number, often the second is larger than the first number. To find a number **n** being expressed as a percent of another number **Q**, we compute the quotient and multiply by 100%.

$$\boxed{n\% = \frac{n}{Q}\text{ x } 100\%}$$

Example 2 What percent is 20 out of 50?

Solution:

$$\frac{20}{50} \text{ x } 100\% = \frac{2000}{50}\%$$
$$= 40\%$$

20 out of 50 is 40%.

Calculate the Number when Percentage is Known

A number can be calculated as a percent of another second number, often the when the percent is known. To find a number **n** being expressed as a percent of another number **Q**, we compute the quotient and multiply by 100%.

$$\boxed{n = x\ \%\text{ of } Q}$$

Example 3 What number is 15% of 200 ?

Solution:

$$n = \frac{15}{100} \text{ x } 200$$
$$= \frac{3}{20} \text{x} 200$$
$$= \frac{600}{20}$$
$$= 30$$

30 is 15% of 200.

Example 4 Find 20% of 8 L.

Solution:

$$n = \frac{20}{100} \text{ x } 8$$
$$= \frac{16}{10}$$
$$= 1.6 \text{ L}$$

20% of 8 L is 1.6 L.

When the percentage is known, you can express the percentage as a vulgar fraction or a decimal number, then multiply by the quantity given. You have to decide when to express as a vulgar fraction or a decimal number.

Say if the quantity given is a power of 10 (100, 1000, 10 000 etc) convert the percentage to a decimal number then multiply.

If the quantity is a multiple of the denominator of the simplified fraction (percentage changed to fraction), then use vulgar fraction.

Example 5 Find 26% of K150.

Solution:

Use fraction.

$$n = \frac{26}{100} \text{ x } 150$$
$$= \frac{13}{50} \text{x}150$$
$$= 13x3$$
$$= 39$$

26% of K150 is K39.

Example 7 Find 7.5% of 100 000 people.

Solution:

Use decimal

$$n = \frac{7.5}{100} \text{ x } 100\,000$$
$$= 0.075 \text{ x } 100\,000$$
$$= 7\,500$$

7.5% of 100 000 people is 7 500 people.

NOW DO PRACTICE EXERCISE 4

Practice Exercise 4

1. Find decimal equivalent of 12 %.

Answer: __________

2. What is the fraction equivalent of 35%?

Answer: __________

3. Write 60kg as a percentage of a tonne.

Answer: __________

4. Work out:

$$\frac{42}{100} \text{ x } 300 \cdots \text{km}$$

Answer: ___________

5. Add the following percentages:

$$8\% + 14\tfrac{1}{2}\% + 6\tfrac{1}{4}\%$$

Answer: ___________

6. Express 0.0125 as a percent.

Answer: ___________

7. Eddie has 20% equity of K3 000 so he can apply for a bank loan to purchase a used car. What is the price of the used car?

Answer: ___________

---

8. Martin needs to raise K10 000 to travel to the Philippines. What percent of the amount has he raised if he now has K4 600 in his bank account?

Answer: ___________

---

9. Solve for A in 0.32A = 160.

Answer: ___________

---

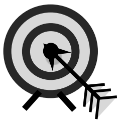
10. Nine girls is 12.5% of the girls population in grade 10. How many girls are doing grade 10?

Answer: ___________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 5: Decimals, Fractions and Percentages

You have revised calculating percentage and finding percentage of quantity in the previous lesson.



In this lesson, you will;

- Convert decimal and fraction to percentage and vice versa.

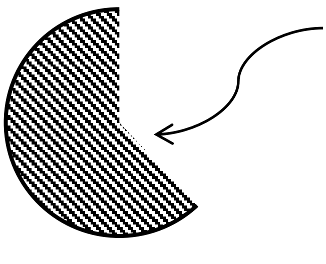
From grade 9, you should by now be able to convert the percentages to its equivalent form in either decimals or fractions. You should also be able to convert from a fraction to a percentage and convert from a decimal to a percentage.

Decimal numbers are as such as 0.34, 1.208 and 0.05. Decimal parts after the decimal points are read individually. Say 14.501 is read as " fourteen point, five-zero-one". We are faced with decimal numbers when we measure quantities such as a mass of an object, when we approximate values of certain roots and when we convert certain vulgar fractions (common) to decimals.

Fractions are numbers as such as $\frac{1}{5}$, $\frac{7}{4}$ and $3\frac{1}{6}$.Where the first is a proper fraction, followed by an improper fraction and a mixed number.

Percentages are numbers as such as $6\frac{1}{2}$%, 12.25% and 30%.

The fraction, decimal and percentage are all expressing part of a whole quantity, therefore all three can be classified as fractions. A distinction can be made on the fraction by naming it as a vulgar fraction.



This shaded sector represents :

- 0.625 of a circle
- $\frac{5}{8}$ of a circle, or
- 62.5% of a circle

Even though percentage is expressed in whole numbers, it indicates number of parts out of 100 parts, therefore they all express part of a whole. So they are all fractions.

Your ability to transform the forms will enable you to compute with confidence without the help of a calculator.

Below is a table showing equivalence of some of the percentages in decimals and fractions.

Table of Equivalence: Percentage, Decimal and Fraction

Percent	Decimal	Fraction
1%	0.01	$\frac{1}{100}$
2%	0.02	$\frac{1}{50}$
3%	0.03	$\frac{3}{100}$
4%	0.04	$\frac{1}{25}$
5%	0.05	$\frac{1}{20}$
6%	0.06	$\frac{3}{50}$
7%	0.07	$\frac{7}{100}$
8%	0.08	$\frac{2}{25}$
9%	0.09	$\frac{9}{100}$
10%	0.1	$\frac{1}{10}$
12 ½ %	0.125	$\frac{1}{8}$
33 $^1/_3$ %	0.333..	$\frac{1}{3}$
70%	0.7	$\frac{7}{10}$
99%	0.99	$\frac{99}{100}$
125%	1.25	$\frac{5}{4}$
200%	2	$\frac{200}{100}$

Converting Decimals to Percentage

To convert a decimal number to a percent, multiply by 100%.

Example 1 Write 0.135 as a percentage.

Solution:

0.135 x 100% = 13.5%
The decimal number 0.135 is equal to 13.5 %

Example 2 Convert 1.08 to a percentage.

Solution:

1.08 x 100% = 108%
The decimal number 1.08 is equal to 108%

Converting Fractions to Percentage

Care must be taken when converting from fraction to percentage. It would be better to write the remainder as a fraction than a decimal. The fraction may not divide exactly, thus give a continuous or repeating decimal number.

Below is table of equivalents of the fractions to guide you in your calculations.

Fraction	Decimal	Percentage
1	1.0	100%
$\frac{1}{2}$	0.5	50%
$\frac{1}{3}$	0.33	$33\frac{1}{3}$ %
$\frac{1}{4}$	0.25	25%
$\frac{1}{5}$	0.2	20%
$\frac{1}{6}$	0.17	$16\frac{2}{3}$ %
$\frac{1}{7}$	0.1	$14\frac{2}{7}$ %
$\frac{1}{8}$	0.125	$12\frac{1}{2}$ %
$\frac{1}{9}$	0.11	$11\frac{1}{9}$ %
$\frac{1}{10}$	0.1	10%

To convert a fraction to a percent, multiply by 100%.

Example 1 Write $\frac{3}{8}$ as a percentage.

Solution:

$$\frac{3}{8} \text{ x } 100\% = \frac{300}{8}\%$$
$$= 37.5\%$$

The fraction $\frac{3}{8}$ is equal to 37.5 %

Example 2 Write $1\frac{3}{16}$ as a percentage.

Solution:

$$1\frac{3}{16} \text{ x } 100\% = \frac{16}{9} \text{ x } 100\%$$
$$= 1.1875 \text{ x } 100\%$$
$$= 118.75\%$$

The fraction $1\frac{3}{16}$ is equal to 118.75 %

Example 3 Write $\frac{13}{8}$ as a percentage.

Solution:

$$\frac{13}{8} \text{ x } 100\% = \frac{130}{8}\%$$
$$= 162.5\%$$

The fraction $\frac{13}{8}$ is equal to 162.5 %

Example 4 When Flannan received 7.5% pay increment, he received a total of K1290 in that fortnight. What was his previous pay?

Solution:

$$\frac{100}{100}S + \frac{7.5}{100}S = 1290$$

$$\frac{107.5}{100}S = 1290$$

$$107.5S = 129000$$

$$S = \frac{129000}{107.5}$$

$$= 1200$$

Converting Percentages to Decimals and Fractions

Percentage	Decimal	Fractions
10%	0.1	$\frac{1}{10}$
20%	0.2	$\frac{1}{5}$
30%	0.3	$\frac{3}{10}$
40%	0.4	$\frac{2}{5}$
50%	0.5	$\frac{1}{2}$
60%	0.6	$\frac{3}{5}$
70%	0.7	$\frac{7}{10}$
80%	0.8	$\frac{4}{5}$
90%	0.9	$\frac{9}{10}$
100%	1.0	$\frac{10}{10}$
110%	1.1	$\frac{11}{10}$

To convert a percent to a fraction, write the percent as a fraction of 100 and simplify.

Example 1 Write 84% as a fraction.

Solution:

84% = 84/100 [divide by HCF; HCF is 4]
= 21/25

84% is equal to 21/25.

To convert a percent to a decimal, write the percent as a fraction of 100 and divide.

Example 2 Write 39.4% as a decimal.

Solution:

39.4% = 39.4/100
= 0.394

39.4% is equal to 0.394.

NOW DO PRACTICE EXERCISE 5

Practice Exercise 5

1. Express 0.45 as a percent.

Answer: __________

2. Change $\frac{5}{12}$ to a percent.

Answer: __________

3. Convert 120% to a

(a) fraction

Answer:__________

(b) decimal

Answer: __________

4. Work out:

(a) 4.75 ÷ 100 =

Answer:___________

(b) 0.063 x 100 =

Answer: ___________

(c) 100 – 17.2 =

Answer: ___________

5. (a) Add the following percentages:

17.5% + 12.25% + 8.75%

Answer: ___________

(b) Round to the nearest whole percent:

6.125%

Answer: ___________

6. In a fund raising event in a school, Villages A,B and C raised 3/5, 0.25 and 20% of the total respectively. Did they reach the total amount needed?

Answer: ___________

---

7. When 7% was added to Rex's pay, he received K406.60. What was his previous pay?

Answer: ___________

---

8. Ron, Sam and Ted contributed money to start a business. Sam paid $^{1}/_{3}$ and Ted paid $^{8}/_{15}$ of the total. What percent of the total amount of money did Ron pay?

Answer: ___________

---

9. When 15% of students was added to a School population, the had a total population of 736. What was his previous population?

Answer: ___________

---

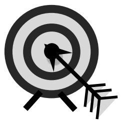
10. When 10% was taken off Koma's wet bean weight of cocoa, he was given a weight of 278kg. What was his actual weght for cocoa beans?

Answer: ___________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 6: Mixed Word Problems

You have revised converting percentage to decimals and fractions and vice versa, in the previous lesson.



In this lesson, you will;

- Use the operations and properties of whole numbers, decimals, fractions and percentages to solve word problems and commercial arithmetic.

From the mathematical knowledge gained in grade 9, you should by now be able to solve the word problems. The problems are related to the previous 5 lessons you have studied.

Some phrases and their translations:

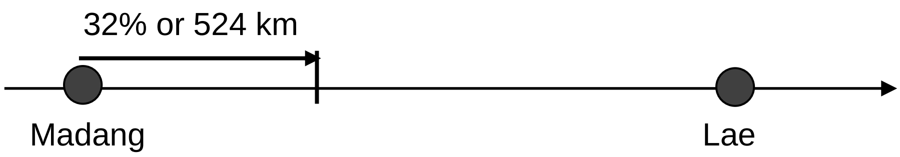
Phrase	Meaning
Twice a number	2 times a number
Trice a number	3 times a number
Square of a number	n x n or n^2
Consecutive numbers	4,5,6 or 29, 30, 31 etc
Consecutive odd numbers	7, 9, 11 or 33, 35, 37
Four less than a number	Subtract 4 from a number (n – 4)
Inverse of a number	1/x or 1/n etc
Three more than a number	3 + n

Your ability to translate the words into algorithmic or algebraic statement will enable you to solve the mixed word problems.

Example 1

Frank travelled 524 km towards Lae from Madang. If the distance covered is 32% of the total distance, how far is Lae from Madang?

Solution:



32%D = 524
0.32D = 524
D = 524/0.32
= 52400/32
= 1 637.5 km

Example 2
Fund raising activities were held to raise money to build a new classroom. Three dance nights raised K16 000, which is one-tenth of the required amount. Villagers' contribution was 60% of the total needed. The school sales of printed T-shirts raised 11/20 of the total needed. What is the cost of the classroom, and have they raised enough money?

Solution:

0.1T = 16 000
T = 16 000/0.1
= **K160 000** Total Cost of Classroom.

60% + 11/20 = 60% + 55%
= 115% Funds raised from T-Shirt sale and by Villagers

115/100 x 160 000 = **K184 000**.

They raised K40 000 more than needed.

Example 3
The sum of the square of a number and one more than the number equals, 4 less than, the square of one more than the number.

Solution:

Let the number be n Added is (n + 1) Equals $(n + 1)^2 - 4$

$$n^2 + (n + 1) = (n + 1)^2 - 4$$
$$n^2 + n + 1 = n^2 + 2n + 1 - 4$$
$$n^2 - n^2 + n - 2n + 1 - 1 + 4 = 0$$
$$- \quad n + 4 = 0$$
$$- n = - 4$$
$$n = 4$$

The number is 4.

Example 4
Three consecutive even numbers add up to 60. What is the largest number of the three consecutive numbers?

Solution:

Let the consecutive even numbers be n, n + 2 and n + 4 . where $n < n + 2 < n + 4$
Now $n + (n + 2) + (n + 4) = 60$
$$3n + 6 = 60$$
$$3n = 54$$
$$n = 18$$
The largest is n + 4 = 18 + 4 = 22.

Therefore 22 is the largest of the consecutive odd numbers.

Example 5

The sum of the inverse of a number and itself is $2\frac{1}{6}$. Find the number.

Solution:

Let the number be n. Thus $\left(\frac{1}{n}\right) + n = 2\frac{1}{6}$

$$\frac{1}{n} + n = \frac{13}{6}$$

$$6 + 6n^2 = 13n$$

$$6n^2 - 13n + 6 = 0$$

$$(3n - 2)(2n - 3) = 0$$

$$n = \frac{2}{3} \text{ or } \frac{3}{2}$$

NOW DO PRACTICE EXERCISE 6

Practice Exercise 6

1. Three consecutive numbers add to 30. Find the three numbers.

Answer: _______________

2. The sum of, half of a number and double the number is 20. Find the number.

Answer: _______________

3. Take 20% of a number from the number, leaves 16. Find the number.

Answer: ________________

4. The sum of a third of a number and 0.6 of the same number equals two less than the number.

Answer: ________________

1. What is the cube of the lowest common multiple of 3 and 6?

Answer: __________________

6. Men **A**, **B** and **C** divide an amount of money in that order as 30%, 40% and 20% each. If the man **B** gets K200, what is the amount man **A** received?

Answer: __________________

7. In a diagnostic test, 420 students passed. If those who passed were 60% of grade 12 population, what is the total grade 12 population?

Answer: __________________

---

8. Three village men A, B and C decided to use fraction, decimal and percentage in that order to be paid equally on the sum of K1 320. Man A wanted a half, man B claimed 0.5 and man C asked for 50%.

Will the amount be enough to entertain each man's demand?

Answer: __________________

---

9. Billy's estimate for a rectangular area of 26 m x 36 m was 900 m^2.

Calculate the correct estimate and find the percentage error.

Answer: __________________

---

10. Calculate the square of the lowest (least) common multiple of 4 and 25.

Answer: __________________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 6.

Summary

- **Real Number** is all the rational numbers and subsets of the rational numbers in a complex number field.
- **Number** is a figure used in counting and calculation of quantities.
- **Whole numbers** are numbers from 1 to infinity.
- **Counting numbers** are numbers from 1 to infinity.
- **Natural Numbers** are numbers {0, 1, 2, 3, ……}
- **Consecutive Numbers** are such numbers as 4, 5, 6 and 100, 101, 102, 103 and so on.
- **Consecutive Even Numbers** are such numbers as 10, 12, 14 and 34, 36, 38, 40 and so on.
- **Consecutive Odd Numbers** are such numbers as 3, 5, 7 and 63, 65, 67, 69 and so on.
- **Cardinal numbers** A1, shelter 6, R3 and so on**.**
- **Ordinal numbers** are 1^{st} , 2^{nd} , 3^{rd} , and so on. They define the order of some form of sequence.
- **Numeral** is a symbol used to represent a number.

- **Multiple** is a number that is divisible by another number equal to or less than itself.
- **Factor** is all the numbers that can divide exactly into another larger number.
- **LCM** is the lowest common multiple or least common multiple of two or more numbers. Such as LCM of 12 and 18 is 36.
- **HCF** is highest common factor, also known as **GCF** or greatest common factor. It is the largest possible number that can divide into two or more numbers. Such as HCF of 12 and 18 is 6.

- **Fractions** include fractions (vulgar), decimals and percentage. They all express part of a whole quantity.
- **Fraction** is a part of a whole number such as, vulgar (common), decimal and percentage.
- **Decimal** is a number relating to 10 or parts of tens, such as decimal fraction in 0.25.
- **Percentages** are numbers expressed as part of a hundred. Per cent means part of a hundred. If we write 20%, the figure 20 is a percent, and the amount worked out from a quantity is the percentage. Percentage is now used interchangeably to mean one or the other.

Answers to Practice Exercises 1- 6

Practice Exercise 1

1. (a) 6
   (b) 15
2. (a) 24
   (b) 56
3. (a) 154
   (b) 25 200
   (c) 72 977
4. (a) K17 654 587
   (b) K18 000 000
5. K20.28
6. 2 063 725
7. Twenty seven thousand and forty six.
8. 2 x 3 x 107
9. 120
10. 4

Practice Exercise 2

1. (a) $\frac{5}{6} = \frac{15}{18}$
   (b) $\frac{2}{7} = \frac{10}{35}$
   (c) $\frac{4}{5} = \frac{32}{40}$
2. (a) $\frac{1}{4}$
   (b) $\frac{1}{3}$
   (c) $\frac{3}{5}$
3. (a) $3\frac{2}{5}$
   (b) $4\frac{1}{7}$
   (c) $4\frac{1}{6}$
4. (a) 13/5
   (b) 37/3
   (c) 102/11
5. $\frac{5}{6}, \frac{11}{15}, \frac{2}{3}, \frac{3}{5}$
6. $\frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$
7. (a) K5.34
   (b) K0.16
8. (a) 5/9
   (b) K216
9. $\frac{5}{8}$ of K400 is larger.
10. (a) 5/8
    (b) K50

Practice Exercise 3

1. (a) 0.2
   (b) 0.23
   (c) 4.04

2. (a) 15
   (b) 14.80
   (c) 0.9
   (d) 1.51

3. (a) 212.0707
   (b) 10.235

4. K5.01

5. (a) 1 700.2
   (b) 706
   (c) 0.04
   (d) 0.04
   (e) 0.00 004

6. (a) 100
   (b) 4

7. (a) 86.151
   (b) 3.9
   (c) 437.5
   (d) 0.625

8. K950

9. K6 384

10. K390

11. (a) K6 700
    (b) K670
    (c) K67
    (d) 84 toea

Practice Exercise 4

1. 0.12
2. 7/20
3. 6%
4. 126 km
5. 28 ¾ %
6. 1.25%
7. K12 000
8. 46%
9. A = 50
10. 72 girls

Practice Exercise 5

1. 45%
2. 41.7%
3. (a) 6/5
   (b) 1.2
4. (a) 0.0475
   (b) 6.3
   (c) 82.8
5. (a) 38.5%
   (b) 6%
6. Total raised 105%; 5% more than needed
7. K380
8. 13 $^{1}/_{3}$%
9. 640 students
10. 278 kg

Practice Exercise 6

1. 9, 10, 11

2. 8

3. 20

4. 30

5. LCM = 6; $6^3 = 216$

6. K150

7. 700 students

8. A: ½ x 13 200 = K660
   B: 0.5 x 13 200 = K660
   C: 50% x 13 200 = K660

   Not possible to meet demand as total is K1980; K660 more than the total amount.

9. 30 m x 40 m = 1 200 m^2.
   Percentage error = (1200 - 900)/1200 x 100%
   = 25%

10. LCM = 100, $100^2 = 10\,000$

TOPIC 2

ESTIMATION

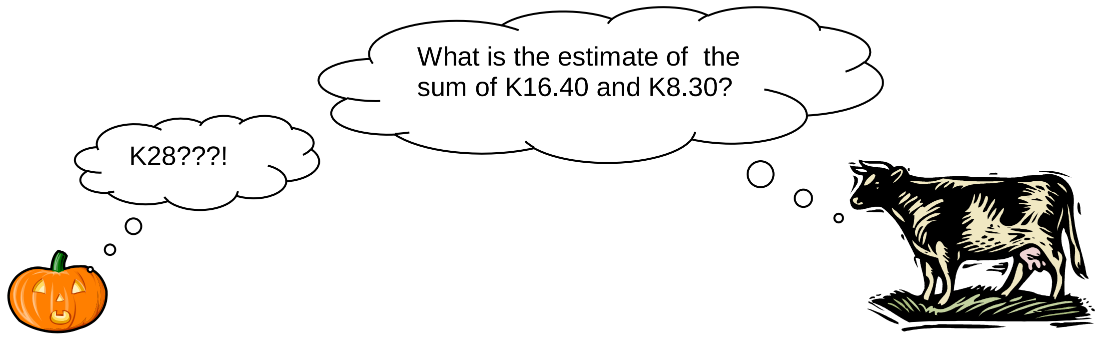
TOPIC 2: ESTIMATION

Introduction

You have done problems involving whole numbers, fractions and decimals and percentages. Estimation involves rounding off and significant figures.

When we round off a number, we state the number correct to the nearest hundreds, tens, ones, tenths and etc, based on the nature of the problem.

Say, when measuring heights of grade 4 to grade 6 pupils, we would write measurements correct to one decimal place such as 143.6 cm. If the exact measure was 143.56 cm or 143.63 cm, we are more concerned with the nearest millimeter so we still write as 143.6 cm. That is the best approximate we can take.



To estimate, involves rounding off of more than one number and includes basic operation.

For example, to estimate the sum and difference of 283 and 48, we would work out the results of 300 + 50 and 300 – 50.

To estimate we round all numbers involved to one non-zero digit number, then we find the sum, difference, product or quotient.

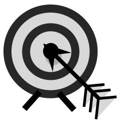
Whilst it is easy to round off to specified number of decimal place value and estimate, we need to take great care when dealing with Significant Figures. There are rules we need to observe.

Given the numbers 30.8 and 0.0308 , both have 3-significant figures but different number of decimal places.

In this topic, you will have a chance to revisit and practice rounding off and estimating, as well as applying basic operatoins with significant figures .

Lesson 7: Rounding Off

You have learned revisited the properties of whole numbers, decimals, fractions and percentages, and worked out further problems by using their properties.



In this lesson, you will
- Explain “Rounding off”
- Round off decimals to a certain number using decimal place.

When we round up or round down, based on whether the digit is greater or less than 5, we are **rounding- off** numbers.

We consider the number in the next place value, if it is equal to or greater than 5 ($n \geq 5$), we add one to the place value that is, rounding up. If the number in the next place value is less than 5 ($n < 5$), we add zero to the place value, that is rounding down.

Example 1

Round-off 247 to the nearest tens.

Solution:

247 is the number we have to round- off; ones is the next place value.
= 250 since $7 > 5$, we add 1 to tens place value.

Example 2

Round-off 24.172 to the nearest tenths.

Solution:

24.172 is the number we have to round- off; hundredths is the next place value.
= 24.17 since $2 < 5$, we add 0 to tenths place value.

Example 3

Round-off 18.3462 correct to one decimal place.

Solution:

18.3462 is the number we have to round- off; 4 is in the next place value.
= 18.3 since $4 < 5$, we add 0 to 3 which is in one decimal place value.

Note: Even though 6 is greater than 5 so 4 in second decimal place inreases by 1, thus it becomes 5 = 5, hence it yields 18.4. No! We always consider the immediate place value after, to which we are to round to. So we restrict ourselves to the place value immediately after the place value to be rounded to. In this case second decimal place, where 4 is in.

Example 4

Add 42.273 and 36.902 write your final answer to the nearest whole number.

Solution:

$$42.273 + 36.902 = 79.175$$
$$= 79 \text{ [since 1 in tenths place value is less than 5]}$$

Example 5

Round-off 42.273 and 36.902 to the nearest whole number and find their difference.

Solution:

$$42.273 - 36.902 = 42 - 37 \quad \text{[since } 2 < 5\text{, and } 9 > 5\text{]}$$
$$= 5$$

Rounding off is often mixed up with approximation and estimation.

In **rounding off**, we restrict ourselves to the place value immediately after the place value we are to round to. Say if we are to round 41.072693 correct to 2 decimal places, we consider third place value digit, which is 2. If we are to round to ones, we consider first decimal place, which is 0, and so on.

Suppose 41.072693 is a result or measurement, we can give the **approximate** correct to certain decimal place values, depending on the nature of the problem. Say if it is a measure in litres, we can approximate to 41.073 L (41 073 mL), which is it is written correct to the nearest millilitre. If it is metres, we approximate our answer to 2 decimal places as 41.07 m; that is, it is written correct to the nearest centimeter (4107 cm).

Estimation occurs when there is a statement of basic operation (+, - , x, ÷), where, we round every number involved to one significant figure before we perform the basic operation.

NOW DO PRACTICE EXERCISE 7

Practice Exercise 7

1. Round – off 294.72 to the nearest tens.

Answer: __________

2. Round – off 0.29472 to the nearest hundredths.

Answer: __________

3. Round – off 0.29472 to the nearest

(a) tenths

Answer: __________

(b) 2d.p.

Answer: __________

4. Work out:

(a) 73 ÷ 6 correct to 2 d.p.

Answer: ________________

(b) 6.32 x 0.44 correct to 1 d.p.

Answer: __________________

(c) 92 400 – 17 263 correct to the nearest thousands.

Answer: __________________

5. Round-off to the nearest tens of thousands and add:

K56 210 437 + K367 471 + K38 967

Answer: ___________________

---

6. Round to two decimal places (dp):

(a) 0.42396

Answer: ___________________

(b) 7.823

Answer: ___________________

(c) 12.805

Answer: ___________________

(d) 560.0936

Answer: ___________________

(e) 0.089348

Answer: ___________________

---

7. Round –off 28 442 mL to the nearest litre.

Answer: ___________________

8. Write (a) $\sqrt{80}$ correct to 2d.p.

Answer: ___________________

(b) $\sqrt{20}$ correct to 4 d.p.

Answer: ___________________

---

9. If π = 3.141 592 653 897 932 …, write its approximate correct to 3 decimal place (d.p).

Answer: ___________________

---

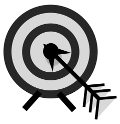
10. Write log 20 (= 1.301 299 566…) correct to 1 decimal place.

Answer: ___________________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 8: Significant Figures

You have learned about rounding - off whole numbers and decimals to a given place value.



In this lesson, you will
- Explain significant figures with confidence
- Distinguish between rounding off and significant figures
- determines the prime factors of a given number
- round off using significant figures.

Say the distance 12.62984 m can be better written correct to two decimal places (dp) or four significant figures, where it is expressed correct to the nearest centimetre.

Suppose we are given a distance of 312.3638 km, it would more appropriate to state the figure correct six significant figures, thus the answer is given correct to the nearest metre. The answer we arrive at is the **approximation** of the exact answer.

Approximate answer is the answer obtained simply by **rounding-off** or writing correct to specified **significant figures**; approximating does not involve any operation.

The number of figures we use to express a measure of a quantity (or coefficient) defines the significance and precision of the measure.

Significant Figures indicate precision or significance of a number or digit in a quantity.

When we say **non-zero digit**, we would like to mean the set of numbers from **1** to **9**. And when we state **sf**, we mean significant figure.

To state a given whole number or a decimal number to a certain number of significant figures, we must remember the rules:

1. count from left to the right.
2. all non-zero digits are significant figures,
3. zeros between non-zero digits are included,
4. zeros before and after the non-zero digits are not included.

Example 1

How many significant figures are in:
(a) 2803
(b) 2.803
(c) 2830
(d) 0.0284

Solution:

(a) 2803 (4sf)	zero is in between non-zero digits so is included.
(b) 2.803 (4sf)	zero is in between non-zero digits so is included.
(c) 2830 (3sf)	zero is after non-zero digits so is NOT included.
(d) 0. 0284 (3sf)	zero is before non-zero digits so is NOT included.

Like in rounding decimals,to write a number to a specified significant figures, we weigh if the significant digit immediate after is equal or greater than 5; or is it less than 5.

If the significant figure is equal to or greater than ($n \geq 5$) we increase the significant figure by 1, and if less than ($n < 5$) the significant figure remains the same.

Example 2
Write the following numbers correct to 3sf:
(a) 26803
(b) 2.08233
(c) 28300
(d) 0.02836

Solution:

(a) 26803	= 26800 (3sf)	[other place values become 0]
(b) 2.08233	= 2.08 (3sf)	[zeros after the sf not necessary to be denoted]
(c) 28300	= 28300 (3sf)	[no change]
(d) 0.02836	= 0.0283 (3sf)	[0 before the first non-zero digit is not included]

Example 3
Write the following correct to appropriate significant figures for:
(a) 2.6803 m
(b) 3.80248 L
(c) 0.28306 kg
(d) 8.3262 cm^2

Solution:

(a) 2.6803 m	= 2.68 m	correct to nearest cm
(b) 3.80248 L	= 3.802 L	correct to nearest mL
(c) 0.28306 kg	= 0.283 kg	correct to nearest g
(d) 8.3262 cm^2	= 8.33 cm^2	correct to nearest mm^2

Example 4
Write the following correct to significant figures indicated:
(a) $\sqrt{10}$ (2sf)
(b) $\sqrt{80}$ cm^2 (4sf)

Solution:

(a) $\sqrt{10} = 3.1622 \approx 3.2$ (2sf)

(b) $\sqrt{80} = 8.94427 \approx 8.94$ cm^2 (3sf)

Note: In example (b) it is more convenient to round to 3s.f. since it is then stated correct to the nearest square millimeter (mm^2).

Significant figures are often stated in **scientific notation** (SN) or **standard form** (SF) as given in the example 3 below. The SN or SF is an ordinary number expressed in the form $\mathbf{A \times 10^N}$, where **A** is the **coefficient** and **N** is the **index**. The coefficient **A** always has a decimal point after the first non-zero digit.

Example 5
Write the following numbers correct to number of significant figures indicated:

(a) 2.6803×10^3 to 2s.f.
(b) 4.0833×10^3 to 3s.f.
(c) 6.325×10^{-3} to 1s.f
(d) 2.836×10^{-1} to 2s.f.

Solution:

(a) $2.6803 \times 10^3 = 2.7 \times 10^3$
(b) $4.0833 \times 10^3 = 4.08 \times 10^3$
(c) $6.325 \times 10^{-3} = 6 \times 10^{-3}$
(d) $2.836 \times 10^{-1} = 2.8 \times 10^{-1}$

Note: When figures are in standard form or scientific notation, we only round the coefficient and retain the powers of 10.

Example 6
How many significant digits are there in 2.0916×10^2 ?

Solution:

A = 2.0916, 6s.f. [zero between non-zero digits is inclusive]

NOW DO PRACTICE EXERCISE 8

Practice Exercise 8

---

1. Write correct to 3sf

   2453

Answer: __________

---

2. How many significant figures has:

(a) 271.189

Answer: __________

(b) 2.51168

Answer: __________

(c) 2911900

Answer: __________

(d) 0.60396

Answer: __________

(e) 0.00171116

Answer: __________

---

3. Write 202.1966 correct to:

(a) 6sf

Answer: __________

(b) 5sf

Answer: __________

(b) 4sf

Answer: __________

(c) 3sf

Answer: __________

4. Write the following to appropriate significant figures:

(a) 12.63 cm

Answer: ________________

(b) 20.1942 m

Answer: __________________

(c) 459.253 km

Answer: __________________

(d) 24.6548 kg

Answer: __________________

(d) 8.0583 L

Answer: __________________

---

5. (a) Express 72.394 correct to 2d.p.

Answer: __________________

(b) Express 72.394 correct to 2 sf.

Answer: __________________

---

6. List the numbers that are not expressed in 3sf in the given list:

6 020
5290
405
126
780
5.204
9.03
0. 327
0. 010 8
0.00351

Answer: __________________

7. How many significant digits are in 3.9942×10^{-3} ?

Answer: ____________________

---

8. Write 9.01342×10^{0} correct to 3 s.f.

Answer: __________________

---

9. Write 1.942×10^{2} correct to 2 s.f.

Answer: __________________

---

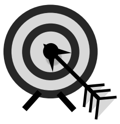
10. Write 8.64×10^{2} cm^{2} correct to 2 s.f.

Answer: __________________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 9: Addition and Subtraction with Significant Figures

You have learned about the properties of whole numbers and work out further problems by using these properties.



In this lesson, you will

- use significant figures in addition and subtraction.

When finding the sums and differences of significant figure (**sf**), we are actually estimating the sum or difference.

The rule for rounding numbers applies. That is we consider the second significant digit and decide if we can round-off (up or down).the first significant digit

So 248 becomes 200; 53 becomes 50; 7349 becomes 7 000. This is due to the significance of the digits 4, 2 and 3 respectively.

Example 1
What is the sum of 346 and 41?

Solution:

$$\begin{aligned} 346 + 41 &= 300 + 40 \\ &= 340 \end{aligned}$$

Example 2
2693 – 1074 = ______

Solution:

$$\begin{aligned} 4\,693 - 1074 &= 5\,000 - 1000 \\ &= 4\,000 \end{aligned}$$

Example 3
9201 + 338 + 520 + 206 = _______

Solution:

$$\begin{aligned} 9201 + 338 + 520 + 206 &= 9\,000 + 300 + 500 + 200 \\ &= 10\,000 \end{aligned}$$

But if your are told to add and or subtract, then write correct to specified significant figures, you are **approximating** the value.

Example 4
Find the sum and write your final answer correct to 4 significant figures:
3.204 + 12.11 + 8.218

Solution:

3.204 + 12.11 + 8.218 = 23.532
= 23.53 (4sf)

(The estimate would be 3 + 12 + 8 = 23)

Example 5

Find the difference and write your final answer correct to 2 significant figures:

26.088 – 7.913 = 18.175
= 18 (2sf)

(The estimate would be 30 - 8 = 22)

When there is no mention of approximation, round or estimate, you perform the normal operation. That is you multiply, divide, add and subtract (including powers) and whatever answer you obtain under that condition of operation is the answer.

Example 6

$5^2 - 8$

Solution:

$5^2 - 8 = 25 - 8$
$= 17$ (2 s.f)

Example 7

28 + 126

Solution

28 + 126 = 154 (3 s.f)

Answers to examples 6 and 7 are neither estimates nor approximates; they are exact answers.

NOW DO PRACTICE EXERCISE 9

Practice Exercise 9

1. Estimate 43.8 + 291.23 =

Answer: __________

2. Estimate 8437 – 834 =

Answer: __________

3. Write the sums of the quantities correct to significant figures indicated:

(a) 9.274 m + 3.38 m + 11.041 m =

Answer: __________ (4 sf)

(b) 0.09274 L + 0.0338 L + 1.1041 L =

Answer: __________ (4 sf)

4. Estimate:

(a) 772 -18

Answer: ________________

(b) 630 - 40

Answer: __________________

(c) 90 000 – 17 023

Answer: __________________

5. Write the sum of the quantities correct to significant figures indicated:

(a) 12.35 kg, 15.324 kg, 17.076 kg, 14.4 kg

Answer: __________________(3sf)

(b) 6.5 m, 68 cm, 85.5 cm, 3 m, 1.58 m

Answer: __________________(3sf)

6. Give the sum correct to the number of significant figures indicated:

(a) $6.639 + 8.24 =$

Answer: ________________(3 sf)

(b) $3.069 + 31.3 =$

Answer: _________________(3 sf)

---

7. Workout estimated cost:

Fish K4.50
Meat K7.85
Rice K4.60
Bread K3.25

Answer: _________________

---

8. Estimate : $6.26 \times 10^2 + 3.2 \times 10^2$

Answer: _________________

---

9. Write $8.036 \times 10^3 - 4.503 \times 10^3$ correct to 2 significant figures.

Answer: _________________

---

10. In a carnival four schools came along with number of athletes as given below:

School A - 363
School B - 341
School C - 403
School D – 449

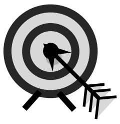
Give the estimate of the athletes population.

Answer: _________________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 10: Multiplication and Division with Significant Figures

You have learned about the properties of whole numbers and work out further problems by using these properties.



In this lesson, you will

- use significant figures in multiplication and division.

When multiplying or dividing we round figures to one non-zero digit number. Then we multiply or divide.

The rule for rounding applies as if equal to or greater than 5, round up; if less than 5 round down. We always make a decision based on the second significant digit.

Example 1
Estimate the product of 23 x 8.

Solution:

$$\begin{aligned} 23 \times 8 &\approx 20 \times 8 \\ &= 160 \end{aligned}$$

Example 2
Give the estimate of the quotient of $319 \div 83$.

Solution:

$$\begin{aligned} 319 \div 83 &\approx 300 \times 80 \\ &= 3.75 \end{aligned}$$

When estimating products in S.F., round the coefficients to one non-zero digit; and find products of the approximated coefficients and powers of ten separately. Then rewrite in S.F. if necessary.

The index rule ($a^m \times a^n = a^{mn}$) applies when dealing with products of powers of 10.

Example 3
Estimate the product of 2.35×10^2 cm x 4.6×10^{-1} .cm.

Solution:

$$\begin{aligned} 2.35 \times 10^2 \times 4.6 \times 10^{-1} &\approx 2 \times 10^2 \text{ cm} \times 5 \times 10^{-1} \text{ cm} \\ &= (2 \times 5) \times 10^{2 + -1} \\ &= 10 \times 10^1 \\ &= 1 \times 10^2 \text{ cm}^2 \end{aligned}$$

Example 4
Estimate the quotient of $3.53 \times 10^2 \div 4.6 \times 10^1$.

Solution:

$$
\begin{aligned}
3.53 \times 10^2 \div 4.6 \times 10^1 \quad &\approx 4 \times 10^2 \div 5 \times 10^1 \\
&= (4 \div 5) \times 10^{2\ -1} \\
&= 0.8 \times 10^1 \\
&= 8 \times 10 \times 10^1 \\
&= 8 \times 10^2
\end{aligned}
$$

Remember that when you estimate, round to one non zero digit before performing any operation. When approximating, you get the exact answer first, then write correct to specified significant figures stated.

Whether you are approximating or estimating, you still round to specified significant figures (or decimal places) as specified. The difference is, in approximation, rounding off comes after the operation; in estimation, rounding off comes before the operation.

When there is no statement of approximation and estimation, just perform the routine operation as stated. The answer you obtain is the answer.

The examples below provide you both, the exact answer and the estimate.

Example 5
$3.3 \times 10^2 \times 4.6 \times 10^1$.

Solution:

$$
\begin{aligned}
3.3 \times 4.6 \times 10^2 \times 10^1 &= 15.18 \times 10^3 \\
&= 1.518 \times 10^4 \text{ (4 s.f.)} \\
&\text{Or } 1518 \text{ (4 s.f.)}
\end{aligned}
$$

$$
\begin{aligned}
\text{Estimate } 3 \times 5 \times 10^2 \times 10^1 &= 15 \times 10^3 \\
&= 1.5 \times 10^4 \text{ (2 s.f)} \\
&\text{Or } 15000 \text{ (2 s.f.)}
\end{aligned}
$$

Example 6
$9.48 \div 0.25 =$

Solution:

$$
\begin{aligned}
9.48 \div 0.25 &= 948 \div 25 \\
&= 37.92 \text{ (4 s.f.)}
\end{aligned}
$$

$$
\begin{aligned}
\text{Estimate } 9 \div 0.3 &= 90 \div 3 \\
&= 30 \text{ (1 s.f.)}
\end{aligned}
$$

Example 7
16.043 + 1.077 + 102.05 =

Solution:

16.043 + 1.077 + 102.05 = 119.07 (5 s.f.)

Estimate 20 + 1 + 100 = 121 (3 s.f.)

Example 8
27.48 – 9.622 =

Solution:

27.48 – 9.622 = 17.858 (5 s.f.)

Estimate 30 – 10 = 20 (1 s.f.)

Example 9
2.3 x 43.5 =

Solution:

2.3 x 43.5 = 100.05 (5 s.f.)

Estimate 2 x 40 = 80 (1 s.f.)

Remember approximation can only be made after normal operation. That is, we add, subtract, multiply and or divide, get the answer, then we approximate by writing the answer correct to specified significant digits.

The approximate answers correct to 3 s.f. for examples 5 to 9 respectively, are:

1.58×10^4 (3 s.f.), 37.9 (3 s.f.), 119 (3 s.f.), 17.9 (3 s.f.) and 100 (3 s.f.)

NOW DO PRACTICE EXERCISE 10

Practice Exercise 10

1. How many significant figures has 3.18×10^3 ?

Answer: __________

2. How many significant figures are in 3.18×10^{-3}?

Answer: __________

3. How many s.f. would the estimated product of 7.04 x 41.26 have?

Answer: ________________

4. How many s.f. would the estimated quotient of 41.53 and 0.83 have?

Answer: _________________

5. Estimate : 2427 x 109

Answer: __________________

6. Estimate : $8263 \div 7509$

Answer: __________________

7. Estimate the area of a rectangle with dimensions of 238 cm x 272 cm.

Answer: __________________

8. Give the estimate of the consumption rate of water given that 3 428 668 L was used in 30 days.

Answer: __________________

9. Estimate 6.344×10^5 km $\div$ 1.722×10^3 hr

Answer: __________________

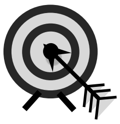
10. Estimate the product $1.826 \times 10^2 \times 3.18 \times 10^{-2}$

Answer: __________________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 11: Word Problems

You have learned about rounding to specified decimal places or significant figures.



In this lesson, you will

- apply the mathematical knowledge and number skills to solve word problems.

Word problems are problems described in English; we are to translate them into mathematical expressions in order to solve the problems.

The **algorithm** of each problem will be easy if the problem is initially correctly translated. Algorithm is the step by step processes taken to arrive at a solution. To be able to do that, you need to learn some of the key words commonly used in mathematical word problems.

Below are some key words and phrases that are used in the word problems which we need to understand their definition or description perfectly well, inorder to apply the appropriate mathematical symbol in our translation process.

Word or Phrase	Mathematical Translation
Add, subtract, divide, multiply	**+, - , ÷ ,X**
sum	+
difference	-
quotient	÷
product	x, ( ), *
Consecutive numbers	n, n + 1 or n-1, n
Of	x
More than	+
Less than, less	-
Square of , square	$(n)^2$
Taken off from	-
Is greater than	>
Is less than	<
Twice, a quarter of, double	x

So when there is a mention of difference and less than, you know that it is to do with subtraction; twice, four times , double, multiple and product is to do with multiplication; consecutive numbers is to do with sequence of counting numbers and so on.

When a word problem is about estimation, you need to apply rules involved in estimating results in an operation. A word problem may be based on a single topic such as estimation, or more than one mathematical topic. When it involves more than one topic, you need to apply skills in the topics involved to set your algorithm.

Now study the following examples.

Example 1
Find the lowest common multiple (LCM) of 12 and 16. Then find the highest common factor (HCF) of 12 and 16. What percent of LCM is the HCF of 12 and 16?

Solution:

LCM = 48, HCF = 4
HCF/LCM x 100 % = 4/48 x 100%
= 1/12 x 100%
= 8.3%

Example 2
Three consecutive numbers in a sequence have a difference of 4. If their sum is 51, find the three numbers.

Solution:

Let the first number be n, the second be n + 4, and the third n + 8.
So the sum is n +(n + 4) + (n + 8) = 51.

$n + (n + 4) + (n + 8) = 51$

$3n + 12 = 51$
$3n = 51 - 12$
$3n = 39$
$n = 39 \div 3$
$n = 13$
since n = 13, n + 4 = 13 + 4 = 17, and n + 8 = 13 + 8 = 21.

The first number is 13, the second is 17 and the third is 21 giving the sequence {13, 17, 21, ….}.

Example 3
An approximate of a distance given to three significant figures is 11.6 km. What is the range in which the distance can be rounded to 11.6 km?

Solution:

The range is **11.55 to 11.64 km.**
(Any figure that does not fall within this range cannot be rounded to 11.6).

Example 4
If the sum of 56.08 and 18.492 is to be written correct to 3 significant figures, what would be the difference between its estimate and its approximate value?

Solution:

Approximate:	56.08 + 18.492	= 74.572
		= 74.6 (3 s.f)
Estimate:	56.08 + 18.492	= 60 + 20
		= 80 (2 s.f)
Difference:	80 – 74.6 = 5.4	

The difference is 5.4

Example 5
Three consecutive even numbers have an estimated sum of 60. What are the three numbers?

Solution:

Even numbers [2,4,6, , 16, 18, 20, 22, 24, 26, …].

There are 3 possible solutions:
16, 18, 20 or 18, 20, 22 or 20, 22, 24 consecutive number sets

Each number in all three sets of solutions can be rounded to 20 before addition.

Understanding basic concepts in a particular topic is very important. When you have that knowledge, you can be able to solve related word problems. If you do not understand the key knowledge in a particular topic, say estimation, you will not be able to solve word problems related to estimation.

NOW DO PRACTICE EXERCISE 11

Practice Exercise 11

1. Find the quotient of the LCM of 6 and 8, divided by HCF of 6 and 8.

Answer: __________

2. Three men A, B and C cut grass to clean school grounds as:
   A: 3 m x 6 m
   B: 4 m x 8 m
   C: 4 m x 7 m

   Estimate the total area of grounds cleared by the three men;

Answer: __________________

3. What is the quotient of the square of 16 by the HCF of 8 and 24?

Answer: __________________

4. The average age of four high school girls is 16. If eight primary school girls ages have the same total, what is the average age of the primary school girls?

Answer: __________________

5. Peter, Qulila and Robert bought a land covering an an area of 40 m x 32 m. They are to share as Peter owns 25%, Qulila 45% and Robert owns the rest. Find the area of land own by Robert.

Answer: __________________

6. Three business partners Au , Teletha and Gari contributed K30 000, K40 000 and K80 000 respectively to start a business. After two years, the business declared a profit of K200 000. How much money will Au get if the money is to be shared in the ratio of their contribution?

Answer: __________________

7.. A demographic survey recorded the following village pollution data of one ethnic group as:

Village A 623 people
Village B 438 people
Village C 661 people
Villade D 574 people
Village E 359 people

Give the estimate of the population of the ethnic group.

Answer: ___________________

8.. A piece-wage earner collected the following weights of cocoa beans each day over a week: 9.8 kg, 11.6 kg, 8.8 kg, 10.7 kg and 6.8 kg. If he is paid at K4/kg, estimate the total amount he will earn.

Answer: ___________________

---

9. Ray bought 5 tins of fish at K5.20 each, 6 tins of meat at K7.20 each, 12 packets of noodles at K1.10 each, 3 packets of chicken at K12.80 each and 2 packets of sugar at K4.30 each.

Estimate the change he gets if he paid K150.

Answer: ___________________

---

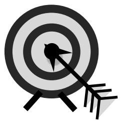
10. Denis measured a length of a rectangular piece of land and gave the approximate distance to be 12.8 m long . If the approximate was the result of rounding up, what are the possible measurement that will yield an approximate of 12.8 m?

Answer: ___________________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 12: The Calculator

You have learned about the how to translate and solve word problems.



In this lesson, you will

- operate a simple calculator by using the basic operational keys.

A calculator has been a useful discovery ever since it was discovered till today. It eases hard work in computation of numbers when appropriate keys are pressed sequentially.

A **calculator** is a hand held electronic device used to compute arithmetic operations.

A Scientific calculator can compute '**order of operations'** all at once, which an ordinary calculator cannot. A scientific calculator would often have the abbreviation **fx** written on it. If the calculator is of the brand **fx350**, it tells us that the calculator has 350 functions it can perform in one set of computation. The higher the fx number, the more the number of functions a scientific calculator can perform.

A **function** is an activity assigned to a key to perform a specific task.

Say, if we enter 2 + 5 = 7, we used four functions in 2, +, 5 and =. The figure 7 is automatically generated by the calculator.

There is also a Graphic Calculator. It can perform all functions of a scientific calculator and also generate statistical and algebraic graphs or images, based on data entered.

Numeric Keys are keys that bear the digits from 0 to 9.

Basic Arithmetic Keys are keys used to manipulate addition, subtraction, multiplication and division of numbers. They are $\boxed{+}$, $\boxed{-}$, $\boxed{x}$ and $\boxed{\div}$ keys, and includes parentheses $\boxed{(}$, $\boxed{)}$, equal $\boxed{=}$ sign , decimal point $\boxed{\bullet}$ and the negative $\boxed{-}$ sign for integers.

These two sets of keys are the usual keys found in any calculator, even the most basic of all models

Function keys such as STORE $\boxed{\text{STO}}$ and RECALL $\boxed{\text{RCL}}$ allow the user to SAVE and RETRIEVE data.

After performing a calculation, you can press $\boxed{\text{STO}}$ if you intend to use or review the calculations or the answer at a latter time.

Pressing $\boxed{\text{RCL}}$ will show the previously saved data. You can modify this by inserting another value, just use the navigation keys to move the cursor where you want to insert data.

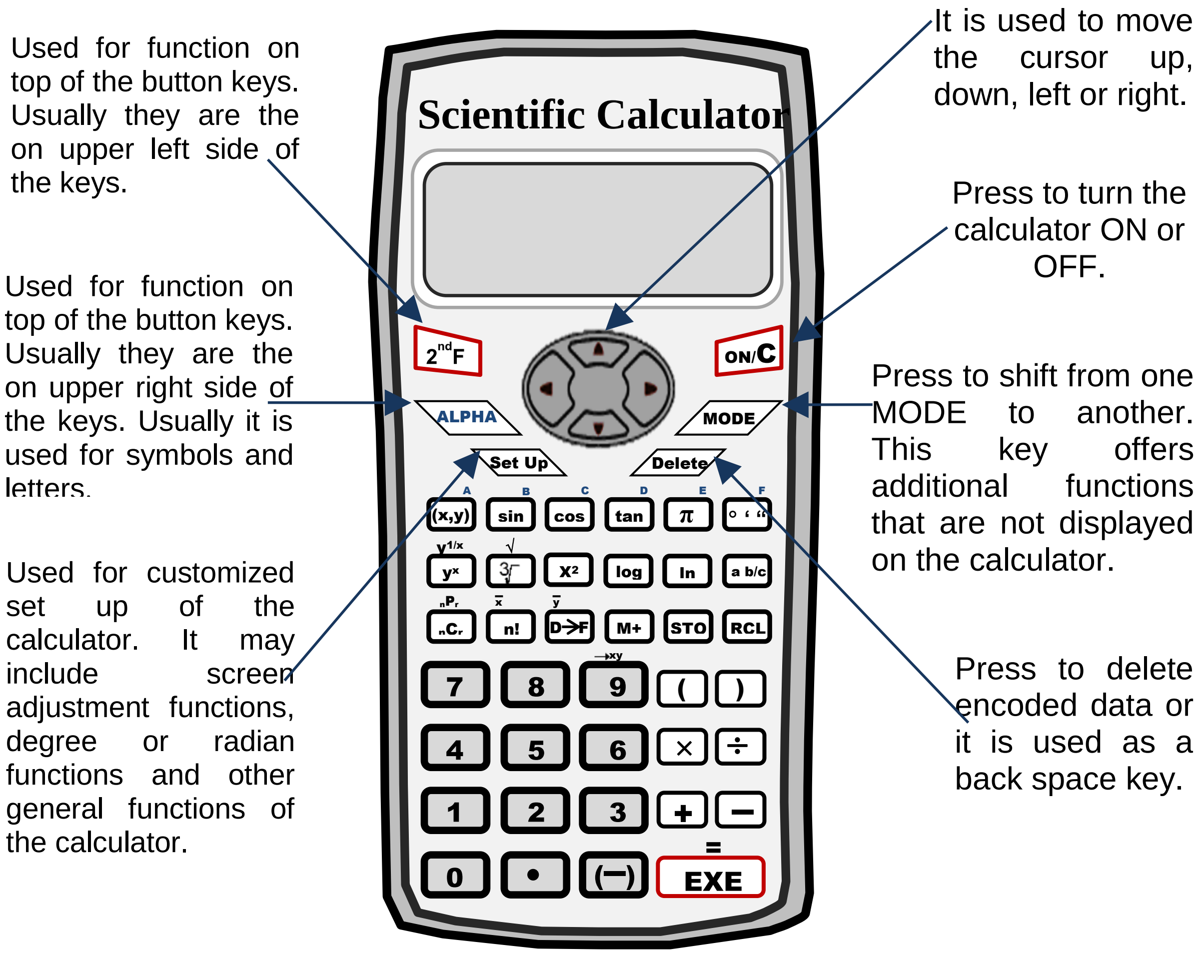
You can also delete or erase data by pressing Delete after moving the cursor next to the data you want to remove.

Some functions and positions of the keys may vary depending on your calculator's brand and model. That is why it is important to read your calculator's manual before using it.

If the function is on the deck of keyboard, press $\boxed{\text{SHIFT}}$ key, then press the button below the function.

Calculators have different capacity as to the number of data to store so always reset your calculator when computing the next problem.

Let us identify some other keys and parts of the calculator that will be used often in calculations. You also need to read the **User's Manual** of the calculator you are using to be more familiar with it.



Calculator Practice

Press the appropriate keys for the algorithm to generate the results given in bold.

Example 1

(8 + 4) ÷ 3 = **4**

Solution:

$\boxed{(}$ $\boxed{8}$ $\boxed{+}$ $\boxed{4}$ $\boxed{)}$ $\boxed{\div}$ $\boxed{3}$ $\boxed{=}$

Example 2

Press [STO] key (while **4** appears on the screen). Then enter [Delete] key, and then the [RCL] key .

Solution:

4 reappears on the screen.

Example 3

Press [RESET] , [=] and the [AC] keys.

Solution:

When you enter [RCL] key, **4** does not reappear.

Example 4

$$\frac{2(-3-4)}{-4} = \mathbf{3.5}$$

Solution:

[-] [4] [STO] [2] [(] [-] [3] [-] [4] [)] [÷] [RCL] [=]

The calculator algorithm will generate the result **3.5**

Example 5

[4] [x^{-1}] [x] [1] [0] [0] [=]

Solution:

The algorithm generates **25**. The [x^{-1}] (reciprocal) key reads 4 as ¼ . Thus it computes ¼ x 100.

Answer is 25

Example 6

$2\left(\frac{6-9}{6}\right)+8=$

Solution:

6 **x^{-1}** ***** **(** **6** **-** **9** **)** ***** **2** **+** **8** **=**

Answer is 7

Example 7

$(2-9)+8^3=$

Solution:

(**2** **-** **9** **)** **+** **8** **^** **3** **=**

Answer is 505

You can enter numbers with basic operation in the sequence as given; the scientific calculator will perform the operation according to BODMAS or PEMDAS rule for order of operations.

Example 8

$2 \times 3 + 4^2 - 5 \div 6 =$

Solution:

2 **x** **3** **+** **4** **^** **2** **-** **5** **÷** **6** **=**

Answer is 21.166666…

With calculators which cannot perform this operation in the given sequence, you have to find **2 x 3**, then **4^2** and then **5 ÷ 6** separately. Use **STO** to store **5 ÷ 6**, then enter 6 + 22 - **RECALL** and you will arrive at the same answer 21.166666….

Power Function

The power function key [^] , if it is not embedded in the calculator, the function may not be available. This is the case with non-scientific calculators.

When it is not available, then use the following process.

Step 1 Enter the number.
Step 2 Enter [*] twice.
Step 3 Enter [=] according to the number of power given, less one.

Example 9
Find 13^4.

Solution:

Step 1 Enter the number [1] [3] so 13 appears on the screen.

Step 2 Enter [x] [x] or [*] [*].

Step 3 Enter [=] [=] [=] Power is 4, less 1 is 3.

Answer 28 561

[1] [3] [x] [x] [=] [=] [=]

Calculator screen [**28 561**]

For **addition** and **subtraction** if you enter equal sign more than once, it repeats the addend or subtrahend. Study the examples below.

[1] [9] [+] [3] [=] [=] [=]

Calculator screen [**28**]

The addend 3 is added each time we enter equal sign.

[6] [1] [-] [7] [=] [=] [=] [=]

Calculator screen [**33**]

The subtrahend 7 is subtracted each time we enter equal sign.

It also works the same way for **division**. If you enter 20 ÷ 4, then equal sign it yields 5, press equal sign twice again it yields 0.3125.

NOW DO PRACTICE EXERCISE 12

Practice Exercise 12

For questions 1 – 5, press the keys as set for the calculator algorithms.

1. $\boxed{9}\ \boxed{+}\ \boxed{3}\ \boxed{-}\ \boxed{-}\ \boxed{5}\ \boxed{-}\ \boxed{4}\ \boxed{+}\ \boxed{2}\ \boxed{=}$

Answer: __________

2. $\boxed{3}\ \boxed{(}\ \boxed{8}\ \boxed{-}\ \boxed{+}\ \boxed{1}\ \boxed{2}\ \boxed{)}\ \boxed{-}\ \boxed{5}\ \boxed{=}$

Answer: __________

3. $\boxed{\sqrt{}}\ \boxed{3}\ \boxed{(}\ \boxed{5}\ \boxed{-}\ \boxed{\sqrt{}}\ \boxed{2}\ \boxed{)}\ \boxed{=}$

Answer: ________________

4. $\boxed{4}\ \boxed{x^2}\ \boxed{+}\ \boxed{3}\ \boxed{x^2}\ \boxed{=}$

Answer: _________________

5. $\boxed{4}\ \boxed{x^{-1}}\ \boxed{x}\ \boxed{3}\ \boxed{+}\ \boxed{5}\ \boxed{x^{-1}}\ \boxed{x}\ \boxed{2}\ \boxed{=}$

Answer: _________________

Evaluate the following using the calculator.

6. $19^2 + \sqrt{625} =$

Answer: __________________

7. 160 – 30 (33 – 35) + 17 =

Answer: __________________

8. 110 – 22 (- 12 + 3 x 4) - 25 =

Answer: __________________

9. 0.2803 x 2/5 =

Answer: __________________

10. 62% x K63 800

Answer: __________________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Summary

- **Estimation** is the sum, difference, quotient or product from rounding quantities. It is found by first rounding to one non-zero digit before any operation takes place.
- **Approximation** writing measurements to certain place value considering its immediate lesser unit; or writing a quantity to required place value in terms of decimal places or significant figures..
- **Round off** is writing numbers correct to certain number of decimal places or significant figures in estimation and approximation. Rounding is done first before operation in estimation, but done later after the operation in approximation
- **Significant Figures** is position of significance of a number.
- Scientific Notation or Standard Form is an ordinary number expressed in the form $\mathbf{A \times 10^{N}}$, where **A** is the **coefficient** and **N** is the **index**. The coefficient **A** always has a decimal point after the first non-zero digit.

- **Calculator** a devise used to compute arithmetic operations.
- **Functions** are activities assigned to computer keys.
- **Fx100** defines the number of functions the calculator can perform. Comparing fx100 and fx530, fx530 can perform more functions than fx100.
- **Repetition of equal sign** will be read by all calculators as the last number entered should be added, subtracted, multiplied by or divided by. Say , 3 – 4 = -1. If we enter = (equal), 4 will be subtracted again and so on.

- **Composite Numbers** are numbers with two or more factors. These numbers c
- **Primes** are numbers with only two factors, 1 and itself.
- **Factors** are numbers that divide exactly into another number.
- **Decimal Place** signifies the specific number of digits to the right of a decimal point.
- **Place Value** is the value of a digits location in a numeral also be expressed as product of their prime factors.
- **Order of Operations** is a defined procedure of sequence of operation in mathematics.

Answers To Practice Exercises 7-12

Practice Exercise 7

1. 290
2. 0.29
3. (a) 0.3
   (b) 0.29
4. (a) 12.7
   (b) 2.8
   (c) 75 000
5. 140 000
6. (a) 0.42
   (b) 7.82
   (c) 12.81
   (d) 560.09
   (e) 0.09
7. 28 000 mL or 28 L
8. (a) 8.94
   (b) 4.47221
9. 3.142
10. 1.3

Practice Exercise 8

1. 2450
2. (a) 6 s.f.
   (b) 6 s.f.
   (c) 5 s.f.
   (d) 5 s.f.
   (e) 6 s.f.
3. (a) 202.197
   (b) 202.20
   (c) 202.2
   (d) 202
4. (a) 12.6 cm
   (b) 20.19 m
   (c) 459.253 km
   (d) 24.655 kg
   (e) 8.058 L
5. (a) 72.39
   (b) 72
6. (a) 780
   (b) 5.204
7. 5 s.f.
8. 9.01×10^0
9. 1.9×10^2
10. 8.6×10^2 cm^2

Practice Exercise 9

1. 340
2. 7 200
3. (a) 23.7 m
   (b) 1.231 L
4. (a) 780
   (b) 560
   (c) 70 000
5. (a) 59.2 kg
   (b) 962 cm or 9.62 m
6. (a) 14.9
   (b) 34.4
7. K21.00
8. 9×10^2
9. 3.5×10^3
10. 15 000 athletes

Practice Exercise 10

1. 3 sf
2. 3 sf
3. 2 sf
4. 1sf
5. 200 000
6. 1
7. 60 000 cm^2
8. 100 000L/day
9. 3×10^2 km/h
10. 6×10^0

Practice Exercise 11

1. 12
2. $80m^2$
3. 32
4. 8 years
5. $384m^2$
6. K40 000
7. 2 700 people
8. K184
9. K21
10. 12.75, 12.76, 12.77, 12.78, 12.79

Practice Exercise 12

1. 15
2. - 17
3. 1.423 494
4. 25
5. 1.5
6. 386
7. 145
8. 85
9. 0.6803
10. K39 556

TOPIC 3

DIRECTED NUMBERS

TOPIC 3: DIRECTED NUMBERS

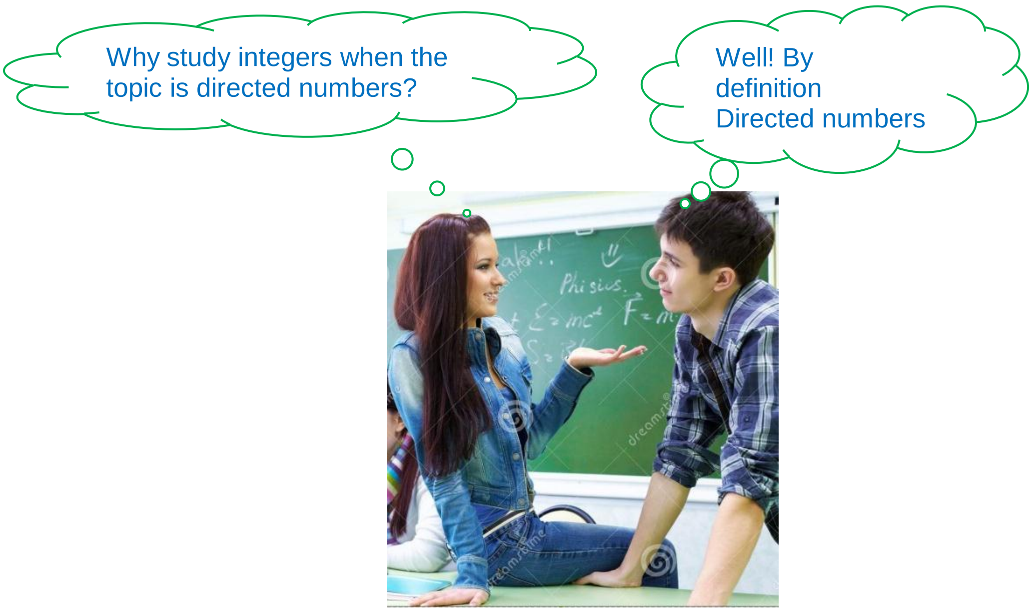
Introduction

You have done problems involving whole numbers, fractions and decimals. You also know that the decimal point separates the whole number part from the fractional part of a number.

In this topic we will study directed numbers. Directed numbers are negative and positive numbers or signed whole or counting numbers. Since zero has no sign at any time we write zero, it is not a directed number.

There exists a need for us to learn the application of negative numbers. When reading a temperature scale below zero or having a debt in cash or kind, we can express those figuratively by a negative number.

We will learn the knowledge and skills in directed numbers through integers; addition and subtraction, multiplication, division and order of operations involving integers.

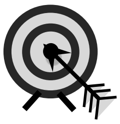


Integers are all negative and positive numbers including zero. Integers are rational numbers, and fall under the real numbers.

In this topic, you will have a chance to revisit and practice directed numbers and be able to use them where appropriate.

Lesson 13: Integers

You have learned about the properties of whole numbers and work out further problems by using these properties.



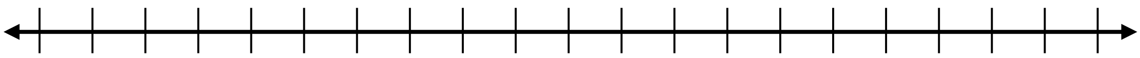
In this lesson, you will

- define and explain integer with confidence
- identify properties of integer
- state examples of integers.

Integers are numbers as such as {… -3, -2, -1, 0, 1, 2, 3, 4, 5, ….}. Integers include zero and is a subset of rational and real numbers.

Directed numbers do not include zero. Integers contains all negative and positive numbers and zero. Integers stretch indefinitely in either direction from zero.

Integer uses $\mathbb{Z}$ as a symbol. The symbol comes from the German word for numbers which is zahlen.



Integers are all negative and positive numbers including zero in the real number field.

Integers can also be expressed on a number line vertically. When we deal with algebraic graphs, the integers are expressed on both the vertical and horizontal number lines known as axes.

Some Properties of Integers

1. The sum of any integer and zero is that integer.

   $a + 0 = a,$ for all $a \in \mathbb{Z}$

2. Every integer has an additive inverse.

   $a + a' = a' + a = 0$ for all $a, a' \in \mathbb{Z}$

3. Additive Identity of integers is 0.

   $a + 0 = a$ for all $a \in \mathbb{Z}$

4. Cancellation property of multiplication

   If $c \neq 0$, then when $ac = bc$, $a = b$

5. Multiplicative Identity of integers is 1

   $a \times 1 = a$ for all $a \in \mathbb{Z}$

6. Integers are associative over multiplication and addition.

$a(bc) = (ab)c$	multiplication	for all $a,b,c \in \mathbb{Z}$
$(a + b) + c = a + (b + c)$	addition	for all $a,b,c \in \mathbb{Z}$

7. Integers are distributive over multiplication.

   $a(b + c) = ab + ac$ for all $a,b,c \in \mathbb{Z}$

8. Integers are commutative over multiplication and addition.

$ab = ba$	multiplication	for all $a,b \in \mathbb{Z}$
$a + b = b + a$	addition	for all $a,b \in \mathbb{Z}$

Now, study the examples provided on the properties of integers.

Example 1

State the property of integer that holds true when $a(b + c) = ab + ac$.

Solution:

$a(b + c) = ab + ac$ distributive property

Example 2

Use commutative property over addition to make this statement true: $-3 + 7 = \ldots$

Solution:

$-3 + 7 = 7 + -3$

Example 3
Evaluate: -6 x k = 5 x - 6

Solution:
By cancellation property ac = bc and c≠ 0,
k = 5

Example 4
Find value of x when 8 + x = 8.

Solution:
By addition identity property, x = 0

Example 5
Find **a** when 5 x 6 = 6 x **a**

Solution:
By commutative property of integers, a = 5

Example 6
Find **a** when 5 (6 + 7) = 6**a** + 35

Solution:
By distributive property of integers, a = 5

Example 7
What property states that 2 + (3 + 6) = (2 + 3) + 6

Solution:
Associative property of addition.

Mastering the knowledge on properties of integers will help you solve more complex number and algebraic problems.

NOW DO PRACTICE EXERCISE 13

Practice Exercise 13

1. Which of the following numbers listed is not and integer?

   $-7,\ 2\sqrt{7},\ 7,\ 0$

Answer: __________________

2. Given that a (b + c) = d and d is not zero, then which of the other three letters cannot be zero as well, to make the statement true?

Answer: __________________

3. Which of the set of numbers are whole numbers?

   Set A: {-3, -2, -1, 0, 1, 2, 3}
   Set B: {0, 1, 2, 3, 4, 5, 6}
   Set C: {1, 2, 3, 4, 5, 6, 7}

Answer: __________________

4. Which of the following integer in the set is also a prime number?

   {8, 11, 14, 17, 20}

Answer: ___________________

5. Which of the following algebraic statements states the commutative property over addition?

   A. $p + q = q + p$ C. $p(q + r) = pq + pr$
   B. $p + (q + r) = (p + q) + r$ D. $pq = qp$

Answer: ___________________

6. Use the integers -3, 2 and 5 to show that integer is associative over addition.

Answer: ___________________

---

7. State the additive inverse of – 20.

Answer: ___________________

---

8. The expression - 4(3 – 7) , by distributive property is equal to

A. - 4 x -3 - - 4 x – 7
B. – 4 x 3 - - 4 x – 7
C. 4 x -3 - 4 x - 7
D. - 4 x 3 - 4 x – 7

Answer: ___________________

---

9. Which of the following set of numbers are not integers?

A. Natural Numbers
B. Whole Numbers
C. Counting Numbers
D. Rational Numbers

Answer: ___________________

---

10. Which of the statement is false, given that -5 and 12 are integers so their product will be:

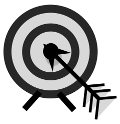
A. An integer
B. A directed number
C. A natural number
D. A rational number

Answer: ___________________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

Lesson 14: Adding and Subtracting Integers

You have learned about the properties of whole numbers and work out further problems by using these properties.



In this lesson, you will

- identify the rules of adding and subtracting integers
- add and subtract integers correctly by following the rules.

When we deal with addition and subtraction of integers, we need to take note that the signs we will have in the arithmetic are numbers signs and the operation signs. The symbols are the same, but their position tells us whether the sign is that of an operation or a number.

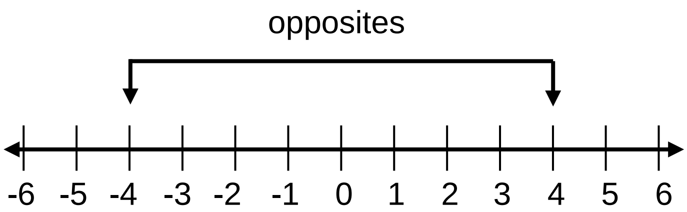
Say, given is $^{+}2 - {}^{-}6$, the operation is subtraction (-), and what is required is we subtract negative 6 from positive 2. So, number signs appear before numbers and operational sign appears after the numbers.

In $^{+}2 + {}^{-}6$, positive 2 is the augend and negative 6 is the addend. When there are more than two numbers to add, it is appropriate to refer to each term as a summand. Say, in the expression $^{+}9 + {}^{-}5 + {}^{+}7 + {}^{-}12$, each term can be called a summand.

In $^{+}2 - {}^{-}6$, positive 2 is the minuend and negative 6 is the subtrahend. Suppose we interchange the positions, then negative 6 becomes minuend and positive 2 becomes the subtrahend. So to say, the first term is the minuend and the second term is the subtrahend.

When adding integers we follow the following rules:

Rule 1: The sum of opposite integers is zero.



Rule 2: The sum of two negative integers is a negative integer.

Example 1
Find the sum of each pair of integers.

Adding Negative Integers	
Integers	**Sum**
$^{-}3 + {}^{-}7 =$	$^{-}10$
$^{-}15 + {}^{-}4 =$	$^{-}21$
$^{-}30 + {}^{-}22 =$	$^{-}52$

Rule 3: The sum of two positive integers is a positive integer.

Example 2
Find the sum of each pair of integers.

Adding Positive Integers	
Integers	**Sum**
$^{+}3 + {}^{+}6 =$	$^{+}9$
$^{+}5 + {}^{+}14 =$	$^{+}19$
$^{+}10 + {}^{+}12 =$	$^{+}22$

Rule 4: The sum of a positive and a negative integer takes the sign of the integer with large absolute value.

Absolute Value

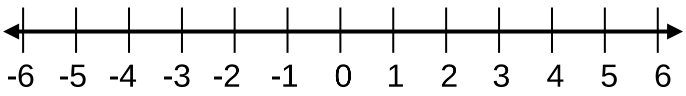
$|-3| = 3$

$|0| = 0$

$|3| = 3$

To add a positive and a negative integer (or a negative and positive integer):

Step 1: Take absolute value of each integer.
Step 2 : Subtract smaller number from the larger number.
Step 3: Write sign of integer with greater absolute value in step1, followed by answer in step 2.



Example 3
Find the sum of $^{+}6$ and $^{-}8$.

Solution:

Step 1: $|^{+}6| = 6$ and $|^{-}8| = 8$
Step 2: $8 – 6 = 2$
Step 3: Answer is $^{-}2$ since $^{-}8$ has a greater absolute value.

Example 4
Find the sum of $^{-}16$ and $^{+}20$.

Solution:

Step 1: $|^{-}16| = 16$ and $|^{+}20| = 20$
Step 2: $20 – 16 = 4$
Step 3: Answer is $^{+}4$ since $^{+}20$ has a greater absolute value.

When subtracting integers we follow the following rules:

Rule 1: Change the subtraction sign with a plus sign.
Rule 2: Change the number sign of the subtrahend.
Rule 3: Apply any addition rule 1 to rule 4 where appropriate..

Example 4
Find the difference of the integers $^{+}2$ and $^{-}5$.

Solution:
$^{+}2 - {}^{-}5 = {}^{+}2 + {}^{+}5$ (rules 1 and 2)
$= {}^{+}7$

Example 5

Find the difference of the integers $^{-}2$ and $^{-}5$.

Solution:

$^{-}2 - {}^{-}5 \quad = {}^{-}2 + {}^{+}5$ (rules 1 and 2)

$= |-2| = 2$ and $|+5| = 5$ (rule 4 in addition)

$= 5 – 2$

$= + 3$

Example 6

Find the difference of the integers $^{-}2$ and $^{+}5$.

Solution:

$^{-}2 - {}^{+}5 = {}^{-}2 + {}^{-}5$ (rules 1 and 2)

$= {}^{-}7$ (addition rule 2)

Now. Suppose that there are more than two integers are to be added or subtracted, work from left to right; deal with two integers at a time and apply rule that is appropriate.

Example 7

Simplify: $^{-}17 + {}^{-}3 + {}^{+}15 - {}^{-}2$

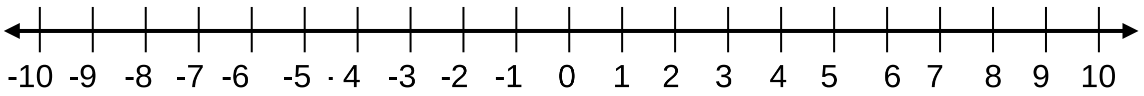
Solution:

$^{-}17 + {}^{-}3 + {}^{+}15 - {}^{-}2 = {}^{-}20 + {}^{+}15 - {}^{-}2$ (sum of two neggatives)

$= {}^{-}5 - {}^{-}2$ (absolute values)

$= - 3$ (absolute values)

Alternatively, when using a number line use the rules:



1. When a number is the same as preceding operation sign, replace both with one plus sign. Say, $^{-}2 - {}^{-}5 = - 2 + 5 = 3$.

   From negative 2, go forward 5 steps, you end at positive 3.

2. When a number sign is not the same as the preceding operation sign, replace both with one minus sign. Say, $^{-}3 - {}^{+}4 = - 3 – 4 = - 7$ and $^{-}3 + {}^{-}4 = - 3 – 4 = 7$.

   From negative 3, go backward 5 steps, you end at negative 7.

However, number line is better used along with Sam the Sentry Method.

Sam the Sentry Method

Step 1	Start on zero facing positive direction.
Step 2	Number sign tells you to walk forward (+) or backward (-).
Step 3	Operational sign tells you to face positive (+) or negative (-) in before your next move.

Example 8

$^{-}9 + ^{-}3 =$

Solution:

I start at 0 facing positive (direction), walk backward 9 steps I stop at -9. Still facing positive, I walk further 3 steps backward and I stop at – 12.

$^{-}9 + ^{-}3 = ^{-}12$

Example 9

$^{-}9 - ^{-}3 =$

Solution:

I start at 0 facing positive (direction), I walk backward 9 steps I stop at -9. I turn around and face negative direction, I walk 3 steps backward and I stop at – 6.

$^{-}9 - ^{-}3 = -9 + ^{+}3 = -|9| - |3| = -6$

Example 10

$9 - ^{-}3 =$

Solution:

I start at 0 facing positive (direction), I walk foreward 9 steps I stop at positive 9. I turn around and face negative direction, I walk 3 steps backward and I stop at $^{+}12$.

Using a number line and Sam the Sentry Method, will never make a mistake. You decide whether to use the different rule given or use Sam the Sentry Mehod.

NOW DO PRACTICE EXERCISE 14

Practice Exercise 14

1. $^{-}6 + {}^{+}6 =$

Answer: _______________

2. $^{-}14 + {}^{-}8 =$

Answer: _______________

3. $^{+}17 + {}^{+}4 =$

Answer: ________________

4. $^{+}36 + {}^{-}19 =$

Answer: _________________

5. $^{-}16 - {}^{+}20 =$

Answer: _________________

6. $^{-}100 + {}^{-}68 - {}^{+}32 + 40 =$

Answer: _________________

7. What is the total given the summands $^{-}7$, $^{+}12$, $^{+}23$ and $^{-}14$?

Answer: ___________________

8. $^{-}6 - {}^{-}9 + {}^{+}4 - {}^{+}8 =$

Answer: ___________________

9. $^{+}12 - {}^{-}32 - {}^{+}24 + {}^{-}8 =$

Answer: ___________________

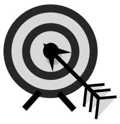
10. What is the difference of $^{-}6$ and $^{-}17$?

Answer: ___________________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

Lesson 15: Multiplying Integers

You have learned about the properties of whole numbers and work out further problems by using these properties.



In this lesson, you will
- enumerate and explain the rules of multiplying integers,
- distinguish between the rules for adding and subtracting, and multiplying integers,
- multiplying integers by following the rules appropriately.

When multiplying integers, we deal with two or more factors. If the factors are only two, then the first can be called a multiplier and the later the multiplicand.

When the multiplier and the multiplicand are both negative or both positive, the product is positive. When the multiplier and the multiplicand have opposite number signs then the product is negative.

In symbols we can illustrate products of multiplier and multiplicand as

$- \times + = {}^{-}$
$- \times - = {}^{+}$
$+ \times - = {}^{-}$
$+ \times + = {}^{+}$

Given that

$a \times b = c$ a is the multiplier,
b is the multiplicand, and
c is the product.

When there are more than two numbers to be multiplied together, we refer to them as factors.

Rules
1. Multiply the numbers.
2. If the multiplier and the multiplicand have the same sign the answer is positive.
3. If the multiplier and the multiplicand have the different signs the answer is negative.

In general, we can say that :

1. Product of even number of negative factors yields a positive product.
2. Product of odd number of negative factors yields a negative product

Now study the examples below.

Example 1
$^{-}12 \times {}^{+}7 =$

Solution
$^{-}12 \times {}^{+}7 = {}^{-}84$ Negative and positive factors; product (answer) is negative.

Example 2
$^{-}12 \times {}^{-}7 =$

Solution
$^{-}12 \times {}^{-}7 = {}^{+}84$ Both are negative factors; product is positive.

Example 3
$^{+}12 \times {}^{+}7 =$

Solution
$^{+}12 \times {}^{+}7 = {}^{-}84$ Both are positive factors; product is positive.

Example 4
$^{-}12 \times {}^{+}7 \times {}^{+}3 =$

Solution
$^{-}12 \times {}^{+}7 \times {}^{+}3 = {}^{-}252$ Odd number of negative factors; answer is negative.

Example 5
$^{-}12 \times {}^{+}7 \times {}^{-}3 =$

Solution
$^{-}12 \times {}^{+}7 \times {}^{-}3 = {}^{+}252$ Negative and positive factors; answer is negative.

Example 6
$({}^{-}12)^2 =$

Solution
$({}^{-}12)^2 = 144$ Even number of negative factors; answer is positive.

Example 7
$({}^{-}8)^3 =$

Solution
$({}^{-}8)^3 = -512$ Odd number of negative factors; answer is negative.

Example 8
$({}^{-}8)^3 \times 0 \times {}^{-}13 \times 5 =$

Solution
$({}^{-}8)^3 \times 0 \times {}^{-}13 \times 5 = 0$ Multiplication Identity a x 0 = 0.

When multiplying, the properties of integers may apply as in example 8. In fact, properties of integers apply in all basic operations, and we can draw from properties of integers to support our reason for the solution.

Below is multiplication table to help you in computing numbers larger than 12.

X	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
1	13	14	15	16	17	18	19	20	21	22	23	24
2	26	28	30	32	34	36	38	40	42	44	46	48
3	39	42	45	48	51	54	57	60	63	66	69	72
4	52	56	60	64	68	72	76	80	84	88	92	96
5	65	70	75	80	85	90	95	100	105	110	115	120
6	78	84	90	96	102	108	114	120	126	132	138	144
7	91	98	105	112	119	126	133	140	147	154	161	168
8	104	112	120	128	136	144	152	160	168	176	184	192
9	117	126	135	144	153	162	171	180	189	198	207	216
10	130	140	150	160	170	180	190	200	210	220	230	240
11	143	154	165	176	187	198	209	220	231	242	253	264
12	156	168	180	192	204	216	228	240	252	264	276	288
13	169	182	195	208	221	234	247	260	273	286	299	312
14	182	196	210	224	238	252	266	280	294	308	322	336
15	195	210	225	240	255	270	285	300	315	330	345	360

NOW DO PRACTICE EXERCISE 15

Practice Exercise 15

1. $^{-}12$ x $^{-}5$ =

Answer: ________________

2. $^{+}50$ x $^{+}3$ =

Answer: ________________

3. $^{-}21$ x $^{+}7$ =

Answer: _________________

4. $^{+}32$ x $^{-}4$ =

Answer: __________________

5. $^{-}3$ x $^{-}4$ x $^{-}5$ =

Answer: ___________________

6. - 5 x 12 x – 7 =

Answer: ____________________

7. $-3 \times +3 \times 0 \times -11 =$

Answer: ___________________

8. $^{+}7 \times {}^{-}4 \times {}^{-}1 \times {}^{+}10 =$

Answer: ___________________

9. What is the product given the factors -2 and – 8?

Answer: ___________________

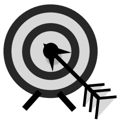
10. Find the product of the sums of , $^{-}7 + {}^{+}4$ and $^{+}3 + {}^{-}3$.

Answer: ___________________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

Lesson 16: Dividing Integers

You have learned about the properties of whole numbers and work out further problems by using these properties.



In this lesson, you will

- enumerate the rules of division of integer and explain each rule
- distinguish the rules for adding, subtracting, multiplying and dividing of integers
- divide integers by following the rules appropriately.

When dividing integers, we deal with a dividend and a divisor. The result of the two is the quotient.

When the dividend and the divisor are both, negative or both positive, the quotient is positive. When the dividend and the divisor have opposite number signs then the quotient is negative.

In symbols we can illustrate division of dividend and divisor as

$- \div + = {}^{-}$
$- \div - = {}^{+}$
$+ \div - = {}^{-}$
$+ \div + = {}^{+}$

Given that

$a \div b = c$

a is dividend,
b is the divisor, and
c is the quotient.

Rules
1. Divide the numbers.
2. If the dividend and the divisor have the same sign the answer is positive.
3. If the dividend and the divisor have the different signs the answer is negative.

Example 1
$^{-}12 \div {}^{+}6 =$

Solution
$^{-}12 \div {}^{+}6 = {}^{-}2$ Negative dividend and positive divisor; quotient (answer) is negative.

Example 2
$^{-}12 \times {}^{-}6 =$

Solution

$^{-}12 \times {}^{-}6 = {}^{+}2$ Both are negative; quotient is positive.

Example 3

$^{+}12 \div {}^{+}6 =$

Solution

$^{+}12 \div {}^{+}6 = {}^{+}2$ Both are positive; quotient is positive.

In division, it is possible to have remainders. That is, if the dividend is not a multiple of the divisor, there is always going to be a **remainder**. The opposite is also true, that is, if the divisor is not a factor of the dividend, there is always going to be a remainder.

Dividend ÷ Divisor = Quotient + Remainder

When faced with such situation, we can express the result as Quotient + Remainder or , the quotient with the remainder as a fraction. The fraction is expressed as the remainder over the divisor.

Example 4

$^{-}12 \div +5 =$

Solution

$^{-}12 \div {}^{+}5 = -2\frac{2}{5}$ Negative dividend and positive divisor; answer is negative.

Example 5

$^{+}12 \div {}^{-}8 =$

Solution

$^{+}12 \div {}^{-}8 = -1 \text{ R} - 4$ Positive dividend and negative divisor; answer is negative, remainder is negative.

Comparing examples 4 and 5, we see slight difference in answers. In example 5, the answer can also be written as – 1 ½ . That is, the fraction of a half does not need to have a negative sign before it. Whereas, when we write as a remainder, we have to indicate the negative sign.

When dealing with large numbers, use the multiplication table on page 118 to help you find quotients faster.

X		17	18	19	20	21	22	23	24
7		119	126	133	140	147	154	161	168
8		136	144	152	160	168	176	184	192
9		153	162	171	180	189	198	207	216
10		170	180	190	200	210	220	230	240
11		187	198	209	220	231	242	253	264
12		204	216	228	240	252	264	276	288
13		221	234	247	260	273	286	299	312
14		238	252	266	280	294	308	322	336
15		255	270	285	300	315	330	345	360

Example 6
$-229 \div 12 =$

Solution:

We can see that there is no product of 229; we take 228, thus the other factor is 19.

So, $-229 \div 12 = -19$ R -1 or $-19\ ^{1}/_{12}$

Example 7
${}^{-}329 \div {}^{-}22 =$

Solution:

We can see that there is no product of 329; we take 322, thus the other factor is 14.

So, ${}^{-}329 \div {}^{-}22 = 14$ R 1 or $14\ ^{1}/_{22}$

NOW DO PRACTICE EXERCISE 16

Practice Exercise 16

1. $121 \div 11 =$

Answer: ________________

2. $^{-}72 \div {}^{+}9 =$

Answer: ________________

3. $^{-}225 \div {}^{-}15 =$

Answer: ________________

4. $^{-}36 \div {}^{+}4 =$

Answer: ________________

5. $^{-}625 \div {}^{-}25 =$

Answer: ________________

6. In the equation $\frac{a}{b} = c$, which of the three letters cannot be zero for the solution to be defined?

Answer: ________________

7. $0 \div {}^{-}6 =$

Answer: ___________________

8. ${}^{-}42 \div {}^{+}13 =$

Answer: ___________________

9. What is the remainder in ${}^{-}103 \div 5$?

Answer: ___________________

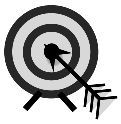
10. Find the dividend given that the quotient is 78,the remainder is 2 and the divisor is ${}^{-}13$.

Answer: ___________________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

Lesson 17: Order of Operations

You have learned about the properties of whole numbers and work out further problems by using these properties.



In this lesson, you will
- identify the BODMAS rules
- discuss and explain BODMAS rules in using integer
- apply BODMAS rules with integers

Order of Operations discusses what operation needs to be performed first, second and so on. It includes brackets and powers, along with the four basic operations. The basic operations are multiplication, division, addition and subtraction. Order of operations may sometimes be referred to as order of precedence.

When we are faced with mixed operation, instead of working from left to right, we perform multiplication or division first. Then we can add or subtract. Given the expression 2 + 4 x 3 – 5, the result we get is 9. It is not 13.

When we are faced with mixed operation, including grouping symbols and powers, we apply the standard or universally accepted rule below.

Steps

1. Clear innermost grouping symbol (Bracket)
2. Deal with power, if any from left to right.
3. Multiply and or divide from left to right.
4. Add and or subtract from left to right.
5. Follow steps 1, 2, 3 and 4 if there are still grouping symbols.
6. Follow steps 2,3 and 4 to get your final answer.

Because we deal with order of basic operations within the bracket first, before multiplication and division, then addition and subtraction, the order of operations is often referred to as BODMAS or BOMDAS.

We are actually following BODMAS rule when we perform the mixed operations by following the six steps given above. BODMAS is the acronym for **B**racket, **O**rder of basic operations, **d**ivision, **m**ultiplication, **a**ddition and **s**ubtraction in that sequence.

When there is power involved, the power is evaluated first before the other four basic operations in multiplication and division, and addition and subtraction.

When there is more than one operation enclosed in the bracket, multiply or divide first, then add and or subtract. Say given (2 + 3 x 6), we multiply (no division) so we get (2 + 18). Now there is only one operation left which is addition. We now add. Thus the expression (2 + 3 x 6) becomes or is equal to 20.

If the bracket contains integers, the positive and negative numbers, we need to be more cautious. Our multiplication follows rules to obtain product of integers, likewise our division follows rules for division of integers. When that is dealt with, rules for addition and or subtraction of integers applies.

Say given ($^-2$ + $^-3$ x $^+6$), we get ($^-2$ + $^+18$) by first multiplying. We are now faced with addition of a negative and a positive number. Since the negative number is smaller, the sum should be positive, thus we get $^+16$.

Some books may use these terms below to explain the BODMAS rule. Only some of the terms were used in this book. BUT you will come across these terms when dealing with polynomial function.

Augend + addend = sum
9 + 5 = 14
9 is augend, 5 is the addend and 14 is the sum.

Summand + summand = total
$^-6$ + 8 = 2
Both $^-6$ and 8 are summands, 2 is the total.

Minuend – subtrahend = difference
10 – 4 = 6
10 is the minuend, 4 is the subtrahend and 6 is the difference.

Multiplicand x Multiplier = product
$^-7$ x $^+3$ = $^-21$
Negative 7 is the multiplicand, positive 3 is the multiplier and negative 21 is the product.

Factor x factor = product
$^-8$ x $^+5$ = 40
Both negative 8 and positive 5 are factors, 40 is the product.

Dividend ÷ divisor = quotient
$^-30$ ÷ 5 = $^-6$
Negative 30 is the dividend, 5 is the divisor and negative 6 is the quotient.

Dividend ÷ divisor = quotient + remainder
43 ÷ 8 = 5 R3
43 is the dividend, 8 is the divisor, 5 is the quotient with remainder 3.

Now study the examples and follow through. You can use the terms above to identify digits in their respective order to help you follow BODMAS rule.

Example 1
Evaluate: $^{-}3 + 6 \times {}^{-}7 - {}^{-}7$

Solution:

$^{-}3 + 6 \times {}^{-}7 - {}^{-}7$	No grouping symbol, go to step 3.
$= -3 + {}^{-}42 - {}^{-}7$	Now add and subtract.
$= {}^{-}45 + 7$	$\lvert -45\rvert = 45$, $\lvert 7\rvert = 7$, $45 - 7 = 38$
$= {}^{-}38$	Negative, since number with greater absolute value is negative.

Example 2
Evaluate: $-3 + 6 \times (-7 - {}^{-}7)$

Solution:

$^{-}3 + 6 \times ({}^{-}7 - {}^{-}7)$	Clear grouping symbol.
$= -3 + 6 \times 0$	Go to Step 3
$= {}^{-}3 + 0$	Go to Step 4
$= {}^{-}3$	

Example 3
Evaluate: $2[-3 + 6 \times (-7 - {}^{-}7) + -2]$

Solution:

$2[-3 + 6 \times (-7 - {}^{-}7) + -2]$	Clear innermost grouping symbol.
$= 2[-3 + 6 \times 0 + -2]$	Go to Step 3 (no power to deal with).
$= 2[-3 + 0 + -2]$	Go to Step 4.
$= 2[-5]$	Clear grouping symbol.
$= -10$	

Alternative rule that maintains the same order can be used is **PEMDAS**. PEMDAS is an acronym for parenthesis, exponents, multiplication and division, and addition and subtraction.

Example 4
Evaluate: $5 + (4 - 2)^2 \times 3 \div 6 - 1 =$

Solution:

$5 + (4 - 2)^2 \times {}^{-}3 \div {}^{-}6 - 1$	$= 5 + 2^2 \times {}^{-}3 \div {}^{-}6 - 1$	Clear Parenthesis
	$= 5 + 4 \times {}^{-}3 \div {}^{-}6 - 1$	Power or Exponent
	$= 5 + 2 - 1$	Multiply and divide
	$= 6$	Add and subtract

Either you apply BODMAS or PEMDAS, the sequence in performing or executing operations is still the same.

NOW DO THE ACTIVITY 17

Practice Exercise 17

Go step by step by BODMAS or PEMDAS rules to evaluate the following:

1. $8 – (^{-}4)^2 - 5$

Answer: _______________

---

2. $^{-}3 – 6\ (^{-}2) - 2$

Answer: _______________

---

3. $7•2 – 5•3$

Answer: ________________

---

4. $(^{-}3)^2 •(5 - 7)^2 – (^{-}9) \div 3$

Answer: ________________

---

5. $4•5 – 10 - 2(1- 2) + {}^{-}5$

Answer: _________________

---

6. $4 \times (12 \times 6 - 4^2) + 9$

Answer: _________________

7. $2 \times (9 \times 5 + 3^2) + 4$

Answer: ___________________

8. $(6 + 3)^2 + (9 - 10 \div 5)$

Answer: ___________________

9. $(9 + 33 - 6) \div 6 - 3^2$

Answer: ___________________

10. $12 \div 3 \times 12 + 10$

Answer: ___________________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

Summary

- **Integer** is a subset of rational numbers. It is and infinite set containing negative and positive numbers and zero. Directed numbers are subset of integers.
- **Rational** numbers are subset of real numbers.
- **Real** numbers is a subset of Complex numbers.

- **Associative** property holds true for multiplication and addition.
- **Distributive** property holds true for multiplication over addition and subtraction.
- **Commutative** property holds true for multiplication and addition.

- **Augend + addend = sum**
- **Summand + summand = total**
- **Minuend – subtrahend = difference**
- **Multiplier x Multiplicand = product**
- **Factor x factor = product**
- **Dividend ÷ divisor = quotient**
- **Dividend ÷ divisor = quotient + remainder**

- **Multiples** of a number X are all numbers that X can divide into exactly.
- **Factors** of a number Y are all numbers that can divide into Y exactly.
- **HCF** is the highest common factor: for 12 = {12, **6**, 4, 3, 2, 1} and 18 = {18, 9, **6**, 3, 2, 1} 6 is the greatest factor that can divide into 12 and 18 exactly.
- **GFC** is the synonym for HCF
- **LCD** is the synonym for LCM. LCD is only used when dealing with denominators of fractions. Otherwise, LCM is commonly required.

- **BODMAS** is acronym for the order of operation: bracket, power, division, multiplication, addition and subtraction.
- **PEMDAS** is acronym for parenthesis, exponent, multiplication, division, addition and subtraction.

Answers to Practice Exercises 13 – 17

Practice Exercise 13

1.	$2\sqrt{7}$	6. (2 + -3) + 5 = 2 + (-3 + 5)
2.	a	7. 20
3.	Set B	8. B
4.	17	9. D
5.	A	10. C

Practice Exercise 14

1. 0	6. - 160
2. -22	7. 14
3. 21	8. -1
4. 7	9. 12
5. -36	10. 11

Practice Exercise 15

1.	60	6. 420
2.	150	7. 0
3.	-147	8. 280
4.	-128	9. 16
5.	60	10. 0

Practice Exercise 16

1. 11	6. b
2. - 8	7. 0
3. 15	8. -3R -3
4. -9	9. R- 3
5. 25	10. 80

Practice Exercise 17

1.	- 13	6. 233
2.	7	7. 112
3.	-1	8. 88
4.	39	9. - 3
5.	7	10. 58

TOPIC 4

NUMBER SEQUENCE AND PATTERNS

TOPIC 4: NUMBER SEQUENCE AND PATTERNS

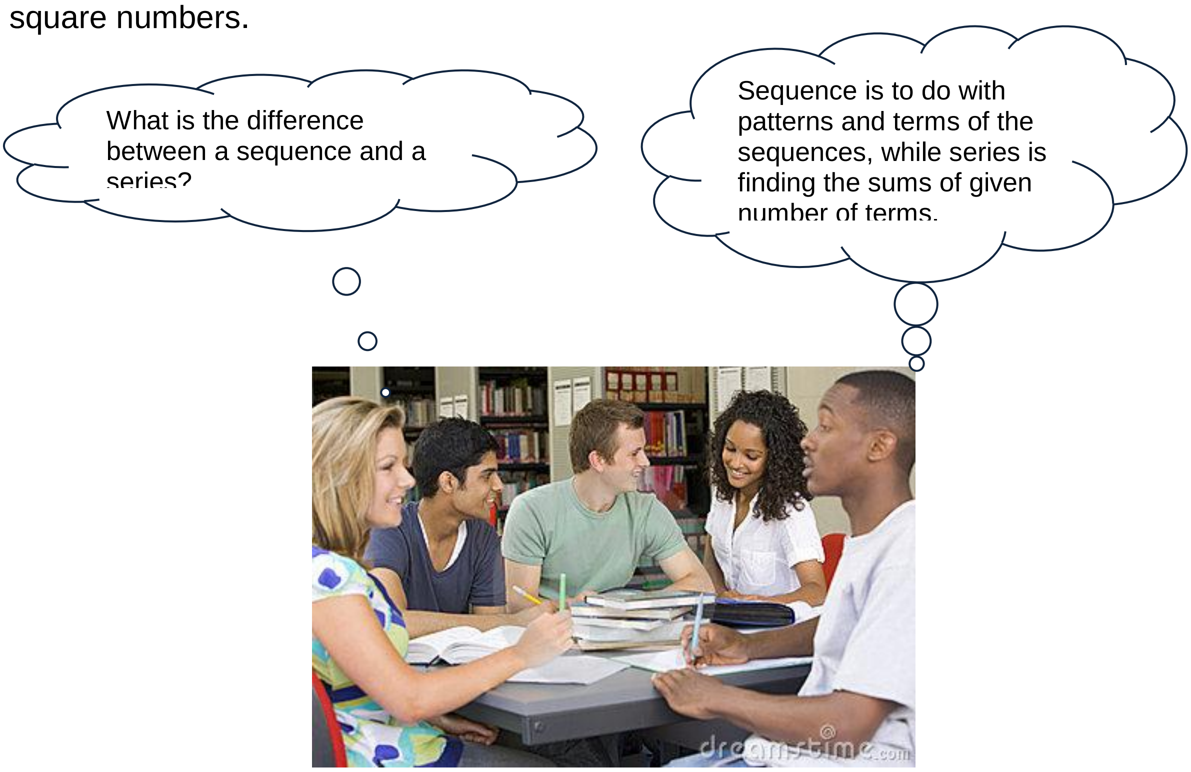
Introduction

You have done problems involving numbers, estimation and directed numbers.

In this topic we will study patterns and number sequence. Some sequences can be demonstrated by regeneration of shapes based on the pattern, while other patterns can be demonstrated by linear or exponential graphs.

Our ability to identify patterns, can enable us to solve queries arising from set building blocks, archaic structures, predicting scenarios based on current trends, and interpreting graphs of trends compiled and displayed graphically.

We will learn the knowledge and skills in patterns and number sequence through number sequences, rules for sequences, varying sequences, graphs and graphs of square numbers.

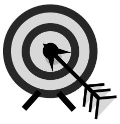


Generally, sequences are categorized as either arithmetic or geometric. The sequence is arithmetic if the pattern is addition of integer, and the sequence is geometric if the pattern is multiplying by and integer, inclusive of powers.

> In this topic, you will have a chance to revisit patterns and number sequence and be able to use them where appropriate.

Lesson 18: Sequences

You have learned about the properties of whole numbers and work out further problems by using these properties.



In this lesson, you will

- Define pattern
- define sequence
- .identify common types of sequences.

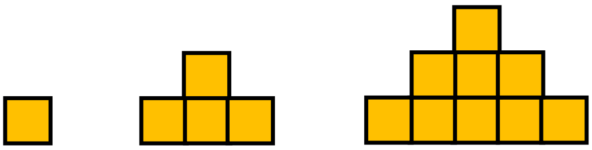
Sequences are generally arithmetic or geometric. However, there are some sequences which do not have a fixed pattern to generate arithmetic of geometric sequences. Sequence is number of things, actions or events arranged or happening in some form of specific order or having a specific connection.

Patterns

A pattern is a regular or repetitive form, order or arrangement of behaviour, objects or numbers. In our study, we will be more concerned with patterns in alignment of objects and number sequence.

Sequence is the order in which the numbers or blocks are arranged based on the pattern. Succession of numbers and building blocks in a sequence have a pattern but may not have a single rule, and may have two rules applied concurrently.

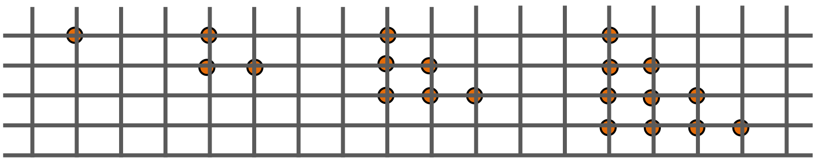
Study the square blocks and identify the pattern.



The pattern here is +1, + 3, + 5. Each time a successive odd number of square blocks is added. And the difference between each successive pattern are 3 and 5. Each pattern is a term of the sequence {1, 4, 9, …}.

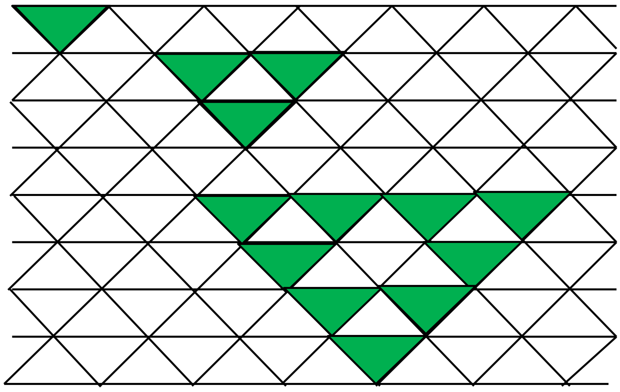
Pattern in this case is used to mean the additional square blocks added and at the same time the individual set of blocks. So when we ask, “ what is the pattern?”, often we mean both. But, when we ask about the pattern in a sequence of numbers, often we mean the rule or what is added to the preceding number to obtain the next number in a sequence.

Below is another pattern. The pattern is a sequence of triangles from the second onward.



If we stop regenerating triangles then the pattern will have the set of sequence as {1, 3, 6, 10}. So starting with one dot, we added 2, then 3, then 4 dots. We were adding consecutive numbers of dots beginning with 2.

Number sequences can be identified by using blocks or grid paper and dots. Using square grid or isometric graph papers, you can develop so many patterns. Below is an example of another pattern drawn on isometric graph paper.

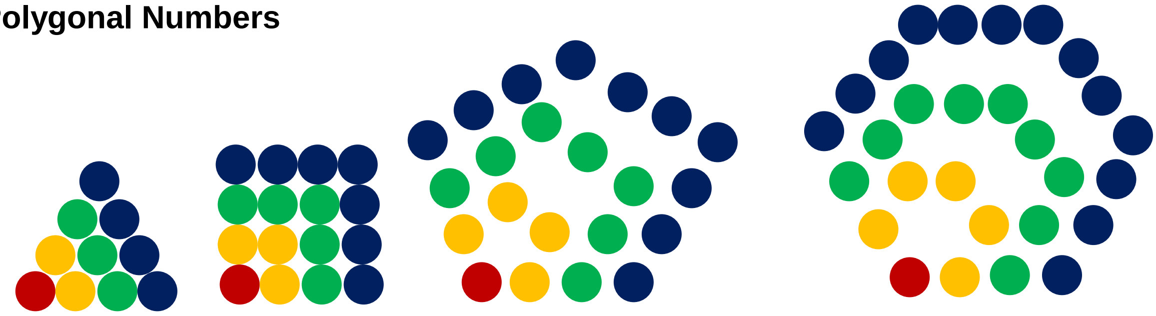


The patterns here give the sequence {1, 3, 9, ..}. The next pattern will have 27 triangles. This will happen as long as we maintain the position and form a complete triangle each time.

A **pattern** is a repetition of event or objects in a particular manner.

Our definition here, may differ to other fields within mathematics such arithmetic and logic and so on.

Polygonal Numbers



Triangular Square Pentagonal Hexagonal

Triangular: 1, 3, 7, 10

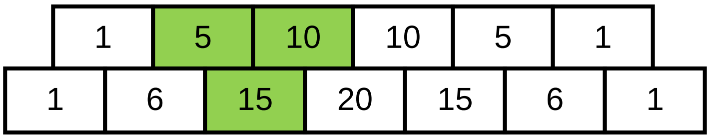
Square : 1, 4, 9, 16

Pentagonal : 1, 5, 12, 22

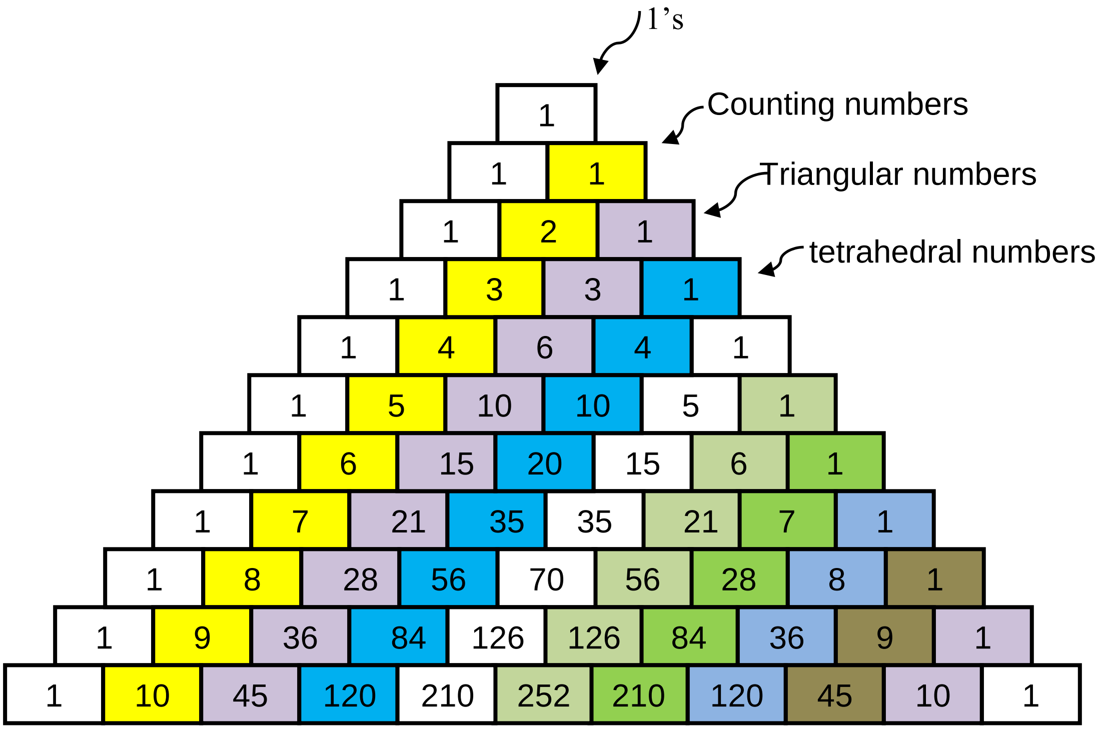
Hexagonal: 1, 6, 15, 28

Patterns in Pascal's Triangle

One of the interesting number patterns is Pascal's Triangle (named after Blaise Pascal, a famous French mathematician and philosopher). Pascal's Triangle has rows one and two, and all the outer boxes with 1. The other boxes are filled by finding the sum of two boxes above it.



Below is a Pascal's Triangle.



The first diagonal generates the 1's. The second, gives the counting numbers. The third, gives the Triangular Numbers. And the fourth, the tetrahedral numbers.

There are also other patterns such as $4^2 = 6 + 10$, $5^2 = 10 + 15$, $6^2 = 15 + 21$, and more if you follow the pattern to obtain the three digits.

The sums of the numbers in any row is equal to 2 to the n^{th} power (2^n).

$2^0 = 1$

$2^1 = 1 + 1 = 2$

$2^2 = 1+2+1 = 4$

$2^3 = 1 + 3 + 3 + 1 = 8$

$2^4 = 1 + 4 + 6 + 4 + 1 = 16$

Binomial Theorem: $(a + b)^n$

Binomial expressions to a higher power of two can easily be found by using nth row of the Pascal's Triangle.

$(a + b)^3 = a^3 + \mathbf{3}a^2b + \mathbf{3}ab^2 + b^3$	1 3 3 1
$(a + b)^4 = a^4 + \mathbf{4}a^3b + \mathbf{6}a^2b^2 + \mathbf{4}ab^3 + b^4$	1 4 6 4 1
$(a + b)^5 = a^5 + \mathbf{5}\, a^4b + \mathbf{10}a^3b^2 + \mathbf{10}a^2b^3 + \mathbf{5}ab^4 + b^5$	1 5 10 10 5 1
$(a + b)^6 = a^6 + \mathbf{6}a^5b + \mathbf{15}a^4b^2 + \mathbf{20}a^3b^3 + \mathbf{15}a^2b^4 + \mathbf{6}ab^5 + b^6$	1 6 15 20 15 6 1

This is quite difficult to follow, but mastering the pattern now will help you in your later study in advanced algebra and mathematics.

Prime Numbers

If the first element in a row is a prime number (0^{th} element of every row is 1) all the other numbers in that row (excluding the 1's) are divisible by it. Say row 7 (1 7 21 35 35 21 7 1), 7, 21 and 35 are all divisible by 7.

Fibonacci Sequence

Fibonacci sequence can be found by adding diagonals to give the sequence 1, 1, 2, 3, 5, 8, ….

Magic 11's

Each row is equal to 11^n when n is the number of row. You will find that the last three digits in multi-digit number is equal to the actual row in Pascal's Triangle.

Row Number	Formula	Multi-Digit number	Actual Row
Row 0	11^0	1	1
Row 1	11^1	11	11
Row 2	11^2	121	121
Row 3	11^3	1331	1331
Row 4	11^4	14641	14641
Row 5	11^5	161051	15101051
Row 6	11^6	1771561	1615201561
Row 7	11^7	19487171	172135352171
Row 8	11^8	214358881	18285670562881

Sequence

A **sequence** is a list of terms or numbers arranged in a definite order.

For number sequences, let us investigate **Fibonacci Sequence**. Fibonacci sequence has {1, 1, 2, 3, 5, 8, 13, 21, 34, 55} as the first ten terms. To find the next term, work out the sum of the two preceding terms.

Suppose we divide each term by the preceding term, we will have this set of numbers {1,2, 1.5, 1.666.., 1.6, 1.625,1.61538.., 1.61904.., 1.6176.. }. Interestingly, the more terms we add and find their respective ratios based on the pattern, we tend to obtain scores getting closer and closer to 1.618 033 9… or approximately 1.618 034, the **Golden Ratio**.

That is, say,

$1 \div 1 = 1$

$2 \div 1 = 2$

$3 \div 2 = 1.5$

$5 \div 3 = 1.666666\ldots$

$8 \div 6 = 1.333333\ldots.$

$13 \div 8 = 1.625$

$21 \div 13 = 1.615384615384615$

$34 \div 21 = 1.619047619047619$

$55 \div 34 = 1.617647058823529$

Fibonacci sequence relates to nature. The sequence can be observed on flowers, vegetables, trees , family tree and so on.

Lukas Numbers were developed by a Mathematician called Lukas based on Fibonacci numbers. An example of a Lukas numbers is 3, 1, 4, 5, 9, 14, 23, 37. Another is 3, 3, 6, 9, 15, 24, 39, 53.

Again, suppose we use the pattern “divide each term by the preceding term” , we will get a series, getting closer and closer to the Golden ratio.

Generally, patterns in sequence can be of an arithmetic or of a geometric progression, and an **explicit rule** can be formulated.

A sequence that does not contain one of the two patterns will require a **recursive rule** to determine their next term. So you look forward to deriving or using one of the three forms of rules to determine terms:

$\mathbf{a; a_{n-1}}$ recursive rule, or

$\mathbf{T_n = a + (n - 1)d}$ explicit rule for arithmetic sequence or progression, or

$\mathbf{T_n = ar^{n-1}}$ explicit rule for geometric sequence or progression.

Recursive Rule: **a; a $_{n-1}$**

9,

Explicit Rule For Arithmetic Sequence: $\mathbf{T_n = a + (n - 1)d}$

9, 14, 19, 24, 29, 34, …

The first term **a** is 9. Common difference **d** is found by subtraction of successive terms. 14 - 9, 19 – 14, 24 – 19, and so on. And **n** is the nth term. To derive explicit rule for T_n, we substitute for **a** and **d.** thus we get:

T_n, = 9 + (n - 1)5

Clearing grouping symbol and simplifying we progress as:

T_n, = 9 + (n - 1)5 = 9 + 5n – 5 = 5n + 4

So the explicit rule for the arithmetic sequence is T_n, = 5n + 4

Explicit Rule For Geometric Sequence: $\mathbf{T_n = ar^{n-1}}$

9, 18, 36, 72, 144, …

The first term **a** is 9. The common ratio **r**, found by multiplying preceding term to get the next term is constant. That is, each term is multiplied by a fixed multiplier to get the next term as follow 9 x **2** = 18, 18 x **2** = 36, 36 x **2** = 72 and so on. We can see that the multiplier is 2. To derive explicit rule for $\mathbf{T_n}$ **,** we substitute for **a** and **r,** thus we get:

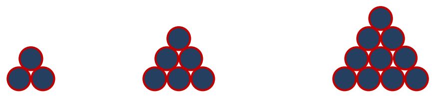
$$
\begin{aligned}
T_n &= ar^{n-1} \\
&= 9 \times (2^{n-1}) \\
&= 9(2^{n-1})
\end{aligned}
$$

So the explicit rule for the geometric sequence is T_n, = $9(2^{n-1})$

NOW DO PRACTICE EXERCISE 18

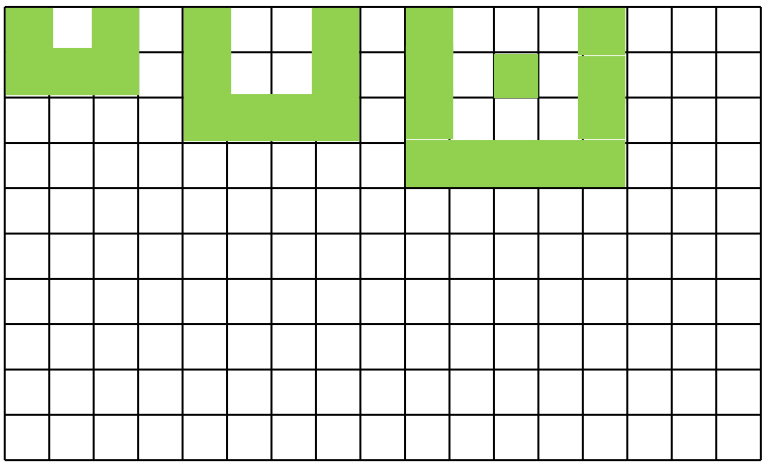
Practice Exercise 18

1. Study the sequence of patterns below and then draw the next pattern.



Answer: __________

2. Draw the next pattern in the grid space below.



Answer: __________

3. Find the pattern and write the next three terms of the Lukas Numbers.

5, 5, 10, 15, 25, __ , __, __

Answer: __________

4. Write the next three terms of the sequence:

17, 15, 18, 16, 19, 17, 20, 18, 21, __ , __ , __ .

Answer: __________

5. What is the pattern of the sequence below?

-7, -2, -4, 1, -1, 4, 2, 7, 5, 10, 8.

Answer: ________________

---

6. Study Pascal's Triangle and write the missing three terms.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 $\boxed{}$ 21 35 $\boxed{}$ $\boxed{}$ 7 1
1 8 28 56 70 56 28 8 1

Answer: __________________

---

7. The table below shows first 9 triangular to decagonal numbers. Add two successive terms of each row, each time, for each sequence and state which polygonal numbers give a sum that is a square number.

Name	n = 1	n =2	n = 3	n = 4	n = 5	n = 6	n = 7	n = 8	n = 9
a = 3	1	3	6	10	15	21	28	36	45
a = 4	1	4	9	16	25	36	49	64	81
a = 5	1	5	12	22	35	51	70	92	117
a = 6	1	6	15	28	45	66	91	120	153
a = 7	1	7	18	34	55	81	112	148	189
a = 8	1	8	21	40	65	96	133	176	225
a = 9	1	9	24	46	75	111	154	204	261
a = 10	1	10	27	52	85	126	165	232	297

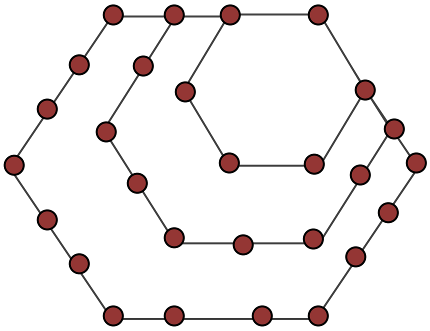
Answer: __________________

8. Given is Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, … If we divide each term by the term before it, we will develop a series of numbers. Find the ratio of the fifth and the sixth terms.

Answer: ________________

---

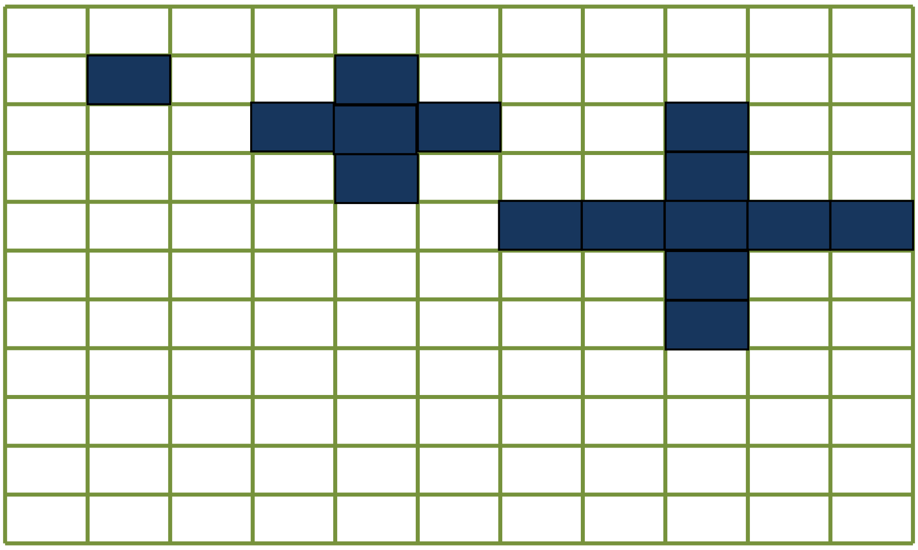
9. Following the pattern, how many dots will the next hexagon hold?



Answer: ________________

---

10. Write the first 5 terms of the sequence given by the patterns.

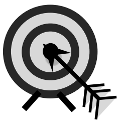


Answer: ________________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 19: A Rule for a Sequence

You have learned about the patterns and number sequence in your previous lesson..



In this lesson, you will

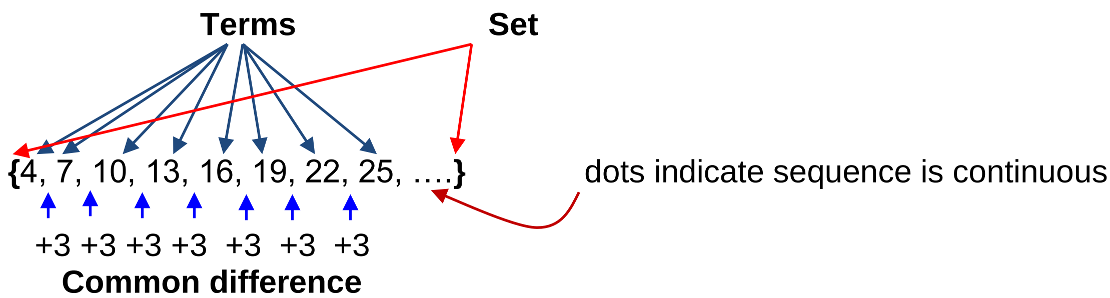
- Calculate and identify the pattern of a certain sequence
- Formulate a rule that fits all results of the identified pattern.

We will try to derive rules for the arithmetic sequence and geometric sequence based on the patterns defined the the terms of a given sequence.

Arithmetic Sequence

In a number sequence, each number is called a **term** of the sequence. The sequence is an arithmetic sequence if the successive term is found by adding or subtracting a fixed number. The fixed number is called the **common difference** of the sequence.

Study the sequence and descriptions below.



So the given sequence begins with 4 as the first term. But the set of the sequence is infinite, it continues. The common difference is 3. We subtract the first term from the second, the second from the third and so on, to find if there exists a common difference. If there is a common difference, then the sequence is arithmetic sequence.

Suppose we define n to be the n^{th} term and T to be the term in general, then T_n specifies the term we are interested in. Using the example above we can arrange as:

n	1	2	3	4	5	6	7	8
T	4	7	10	13	16	19	22	25

We can now write a explicit formula for the arithmetic sequence. The arithmetic sequence obeys the computing of each term as $\mathbf{T_n = a + (n - 1)d}$. The formula tells us that succeeding term is calculated from the previous term. Say, from the table, the second term 7, is calculated from the first term which is 4, and so on. And **a** represents the first term in a given arithmetic sequence, **n** the n^{th} term, **d** the common difference and **T** the term we need to find. In general, we could write arithmetic sequence like this:

{a, a + d, a + 2d, a + 3d, a + 4d, …}

Now using a = 4, and d = 3 we can derive a general rule for the sequence. Substituting into $\mathbf{T_n = a + (n - 1)d}$, we get $T_n = 4 + (n - 1)3$. Simplifying the RHS, we follow as:

$$
\begin{aligned}
T_n &= 4 + (n - 1)3 \\
&= 4 + 3n - 3 \\
&= 1 + 3n \\
T_n &= 3n + 1
\end{aligned}
$$

We can now calculate the 9^{th} term and onwards using the rule or formula. Let us calculate the 10^{th} term of the sequence. That is, a = 4, d = 3 and n = 10.

$$
\begin{aligned}
T_{10} &= a + (n - 1)d \\
&= 4 + (10 - 1)3 \\
&= 4 + (9)3 \\
&= 4 + 27 \\
&= 31
\end{aligned}
$$

So the 10^{th} term of the sequence is 31.

Two or more sequences may have the same rule, but their pattern will be different. Their pattern will be based on the first term **a** of the sequence.

Example 1

Find the general rule in the given sequence {-6, -2, 2, 6, 10,14, 18 …}.

Solution:

n	1	2	3	4	5	6	7
T_n	-6	-2	2	6	10	14	18

Common difference: -2 – -6 = 4, 2 - - 2 = 4, 6 - 2 = 4, so the common difference is 4. First term: a = - 6

General Rule: $T_n = a + (n - 1)d$

$= - 6 + (n - 1)4$

$T_n = - 6 + 4n - 4$

$T_n = 4n – 10$

$T_{10} = 5 – 3(10) = 5 – 30 = - 25$

Therefore the general rule is $T_n = 4n - 10$.

Example 2

Find the general rule in the given sequence and calculate the 10th term of the sequence {2, -1, -4, -7, -10, -13, …}.

Solution:

n	1	2	3	4	5	6
T_n	2	-1	-4	-7	-10	-13

Common difference: -1 – 2 = -3, - 4 - - 1 = - 3, -7 - - 4 = -3, so the common difference is – 3.

First term: a = 2

General Rule: $Tn = a + (n - 1)d$

$= 2 + (n - 1)-3$

$Tn = 2 + -3n + 3$

$T_n = 5 – 3n$

$T10 = 5 – 3(10) = 5 – 30 = - 25$

Therefore the general rule is $T_n = 5 – 3n$ and the 10th term is – 25.

The Sum of Arithmetic Series

The sum of the terms (series) in an arithmetic sequence is found by adding:

$a + (a + d) + (a + 2d) + (a + 3d) +(a + 4d) + \;….+ (a + (n - 1)d)$

For short, we use the formula $\mathbf{S_n = \frac{1}{2} n [2a + (n -1)d]}$. Where $\mathbf{S_n}$ is the sum of the terms, **a** is the first term, **d** is the common difference and **n** is the nth term.

The rule is derived by adding

a + (a + d) + (a + 2d) + (a + 3d) +(a + 4d) + ….+ (a + (n - 1)d) to its reverse order.

$S = a + (a + d) + \ldots + (a + (n - 1)d$
$S = (a + (n - 1)d + \ldots + a + (a + d)$

By algebraic manipulation you will get

$2S = n (2a + (n - 1)d)$

We divide by 2 since we are finding S (not 2S), so we get $\mathbf{S = \frac{1}{2} n [2a + (n -1)d]}$.

Example 3
Given the sequence {4, 9, 14, 19, 24, …} find:
(a) The 8^{th} term.
(b) The sum of the eight terms.

Solution:

(a) a = 4, d = 5 ;

$$\begin{aligned} T_n &= 4 + (n - 1)5 \\ T_8 &= 4 + (8 - 1)5 \\ &= 4 + (7)5 \\ &= 39 \end{aligned}$$

Therefore, the 8^{th} term is 39.

(b) a = 4, d = 5, n = 8

$$\begin{aligned} S_8 &= \frac{1}{2}.8\ [2.4 + (8 -1)5] \\ &= 4\ [8 + (7)5] \\ &= 4\ [8 + 35] \\ &= 4\ [43] \\ &= 173 \end{aligned}$$

Therefore, the sum of first 8 terms is 173.

The 8 terms are 4, 9, 14, 19, 24,29, 34, 39. Try adding these numbers with or without a calculator, you will still get the sum of 173.

If we are given the first term, **a**, the n^{th} term and the common difference, **d**, without the sequence, we can still compute the n^{th} term and the n^{th} series.

Example 4

A certain sequence has 6 as the first term, and a common difference of 4. Find :

(a) The 10th term.

(b) The sum of the eight terms.

Solution:

(c) $a = 6$, $d = 4$; $T_n = 6 + (n - 1)4$

$T_{10} = 6 + (10 - 1)4$

$= 6 + (9)4$

$= 40$

Therefore, the 10th term is 40.

(d) $a = 6$, $d = 4$, $n = 8$ $S_8 = \frac{1}{2}.8\,[2.6 + (8 - 1)4]$

$= 4\,[12 + (7)4]$

$= 4\,[12 + 28]$

$= 4\,[40]$

$= 160$

Therefore, the sum of first 8 terms is 160.

Geometric Sequence

The sequence is an geometric sequence if the successive term is found by multiplying (or dividing) a fixed number. The fixed number is called the **ratio** of the sequence.

Study the sequence below.

3, 6, 12, 24, 48

So the given sequence begins with 3 as the first term.. The common ratio is 2. We multiply the first term by 2, to get the second term, the sec. If there is a common difference, then the second term by 2 to get the third term.

Suppose we define n to be the n^{th} term and T to be the term in general, then T_n specifies the term we are interested in. Using the example above we can arrange as:

n^{th} term	1	2	3	4	5
Term	3,	6,	12,	24,	48

So the terms were generated as:

3, **3** x 2, **3** x 2 x 2, **3** x 2 x 2 x 2, **3** x 2 x 2 x 2 x 2

Which can be algebraically expressed as:

$\mathbf{a}$, $\mathbf{a}r$, $\mathbf{a}r^2$, $\mathbf{a}r^3$, $\mathbf{a}r^4$

We can observe that the power of **r** is one less than the terms' position, **n**, in the sequence. So the power of **r** can be generalized as (**n - 1**).

So we can now express the rule to obtain the n^th^ term as $\mathbf{T_n = ar^{n-1}}$.

Example 5

Find the 6^{th} term of the geometric sequence 2, 6, 18. 54, ….

Solution:

In 2, 6, 18. 54, …., a = 2, r = 3

$T_6 = \mathbf{ar^{n-1}}$.

$T_6 = 2 \times 3^{(6-1)}$

$= 2 \times 3^5$

$= 2 \times 241$

$= 482$

This example is quite hard to follow when you do not have a calculator.

Example 6

Find the common ratio of the geometric sequence with the first term a = 3 and the fourth term $T_4 = 192$.

Solution:

GP {3___,___,192, …} a = 3, r = ? and $T_4 = 192$

$192 = \mathbf{3r^{4-1}}$.

$192 = 3r^3$

$192 \div 3 = r^3$

$64 = r^3$

$r^3 = 64$

$r = \sqrt[3]{64}$ **So the common ratio is 4.**

$r = 4$

In the example 5, the explicit rule for the GP sequence can now be stated as

$T_n = 3\,(4)^{n-1}$ or $T_n = 3 \times 4^{n-1}$.

We can use the formula to find the 5th and 6th terms, and onward.

Series

To find the sum of a certain number of terms of a geometric sequence (partial sum), we use the rule:

$\mathbf{S_n} = \frac{a(1-r^n)}{1-r}$**,** where $\mathbf{S_n}$ is the sum of nth terms, **a** is the first term and **n** is the nth term.

Example 6

Find the sum of the first 8 terms geometric sequence 2, 6, 18. 54, ….

Solution:

In GP {2, 6, 18. 54, ….}, a = 2, r = 3 , S_8

$$S_8 = \frac{a(1-r^n)}{1-r}$$

$$S_8 = \frac{2(1-3^8)}{1-3}$$

$$= \frac{2(1-6561)}{-2}$$

$$= -1(-6560)$$

$$= 6560$$

The partial sum of the of the geometric series is 6560.

The Arithmetic Progression has the rules

$T_n = a + (n - 1)d$

$S_n = ½\, n\,(2a + (n - 1)d)$

The Arithmetic Progression has the rules

$T_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r}$$

NOW DO PRACTICE EXERCISE 19

Practice Exercise 19

1. What is the first term in the arithmetic sequence 12, 6, 7, 3.5, 4.5, …

Answer: __________

2. What is the common difference in the sequence {4, 2, 0, -2, -4, -6}

Answer: __________

3. What is the 5th term of the sequence -8, -3, 2, 7, 12, 17, 22, 27, 32,..

Answer: __________

4 Find the pattern and complete the sequence {__, 15, 23, __, 31, __, 47, 55}

Answer: __________

5. Find the sum of the first 10 terms of the sequence 5, 8, 11, 14, ..

Answer: ________________

6. Calculate the 8th term of the geometric sequence 2, 4, 16, 64, 254, ….

Answer: ___________________

7. What is the common ratio of the geometric sequence – 2, 4, - 8, 16, -32, 64, ..

Answer: ___________________

---

8. Calculate the sum of the first 8 terms of the geometric sequence when the first term is 3 and the common ratio is 3.

Answer: ___________________

---

9. Is the sequence 200, 300, 400, 500, 600, .. arithmetic or geometric?

Answer: ___________________

---

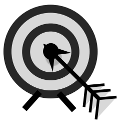
10. How many terms of the sequence – 6, - 1, 4, 9, … will add up to 63?

Answer: ___________________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 20: More Sequences

You have learned about deriving rules or formula of a given arithmetic and geometric sequences.



In this lesson, you will

- Calculate and identify the pattern of certain sequence
- Formulate a rule that fits all the results of identified pattern

Certain sequences (not all) can be defined (expressed) in a "recursive" form. In a **Recursive Formula**, each term is defined as a function of its preceding term. Recursive forms work with term(s) immediately in front of the term being examined. A recursive formula is written with two parts: a statement of the first term along with the formula relating successive terms.

> A recursive formula designates the starting term, a_1, and the n^{th} term of the sequence , a_n, as an expression containing the previous term (the term before it), a_{n-1}.

The process of recursion can be thought of as climbing a ladder. To get to the third rung (step), you must step on the second rung. Each rung on the ladder depends on the rung below it.

The first rung is a_1, then the second rung is a_2. Thus $a_2 = a_1$ + "step up". And $a_3 = a_2$ + "step up", and $a_4 = a_3$ + "step up" and so on. Hence we can generalize as $\mathbf{a_n = a_{n-1}}$ **+ step up** when you are on the n^{th} rung**.**

The table below gives statements naming the same sequence.

Given Term	Term in front of Given Term
a_4	a_3
a_n	a_{n-1}
a_{n+1}	a_n
a_{n+4}	a_{n+3}
$f(6)$	$f(5)$
$f(n)$	$f(n - 1)$
$f(n+1)$	$f(n)$

To find recursive formula for an **arithmetic** sequence:

1. Determine if the sequence is arithmetic (is the next successive term found by adding or subtracting the same number)
2. Find the common difference (number used to add or subtract to get the next term)
3. Create a recursive formula by stating the first term, and then stating the formula to be the common difference plus the previous term.

Formula a_1 = first term; $T_n = a_{n-1} + d$ (subscript notation)
or $f(1)$ = first term; $f(n) = f(n - 1) + d$ (function notation)

Example 1

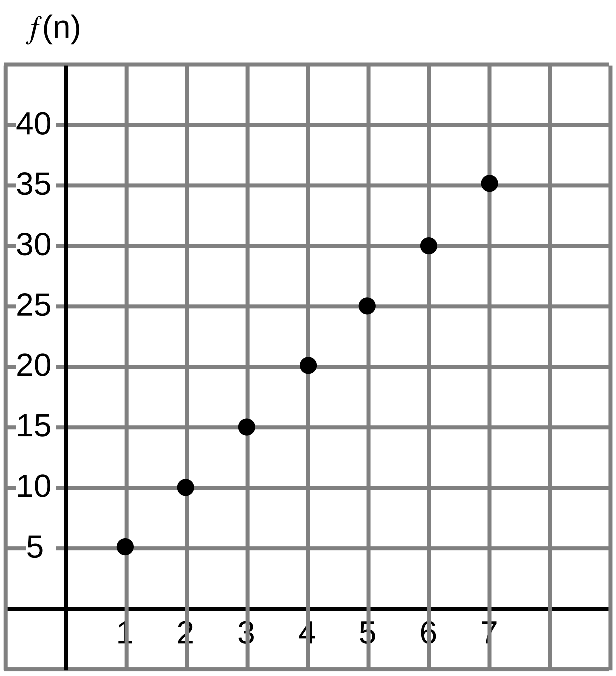
Derive the recursive formula for the sequence {10, 15, 20, 25, 30, 35, …}.

Solution:

Term Number	1	2	3	4	5	6	n
Term	10	15	20	25	30	35	…
Subscript Notation	a_1	a_2	a_3	a_4	a_5	a_6	a_n
Function Notation	$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$	$f(n)$

Formula $a_1 = 10$; and $T_n = a_{n-1} + 5$ (subscript notation)
$f(1) = 10$; and $f(n) = f(n - 1) + 5$ (function notation)

We can illustrate the sequence as given below.



To find recursive formula for a **geometric** sequence:

1. Determine if the sequence is geometric (is the next successive term multiplied or divided by the previous term by the same number)
2. Find the common ratio (number used to multiply or divide by to get the next term)
3. Create a recursive formula by stating the first term, and then stating the formula to be the common ratio times the previous term.

Formula a1 = first term; $T_n = r \cdot a_{n-1}$
or $f(1)$ = first term; $f(n) = r \cdot f(n - 1)$

Example 2

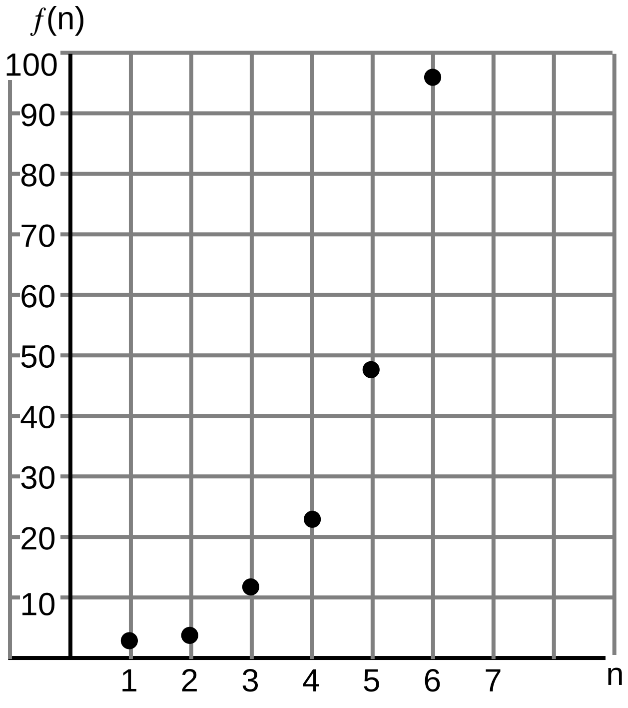
Derive the recursive formula for the sequence {3, 6, 12, 24, 48, 96, …}.

Solution:

Term Number	1	2	3	4	5	6	n
Term	3	6	12	24	48	96	…
Subscript Notation	a_1	a_2	a_3	a_4	a_5	a_6	a_n
Function Notation	$f(1)$	$f(2)$	$f(3)$	$f(4)$	$f(5)$	$f(6)$	$f(n)$

Formula $a_1 = 3$; and $T_n = 2 \cdot a_{n-1}$ (subscript notation)
$f(1) = 10$; and $f(n) = 2 \cdot f(n - 1)$ (function notation)

Graph of geometric sequence.



Example 3
Find the rule for Fibonacci sequence {0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, …}.

Solution:

The sequence is neither arithmetic nor geometric. Simply because, there is neither a common difference nor common ratio.

Study and work out the pattern in the sequence: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ….

The pattern begins after the second term.

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55

0 + 1, 1 + 1, 2 + 1, 3 + 2, 5 + 3, 8 + 5, 13 + 8, …

Formula $a_1 = 0$; $a_2 = 1$; $a_n = a_{n-1} + a_{n-2}$ or $f(n) = f(n - 1) + f(n - 2)$

In this case, it is necessary to state the first and second terms before stating the formula.

We can write either a recursive formula or explicit formula to help us find other terms of sequences. The explicit formula is as given in lesson 19. Where we derive rules of geometric and arithmetic sequences in the forms $T_n = ar^{n-1}$ and $T_n = a + (n - 1)d$ respectively.

Recursive rules help provide to you the basis to be able to use spreadsheet on a computer or as an alternative to explicit rule. It also exposes to you how you can interpret some of the figurate numbers formulas such as Fibonacci numbers and polygonal numbers.

NOW DO PRACTICE EXERCISE 20

Practice Exercise 20

1. Find the recursive rule for the sequence {10, 12, 14, 16, 18, …}.

Answer: __________

2. Find the recursive rule for the sequence {-10, - 8, - 6, - 4, -2, …}.

Answer: __________

3. Find the recursive rule for the sequence {7, 9, 11, 13, 15, …} and find the 7th term of the sequence.

Answer: __________

4. Find the recursive rule for the sequence {-2, 4, - 8, 16, -32, …}.

Answer: __________

5. Find the recursive rule for the sequence {6, 12, 24, 48, 96, ….}.

Answer: ________________

6. Find the recursive rule for the sequence {1, 2, 4, 8, 16, …} and find the 8th term of the sequence.

Answer: __________________

7. What is the common difference in the sequence {- 9, - 5, -1, 3, 7, …}.

Answer: ___________________

---

8. What is the common ratio in the sequence {- 9, 18, - 36, 72, - 144, …}.

Answer: ___________________

---

9. Write the first 5 terms of the sequence when the recursive rule is:
$a_1 = 12$; $a_n = a_{n-1} + 7$

Answer: ___________________

---

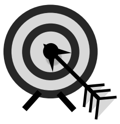
10. Write the first 5 terms of the sequence when the recursive rule is:
$a_1 = 12$; $a_n = 3 \bullet a_{n-1}$

Answer: ___________________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 21: Patterns in Graphs

You have learned about finding recursive rules in arithmetic and geometric sequences.



In this lesson, you will
- Graph linear sequence
- Identify and state the pattern on the linear graph.

The arithmetic sequence is similar to linear sequences. The explicit rule of the sequence is of the form $T_n = a + (n - 1)d$ and linear sequence graphs are of the form $y = c + mx$.

Below is the table showing the relationship.

Sequence Formula	Linear Graph Formula
T_n	y
a	c
d	m
n - 1	x

In linear graph of the form **y = mx + c**, the pattern is defined by the **m** or the slope of the linear graph. The slope **m**, can be any value but **not** 0.

Say, given a table of values such as the one below, the n^{th} term is not specified by any of the two sets of values.

x	2	4	6	8	10
y	7	11	15	19	23

The x-values and y-values are both arithmetic sequences, however, x-values are independent and y-values are dependent on x values.

What we need to do is identify the pattern connecting the two sequences.We are not interested in sequence of x-values nor y-values. We are more concerned with connectivity of the two separate sequences.

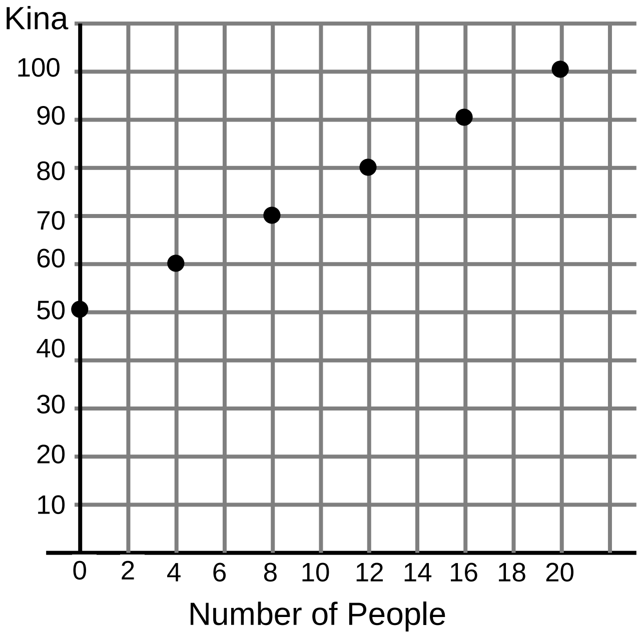
The pattern is found by calculating the slope **m** which is defined by $m = \frac{y_2 - y_1}{x_2 - x_1}$ and **c**, the y - intercept. Y- intercept is the y-value at which the linear graph meets or cuts through the y-axis. Once we have defined or calculated the two values, **m** and **c**, and substitute their values into the standard form y = mx + c, we have derived the explicit formula for the particular sets of numbers. We can evaluate y-value given its corresponding x-value, or evaluate x-value given its corresponding y-value.

The explicit formula for the linear graph can be derived from:

- the table,
- a graph, and
- given two points (coordinates of two points).

Example 1

Write the formula for the linear graph below, describing daily cost per person by a hotel.



Solution:

Sequence of x: 0, 4, 8, 12, 16, 20

Sequence of y: 50, 60, 70, 80, 90, 100

Table of values.

Number of People (x)	0	4	8	12	16	20
Amount in Kina (y)	50	60	70	80	90	100

The minimum cost is more than K50.

Pattern: x increases by adding 4

y increases by adding 10

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{70 - 60}{8 - 4} = \frac{10}{4} = 2.5 \quad ; c = 50$$

Rule: $y = 2.5x + 50$

We can now evaluate cost for accommodating one person to infinite number of people being accommodated. We have derived the explicit rule or formula for the linear graph.

Suppose we write the recursive rule, we will be required to write two sets of recursive rules. One set will be for x-values and the other set for y-values. In this case, the rules are:

$x_1 = 0$; $x_n = x_{n-1} + 4$, and
$y_1 = 50$; $y_n = y_{n-1} + 10$

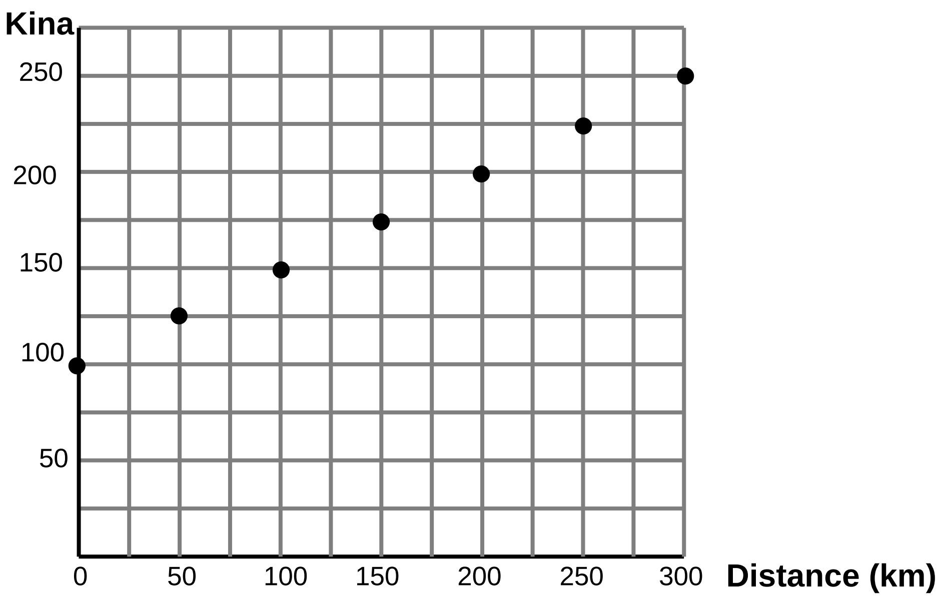
Now, to find the n^{th} term. we have to find the n^{th} term of x-value separate from the n^{th} term of the y-values.

Example 2

A car rental charge is K100 per day plus K0.50 per kilometer travelled within town. Complete a table of values for 300 km travelled on the day and plot its graph.

Solution:

Distance in km	0	50	100	150	200	250	300
Cost in Kina (K)	100	125	150	175	200	225	250



Example 3

When digging into the earth, the temperature rises according to the following linear equation:

$$t = 15 + 0.01h.$$

t is the increase in temperature in degrees and **h** is the depth in metre. Complete the table of values for the first 10 m of depth.

Solution:

h	1	2	3	4	5	6	7	8	9	10
t	15.01	15.02	15.03	15.04	15.05	15.06	15.07	15.08	15.09	15.1

Example 4

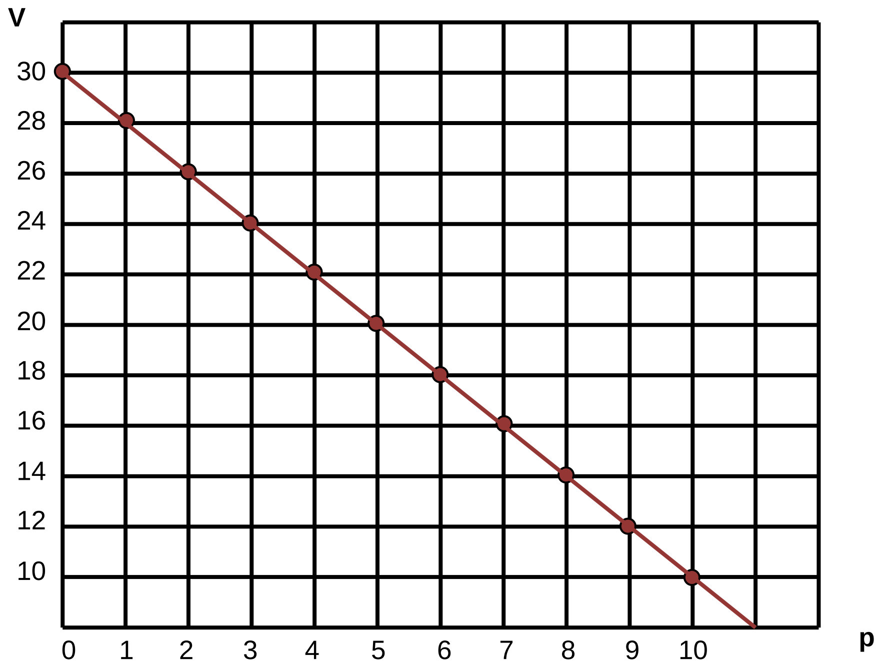
Volume of tank decreases according to the following linear equation:

$$V = 30 – 2p.$$

V is the decrease in volume in litres and **p** is the number of people. Complete the table of values for the first 10 people's use of water. Then plot the graph of the problem.

Solution:

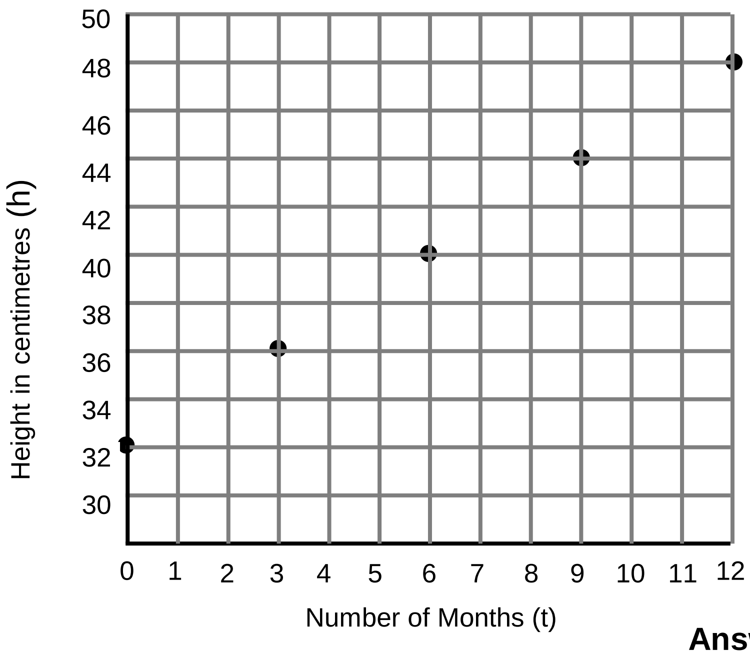
h	0	1	2	3	4	5	6	7	8	9	10
t	30	28	26	24	22	20	18	16	14	12	10



NOW DO PRACTICE EXERCISE 21

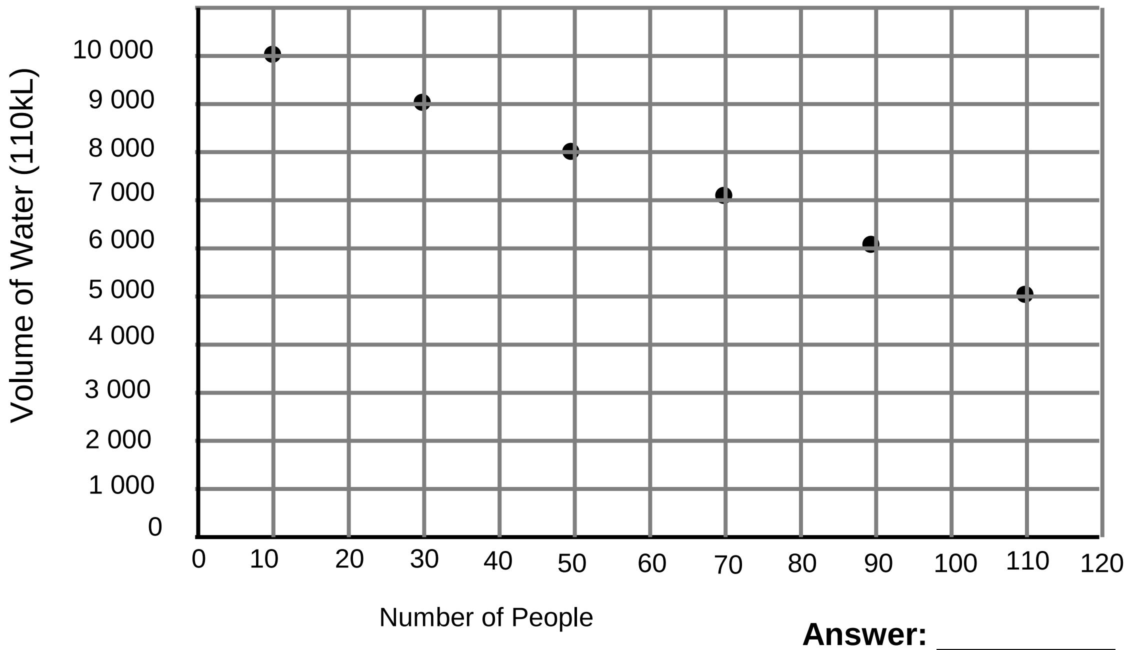
Practice Exercise 21

1. Write equation for the given linear graph of a plant growth.



Answer: ___________

2. Write linear equation of the graph below using **k** for kilolitres and **n** for number of people.



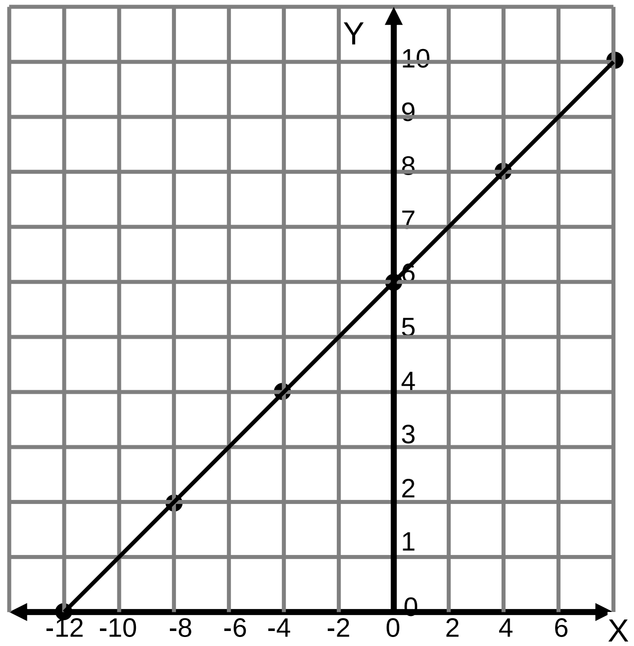
Answer: ___________

3. Derive a linear equation for the given table of values.

Distance (d)	0	3	7	12	15	20
Cost (K)	200	230	270	320	350	400

Answer: ___________

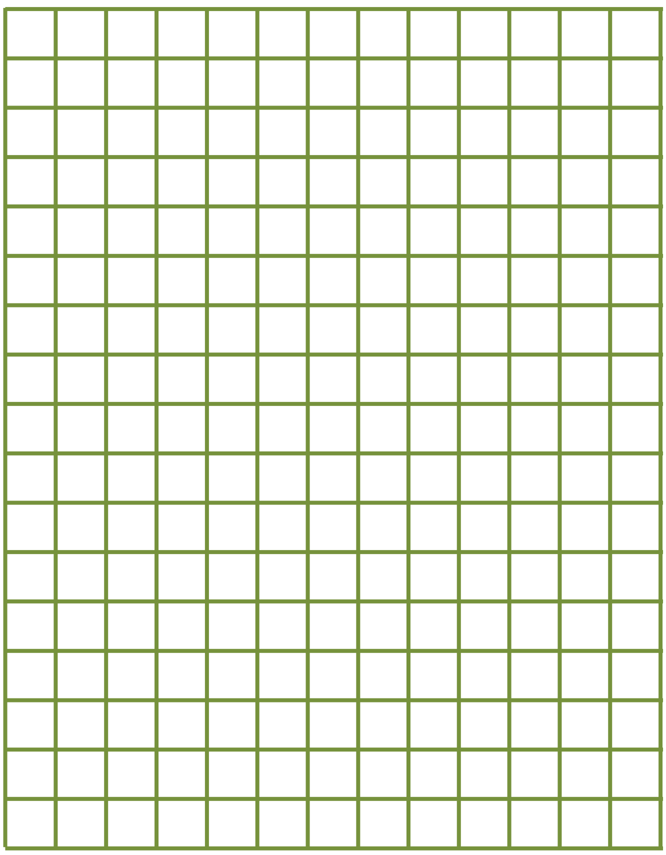
4. Write linear equation of the relationship between X and Y.



Answer: __________

---

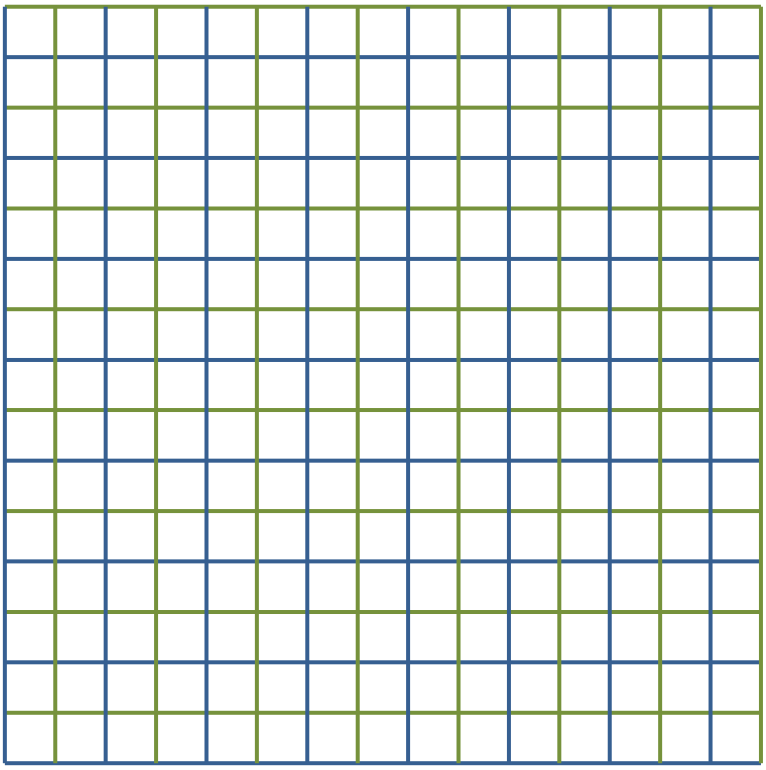
5. Plot the Triangular Number sequence {1, 3, 6, 10} against the Square Number sequence {1, 4, 9, 16}. Do you get a linear relationship?



Answer: ________________

6. The table shows the height (h) of the burning candle every minute (t) over 50 minutes. Plot the points to help you write the linear equation.

t	0	5	10	15	20	25	30	35	40	45	50
h	20	18	16	14	12	10	8	6	4	2	0



Answer: ___________________

---

7. A supervisor pays K20 to each person who turns up for work. He then pays K30 for each hour of work. Write the linear equation for total amount (P) received in relation to the time (t) worked.

Answer: ___________________

---

8 A taxi driver charges K5 for any trip. He then charges K0.60 for each kilometer. How much will Ronan pay if he travels 10 km by taxi?

Answer: ___________________

9. Plot the graph of the given table of values.

X	-3	-2	-1	0	1	2	3
y	5	5	5	5	5	5	5

What is the linear equation of this graph?

Answer: ___________________

10. Plot graph of the given table of values

d	1	2	3	4
h	1	2	4	8

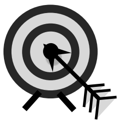
Is the graph that of an arithmetic progression or a geometric progression?

Answer: ___________________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 22: The Graph of Square Numbers

You have learned about the deriving explicit formula in linear sequences in lesson 21.

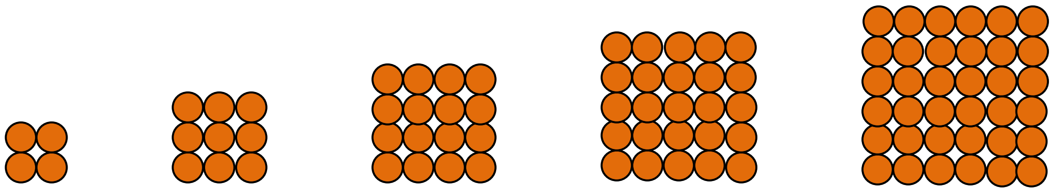


In this lesson, you will

- Graph the square of positive and negative numbers
- Find square numbers using graph

When you multiply a whole number times itself, the resulting product is called a square number or a perfect square. So 1, 4, 9, 16, 25, 36, 49, 64, 81, 100 are the first ten perfect squares or square numbers.

The name comes from the fact that these particular numbers of objects can be arranged to fill a perfect square. The numbers that can be shown as an array of n rows and n columns: n x n = n^2.



2 x 2	3 x 3	4 x 4	5 x 5	6 x 6
4	9	16	25	36

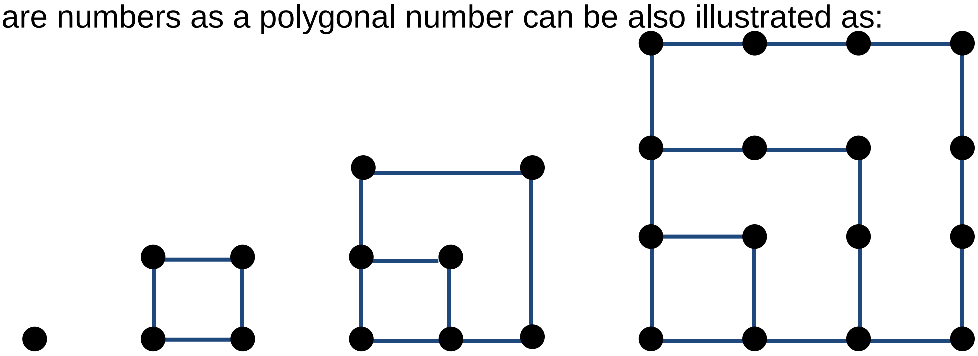
Square numbers are represented by $2*n - 1 = n^2$ in terms of a polygonal number. This rule can be derived from polygonal numbers general formula which states:

$$p_a(n) = \frac{n*[2+(n-1)(a-2)]}{2}$$

where **a** is the name (such as a = 4 for square numbers) and **n** is the n^{th} term.

While n^2 generates all perfect squares, 2*n -1 gives other numbers apart from square numbers. So use n^2 to find square numbers.

Square numbers as a polygonal number can be also illustrated as:



A square number is a number of the form **n x n** or $\mathbf{n^2}$, where n is any whole number.

If we add consecutive odd numbers, we will get perfect squares, such as:

1 + 3 =	4
1 + 3 + 5 =	9
1 + 3 + 5 + 7 =	16
1 + 3 + 5 + 7 + 9 =	25
1 + 3 + 5 + 7 + 9 + 11 =	36
1 + 3 + 5 + 7 + 9 + 11 + 13 =	49
1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 =	64
1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 =	81
1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 =	100

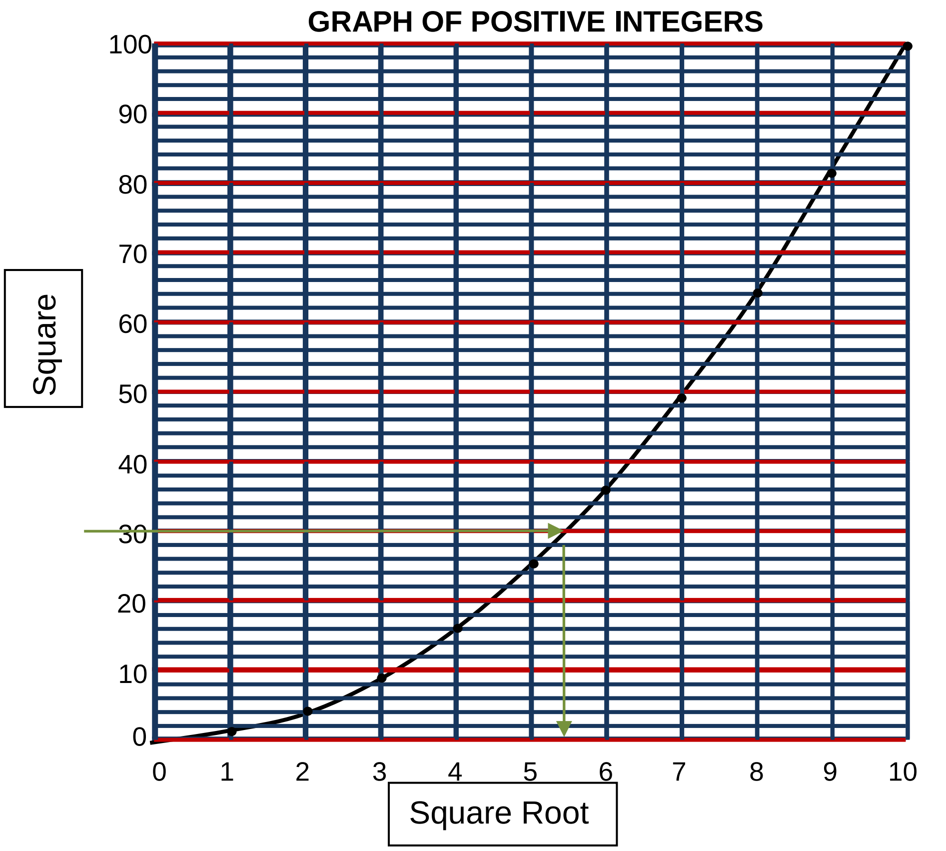
When we continue, we will still obtain a sum that is a perfect square.

Graphs of Positive and Negative Integers

To plot graph of the square numbers, we complete a table of values for the first 10 counting numbers and their opposites.

x	1	2	3	4	5	6	7	8	9	10
-x	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10
$y = x^2$	1	4	9	16	25	36	49	64	81	100

We need a graph paper that has smaller calibration than 1 cm by 1 cm, to be able to show. The scale shall be 1:10 for vertical axis and 1:1 for the horizontal axis. So the perfect squares can be plotted against their root.

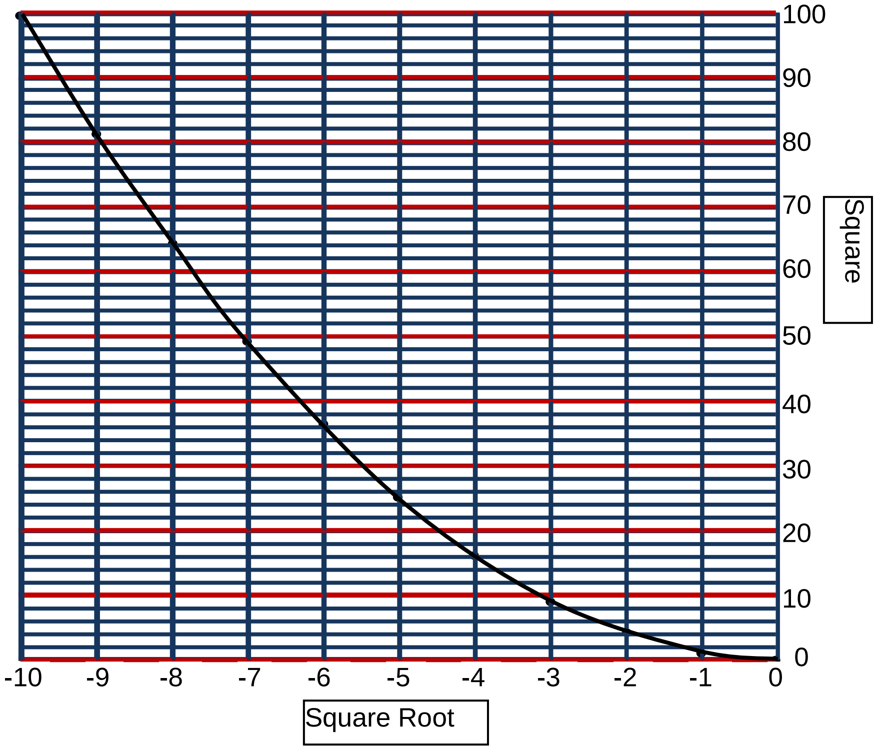


The graph can be used for finding the square roots and the square numbers. When we are given the n^{th} term, we will be required to find its square. And when we are given the perfect square, we will be asked to find its square root.

We can also use the same graph to find approximate values of square roots of numbers which are not perfect squares, such as 30. From the graph, the approximate value of square root of 30 is 5.4.

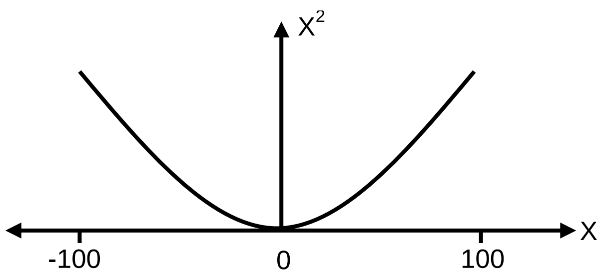
We know that product of two negatives is positive, thus when **n** is negative, $\mathbf{n^2}$ is positive. When we plot the points for negative integers, the graph we obtain seems a reflection of the square of positive integers.

GRAPH OF NEGATIVE INTEGERS



Again, the same graph can be used to find squares of numbers and square root of numbers. On both graphs we have limited ourselves to values from -10 to -1 and 1 to 10 (square roots) and between 1 to 100 for the perfect squares.

Suppose we plot $y = x^2$, we will observe that it is a quadratic curve with minimum point (0, 0).



The same graph can be used to find perfect squares and square roots. However, the graphs have their limits. The graphs do not give perfect squares of numbers less than – 10 and greater than 10.

For the perfect squares of large numbers, it may not be possible to plot on a graph. As the numbers get larger, their squares get larger times itself. Thus, a table of squares is more convenient to use than the graph of squares.

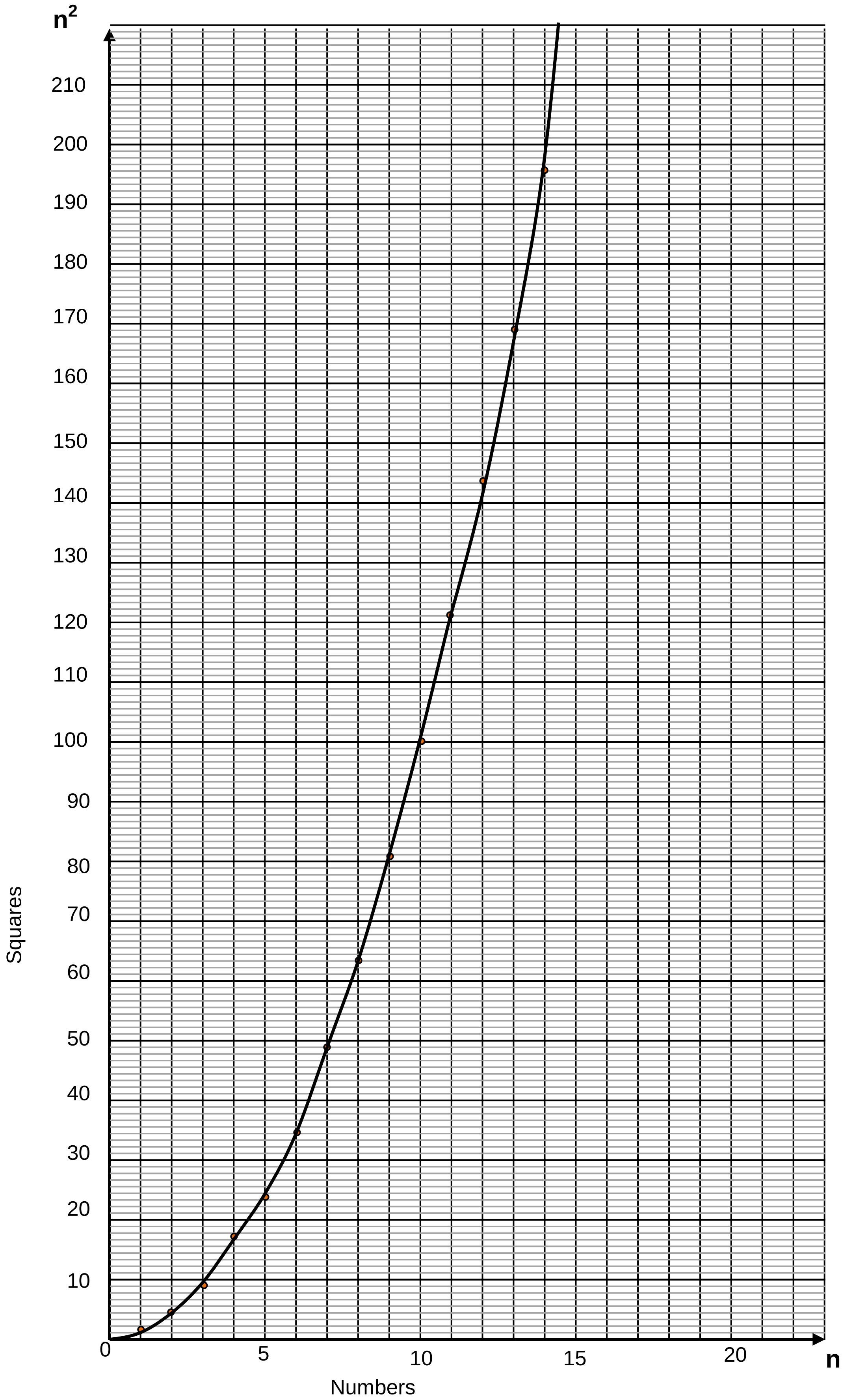
TABLE OF SQUARES

n	n^2	n	n^2	n	n^2	n	n^2	n	n^2
1	1	11	121	21	441	31	961	41	1681
2	4	12	144	22	484	32	1024	42	1764
3	9	13	169	23	529	33	1089	43	1849
4	16	14	196	24	576	34	1156	44	1936
5	25	15	225	25	625	35	1225	45	2025
6	36	16	256	26	676	36	1296	46	2116
7	49	17	289	27	729	37	1369	47	2209
8	64	18	324	28	784	38	1444	48	2304
9	81	19	361	29	841	39	1521	49	2401
10	100	20	400	30	900	40	1600	50	2500

n	n^2	n	n^2	n	n^2	n	n^2	n	n^2
51	2601	61	3721	71	5041	81	6561	91	8281
52	2704	62	3844	72	5184	82	6724	92	8464
53	2809	63	3969	73	5329	83	6889	93	8649
54	2916	64	4096	74	5476	84	7056	94	8836
55	3025	65	4225	75	5625	85	7225	95	9025
56	3136	66	4356	76	5776	86	7396	96	9216
57	3249	67	4489	77	5929	87	7565	97	9409
58	3364	68	4624	78	6084	88	7744	98	9604
59	3481	69	4761	79	6241	89	7921	99	9801
60	3600	70	4900	80	6400	90	8100	100	10000

However, the graph of squares is more convenient and can be used to estimate square roots of numbers that are not perfect squares as illustrated above.

GRAPH OF SQUARES



Practice Exercise 22

1. Use the graph on page 152 to estimate the square root of 50.

Answer: __________

2. $1.2^2 =$

Answer: __________

3. $(-19)^2 =$

Answer: __________

4. $200^2 =$

Answer: __________

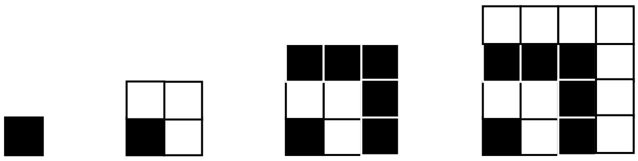
5. $\left(-\frac{3}{8}\right)^2 =$

Answer: ________________

6. Use the graph on page 153 to estimate $\sqrt{82}$.

Answer: ___________________

7. Draw the next pattern.



Answer: ___________________

8 $5^2 + 12^2 =$

Answer: ___________________

9. $17^2 - 15^2 =$

Answer: ___________________

10. When we add the sums of each row, will they add up to 105?

$$\begin{array}{ll} 1 & = \\ 1 + 4 & = \\ 1 + 4 + 9 & = \\ 1 + 4 + 9 + 16 & = \\ \underline{1 + 4 + 9 + 16 + 25} & = \\ 5 + 16 + 27 + 32 + 25 & = 105 \end{array}$$

Answer: ___________________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Summary

- **Pattern** is a repetition of event or objects in a particular manner.
- **Polygonal numbers** are numbers derived by the general formula

  $p_a(n) = \frac{n*[2+(n-1)(a-2)]}{2}$, where **a** defines the name and **n** is the nth term. They are such numbers as Square, Triangular, tetrahedral, pentagonal and hexagonal numbers.
- **Term** is one element of a sequence. Successive terms can be found using recursive or explicit rule. For Arithmetic and Geometric sequences we use

  AP $T_n = a + (n - 1)d$

  GP $T_n = ar^{n-1}$

- **Sequence** is a list of terms or numbers arranged in a definite order.
- **Series** is the sum of sequences. Sums are found by:

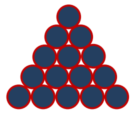
  AP $S_n = ½ [2a + (n - 1)d]$, and

  GP $S_n = \frac{a(1-r)^{n-1}}{1-r}$ [Partial Sum]
- **Recursive** formula designates the starting term, a_1, and the n^{th} term of the sequence , a_n, as an expression containing the previous term (the term before it), a_{n-1}. Figurative numbers are often found by recursive formula.
- **Explicit** formula provides overall rule from which you can compute any term of an arithmetic or geometric sequence.
- **Arithmetic sequence** if successive term of the sequence is found by adding (or subtracting) a fixed number. The fixed number is called common difference.
- **Geometric sequence** if successive term of the sequence is found by multiplying (or dividing) a fixed number. The fixed number is called common ratio.
- **Perfect square** are numbers such that they can be arranged in a way that their n^{th} row is the same as their n^{th} column. Or simply saying, are product of a number multiplied by itself.
- **Linear Graphs** have patterns similar to arithmetic sequence.
- **Pascals' Triangle** is a triangle with each row beginning and ending with 1; while the other terms of each row are found by adding two terms above it. Interestingly, pascal's triangle helps in solving binomial expansion and diagonals contain figurative or polygonal numbers among other patterns.

Answers to Practice Exercises 18- 22

Practice Exercise 18

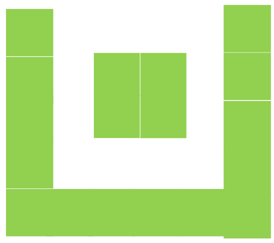
1. (see diagram A1)
2. (see diagram A2)
3. 40, 65, 105
4. 19, 22, 20
5. +5, - 2
6. 7,…,35,21
7. a = 3
8. 1.6
9. 25
10. 1, 4, 9, 13, 17



A1

Practice Exercise 19

1.12
2. – 2
3.12
4. +8 (7, ..,31, .., 39)
5. 144
6. $T_8 = 510$
7. r = 2
8. 9840
9. arithmetic
10. 8 terms



A2

Practice Exercise 20

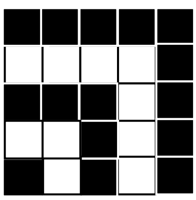
1. $a = 10;\ a_{n-1} + 2$
2. $a = -10;\ a_{n-1} + 2$
3. $a = 7;\ a_{n-1} + 2,\ n_7 = 19$
4. $a = -2;\ 2\bullet a_{n-1}$
5. $a = 6;\ 2\bullet a_{n-1}$
6. $a = -2;\ 2\bullet a_{n-1}$
7. + 4
8. - 2
9. 12, 19, 26, 33, 40
10. 12, 36, 108, 324, 972

Practice Exercise 21

1. $h = \frac{4}{3}t + 32$
2. k = 10 500 – 50n
3. K = 10d + 200
4. y = ½ x + 6
5. No
6. $h = -\frac{2}{5}t + 20$
7. P = 30t + 20
8. K11.00
9. y = 5
10. Geometric Progression

Practice Exercise 22

1. 12.5
2. 1.44
3. 361
4. 40 000
5. $\frac{9}{64}$
6. 9.1
7. (see diagram A3)
8. 13^2 or 169
9. 8^2 or 64
10. Yes



A3

REFERENCES

http://mathbitsnotebook.com/Algebra1/Functions?FNSequenceFunctionRecursive.html

http://www.mathsisfun.com/algebra/sequences-arithmetic.html

http://www.mathsisfun.com/algebra/sequences-sums-arithmetic.html

http://www.regentsprep.org/regents/math/altrig/atp3/recursive.html

http://www.mathsisfun.com/numbers/evolution-of-numbers.html

www.mathscentre.ac.uk

http://www.purplemath.com.modules/numtypes.html

http://www.cut-the-knot.org/arithmetic/combinatorics/PascalTriangleProperties.shtml