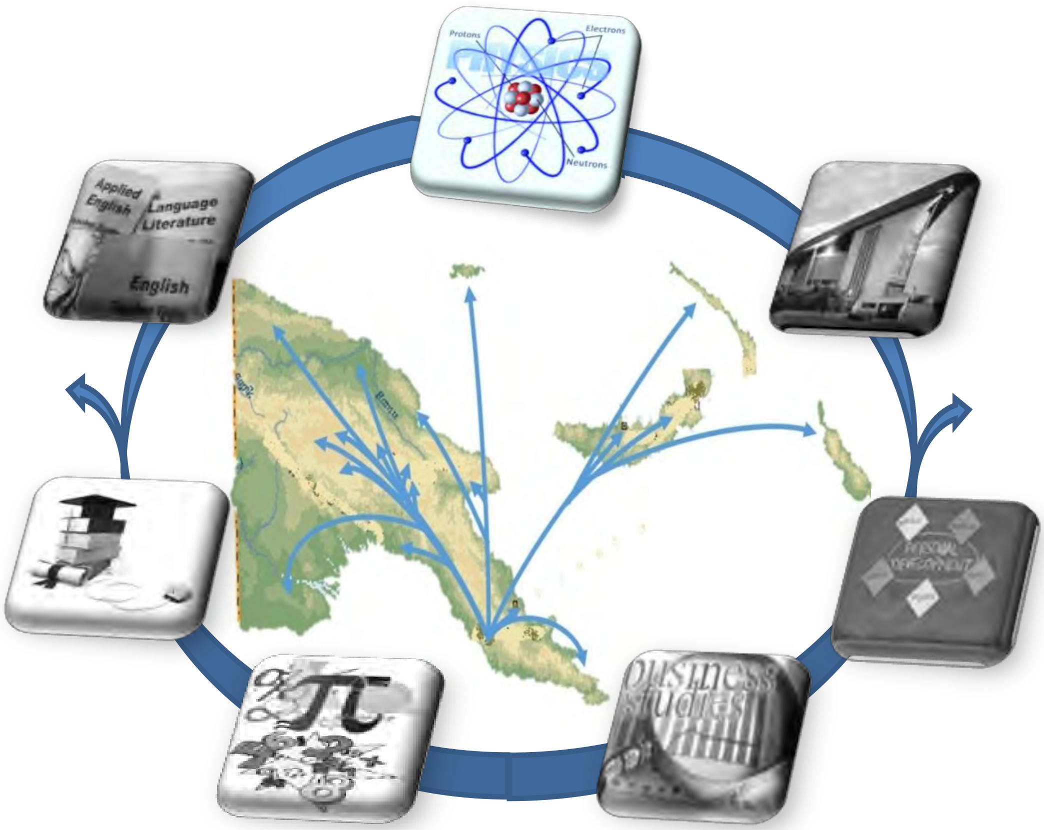
DEPARTMENT OF EDUCATION

GRADE 11

PHYSICS

MODULE 2



MOTION (KINEMATICS)

PUBLISHED BY FLEXIBLE OPEN AND DISTANCE EDUCATION
PRIVATE MAIL BAG, P.O. WAIGANI, NCD
FOR DEPARTMENT OF EDUCATION
PAPUA NEW GUINEA
2017

Writer
Tommy Sariaman

Content Editors
Science Department
Subject Review Committee

Language Editor
Dr. Mirzi. L. Betasolo

Course Format Editor
Elizabeth. W. Aimundi

GRADE 11

PHYSICS

MODULE 2

(MOTION) KINEMATICS

IN THIS MODULE YOU WILL LEARN ABOUT:

Acknowledgement

We acknowledge the contribution of all Lower and Upper Secondary teachers who in one way or another helped to develop this Course.

Our profound gratitude goes to the former Principal of FODE, Mr. Demas Tongogo for leading FODE towards this great achievement.

Special thanks are given to the staff of the Science Department of FODE who played active roles in coordinating writing workshops, outsourcing of module writing and editing processes involving selected teachers of Central Province and NCD.

We also acknowledge the professional guidance and services provided through-out the processes of writing by the members of:

Science Subject Review Committee-FODE
Academic Advisory Committee-FODE
Science Department- CDAD

This book was developed with the invaluable support and co-funding of the GO-PNG and World Bank.

DIANA TEIT AKIS
Principal-FODE

Flexible Open and Distance Education
Papua New Guinea

Published in 2017

Printed by the Flexible, Open and Distance Education
ISBN 978-9980-89-540-0
National Library Services of Papua New Guinea

TABLE OF CONTENTS

SECRETARY'S MESSAGE

Achieving a better future by individual students, their families, communities or the nation as a whole, depends on the kind of curriculum and the way it is delivered.

This course is part of the new Flexible, Open and Distance Education curriculum. The learning outcomes are student-centred and allows for them to be demonstrated and assessed.

It maintains the rationale, goals, aims and principles of the National Curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision by Flexible, Open and Distance Education as an alternative pathway of formal education.

The Course promotes Papua New Guinea values and beliefs which are found in our constitution, Government policies and reports. It is developed in line with the National Education Plan (2005 – 2014) and addresses an increase in the number of school leavers affected by lack of access into secondary and higher educational institutions.

Flexible, Open and Distance Education is guided by the Department of Education's Mission which is fivefold;

- To develop and encourage an education system which satisfies the requirements of Papua New Guinea and its people
- To establish, preserve, and improve standards of education throughout Papua New Guinea
- To make the benefits of such education available as widely as possible to all of the people
- To make education accessible to the physically, mentally and socially handicapped as well as to those who are educationally disadvantaged

The College is enhanced to provide alternative and comparable path ways for students and adults to complete their education, through one system, two path ways and same learning outcomes.

It is our vision that Papua New Guineans harness all appropriate and affordable technologies to pursue this program.

I commend all those teachers, curriculum writers, university lecturers and many others who have contributed so much in developing this course.

UKE KOMBRA, PhD
Secretary for Education

MODULE 11.2 Motion (Kinematics)

Introduction

In this module, you are introduced to motion (kinematics) without considering what caused it. There are fundamental principles governing motion, and they can be used to explain the motion of an object. Motion of objects and people can be quite complex, so we usually simplify it in a number of ways. In this module, you will study straight line motion and then you will look at motion under constant acceleration. Motion in two dimensions will be studied with the use of vectors. This module has four (4) four topics. The first topic explains the characteristics of motion. It describes the different types of motion that we see and experience every day. The next topic is all about reading and plotting graphs. Motion in straight line, help us calculate the distance, displacement, acceleration speed, and velocity of an object moving along the surface or falling downwards. The equations of motion, you will study under linear motion can be applied in two dimensional motion. The projectile, circular and rotary motions are the topics that explain moving objects in the circular path. The learning activities and their answers are given right after you study each topic to help understanding it better. However, there may be some words that may be slightly difficult. If you cannot understand a word or words, please do consult your tutor.

This module is also a follow up to the first module on Measurement. It requires reasoning skills and your knowledge of mathematics to help you solve word problems. You will also need to have a scientific calculator to assist you in your calculations. Always remember to take note of any equation and formulas that you come across in the module for use in other modules. The more you use them you will go through the module with ease. At the end of the module, you will find a table of units containing some quantities with their corresponding units that should help to solve word problems. The picture below shows examples of motions we see every day.

Busy traffic

Cyclists moving around a bend

Learning Outcomes

After going through this module, you are expected to:

- describe the characteristics of linear motion including distance, displacement, speed, velocity, acceleration, instantaneous and average speed, velocity and acceleration, and calculate these quantities from a given set of experimental data.
- emphasise the correct units of the quantities described.
- perform vector analysis on displacement, velocity and acceleration.
- draw and interpret distance–time, speed–time, velocity–time, displacement–time and acceleration–time graphs.
- conduct ticker timer experiments for uniform and accelerated motions and plot the relevant graphs.
- describe the motion of an object (uniform and non-uniform) from a ticker tape and record graphs.
- determine unknown parameters from graphs of motion; for example, area under displacement-time graph represents average velocity of the moving object.
- solve worded problems with the aid of diagrams.
- derive and explain the following equations of linear motion using graphs where necessary.
- apply the equations to calculate parameters such as distance, displacement, speed, velocity, acceleration and time.
- translate the above equations of linear motion to one-dimensional vertical motion of an object under the influence of gravity, as the motion of this object, is an example of a linear motion.
- apply equations of motion to one-dimensional free-fall motion (vertical) and solve problems associated with this motion.
- draw graphs of motion in two dimensions from a given set of experimental data and worked examples on a two-dimension projectile motion.
- describe the projectile motion in two separate directions, vertical and horizontal directions, and emphasise the essential facts about the motion along each direction.
- derive the relevant equations of motion along each direction.
- solve word problems on projectile motion.
- describe quantities of circular motion and their units including angular displacement (θ), velocity (ω) and acceleration (α).
- write the equations of circular motion analogous to the equations of linear motion.
- explain the relationships between linear and angular quantities.
- solve worked examples on circular motion.

Time Frame

Suggested allotment time: **10 weeks**

This module should be completed within ten (10) weeks.

If you set an average of 3 hours per day, you should be able to complete the module comfortably by the end of the assigned week.

Try to do all the learning activities and compare your answers with the ones provided at the end of the module. If you do not get a particular question right in the first attempt, you should not get discouraged but instead, go back and attempt it again. If you still do not get it right after several attempts, then you should seek help from your friend or even your tutor.

DO NOT LEAVE ANY QUESTION UNANSWERED.

11.2.1 Characteristics of Motion

There are different types of motion occurring around us all the time. Some examples are the movement of people walking, athletes running on tracks, cars being driven along roads, aeroplanes flying in the sky, footballs being kicked, compact discs rotating on CD players, trains travelling along tracks, mail being sorted and basketball falling through the ring and so on. This movements form an important part of everyday life.

Figure 1 Different types of motion

Movement involves a change in position in a certain time. Therefore, quantities like distance, time, speed and acceleration with their measurements must be considered when describing the motion of an object. Our knowledge of Physics enables us to analyze the motion of an object precisely. For example, aeroplane pilots need to know their exact position in the air, how long it will take to fly to their destination, how much fuel to take and the effect of the wind on their speed and direction of flight.

Manufacturers of motor vehicles need to know whether their vehicles will stop within certain limits, accelerate and use fuel at acceptable rates, withstand collision and have sufficient power to climb steep hills and overtake other vehicles. Traffic accident investigation also requires knowledge of motion. Physics can be used to reconstruct an accident and to determine the speed and direction of the vehicles before the collision.

In the study of motion, we find ways to specify the position of the object at any time.

Graphing is the most common method used to explain the characteristics of motion. The geographical system of the north, south, east and west is also convenient to use. A number line such as the x-axis marked in units allows us to specify the position along a straight line. The zero can represent a starting point, with the positive and negative numbers giving the direction. The positions of a point on the line are described by a number, a unit, and a sign to indicate the direction. The direction is the line itself indicating which way, positive or negative along the line. We usually use direction to make sense of how the motion is going. As direction is involved, we say that position is a vector quantity. A common symbol for position is $\vec{x}$ where the half arrow shows that we are dealing with a vector quantity.

Displacement is a change in position in a certain direction. It is a change that is obtained by subtracting or adding one vector from or to another, the result is a vector. The average speed of a moving object is the distance travelled divided by the time taken. Therefore, displacement divided by the time taken gives the average displacement per unit time which is the velocity. Velocity is a vector quantity because the information about direction is in the displacement.

Distance, Displacement, Speed and Velocity

Motion in one dimension was described using the quantities displacement (s), time (t), velocity (v) and acceleration (a). It is also true for motion in two dimensions but the vector nature of some of the quantities needs further clarification. It is also important to redefine the quantities covered in the last unit regarding their vector or scalar nature.

Distance and displacement are both quantities describing length. However, a distance is a scalar while displacement is a vector, requiring a direction. Both quantities use the symbol, s, and both units of measurements are in metres.

Example, d = 10m: Distance
s =10m east: Displacement

Distance is the length of actual path travel from the starting point to the finishing point. It has a magnitude (size) but no direction.

A girl walks 5 metres from point A to point B in the classroom and backs to point A another 5 metres. The distance travelled by the girl from point A to B and from B to A is 10 metres, but does not give the girl's direction. Since distance is a scalar quantity and does not give direction, the total length she travelled is 10 metres (5m +5m =10m).

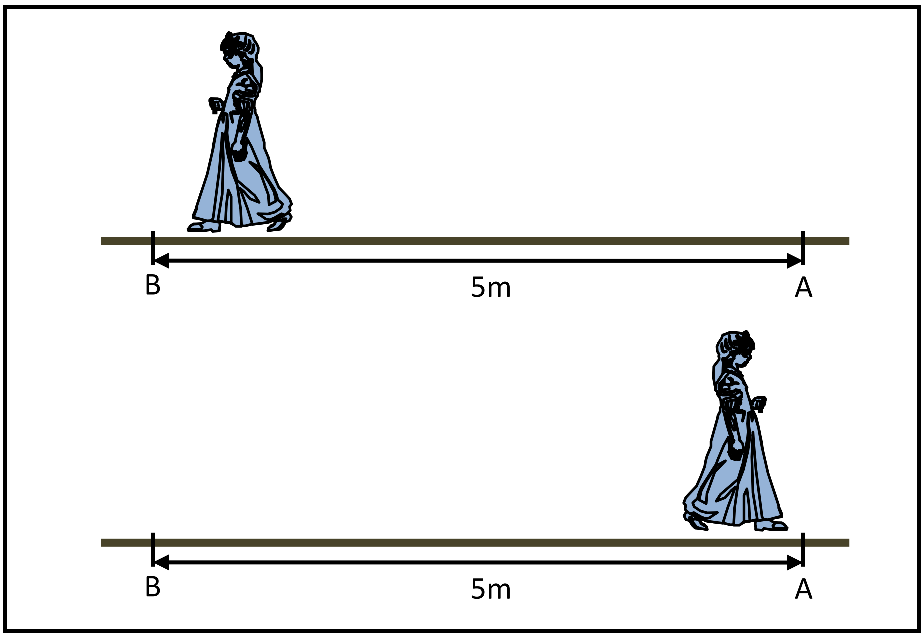


Figure 2 Girl walking in a 5m length

Displacement is the distance in a straight line in a certain direction from the starting point. It has magnitude (size) and direction. To understand this, we can simply study the diagram in Figure 3 below. Consider a girl walking west (**point B**) 5m in the classroom and then back to east (**point A)** 5 metres. The displacement of the girl will be zero. This is because the girl walks 5m in the west direction which is negative and the east direction to be positive. This gives us negative 5m plus positive 5m east which will cancel out giving us a net displacement of zero. (-5 +5 = 0 m)

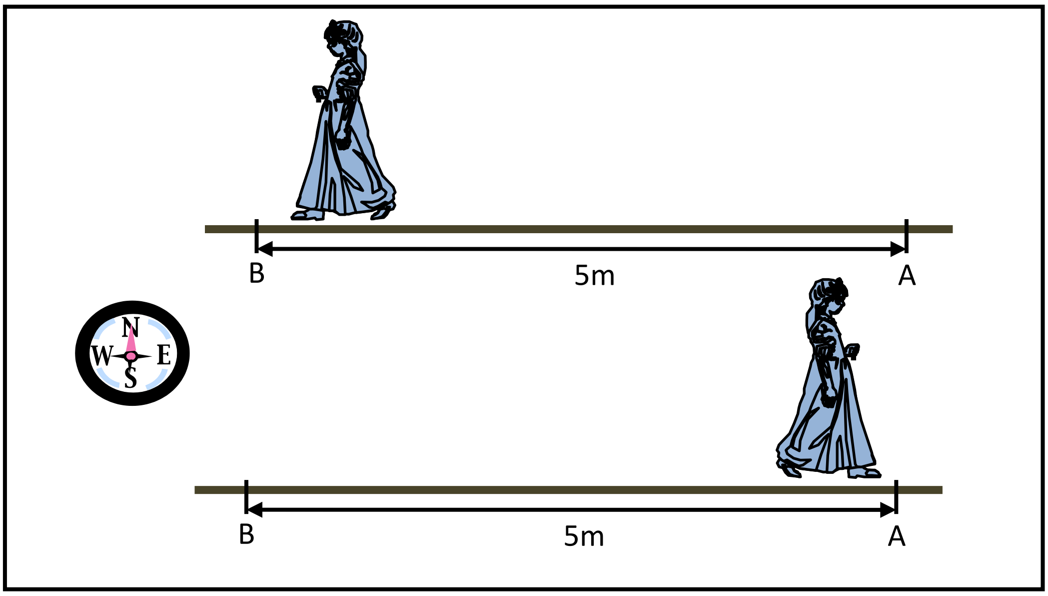


Figure 3 Girl walking 5m West and then 5m East

Displacement is a vector that joins the initial position to the final position of an object.

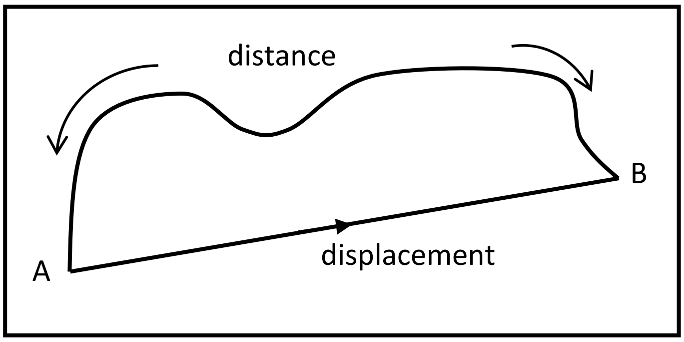


Figure 4 Displacement and distance illustration

Displacement is that vector from the beginning position to the end position of a motion.

Example 1

A body moves from A to C via B. It has travelled a total distance of 16m.

Distance = 10m+ 6m

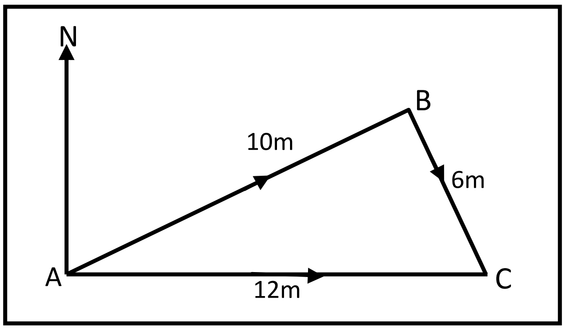
= 16m

Resultant =$-\sqrt{10^2 + 6^2}$

$R^2 = -\sqrt{100m+ 36m}$

$R^2 = -\sqrt{136m}$

R= 11.67m East



Displacement representation

The distance travelled is 16m. However, its displacement from the starting position is 12m to the East or displacement is equal to 12m east. The displacement is calculated using **Pythagoras theorem** which you will come across later in the unit. Displacement is a vector quantity. For straight line motion, positive displacement shows forwards motion while a negative displacement shows backward motion. Displacement can also be referred to by compass direction.

Positive displacement is always to the right and negative displacement is always to the left.

Now consider a person running around a circular track. The **circumference** of the circular track is always the distance travelled, which is calculated by using **c = 2πr** and the displacement would be the diameter of the track. This is because the displacement is always in a straight line in a certain direction.

In the Figure 5, the circular track is 400m and the radius is 63.69m, thus giving us the diameter of 127.38 metres. Using the formula above as **400/2x3.14 =63.69 x 2 =127.38m**.

Therefore the diameter is 127.38m which is now the runner's displacement if he runs halfway around the track. If the runner starts and completes the race, then the displacement is zero.

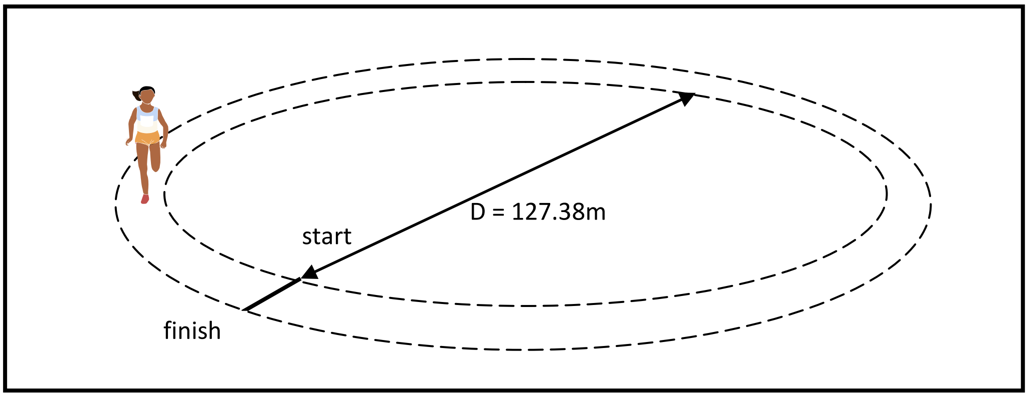


Figure 4 Displacement and distance representation in circular track

We will use letter **d** to represent the total distance travelled and **s** for the change in displacement.

The change in displacement is defined by;

s = final displacement – initial displacement

If the motion consists of many parts, then the change in displacement is the sum of the displacement in each part of the motion. Take for example, if an object starts at an origin let us say 15m and then by -5m then by 2m, the change in displacement is 15 – 5 + 2 = 12m. The final displacement is 12 + 0 = 12m.

Measurement of time
Time is used to measure speed and velocity.
Time, t, is a scalar and is measured in seconds.
For example, t = 2min 10s which is equal to 130s.

Speed and velocity
In the last unit, you learnt about the difference between scalar and vector quantities. Now that we are studying motion, you expect to meet more vector quantities because object has motions in particular directions. We use speed as a scalar which has a magnitude (size) and velocity as a vector that has magnitude and direction. So velocity is both the speed and direction of the motion of an object.

We can use some terms to describe the motion of a motor vehicle. The terms commonly used include speed, velocity, acceleration and distance. The everyday use of these terms is often different from the scientific usage. For example, most people use the words speed and velocity to mean the same thing. But to the scientist, the two terms have slightly different meanings.

When we talk about an object's speed, we have no idea about its direction since it is a scalar. We can say the speed is the time rate of change of distance. Velocity, on the other hand, is the time rate of change of displacement. Both quantities use the symbol, v, and both units of measurement are in metres per second, ms^{-1}.

For example $v = 5ms^{-1}$: Speed
$v = 5\ ms^{-1}$ East: Velocity.

- **Speed is the distance travelled in unit time.**
- **Velocity is the distance travelled in particular direction in unit time.**

Velocity is a vector and is a derived quantity. It is related to the quantities displacement and time and is build up from the fundamental quantities of length and time.

The average velocity of a moving object is found by dividing total displacement by the total time taken.

$$\mathbf{v_{av}} \quad \frac{\mathbf{s}}{\mathbf{t}}$$

From the equation, the dimensions of the average velocity are LT^{-1}. While the average velocity is a vector quantity, and therefore has direction and magnitude, average speed is a scalar quantity and is found by dividing the total distance by the total time.

Average velocity

Two very simple required measurements for the calculation of an average velocity are:

- displacement(s) of an object and
- time (t) for this displacement to occur.

Displacement, s, is the straight line distance from a reference position.

The average velocity is given the symbol $\mathbf{v_{av}}$**.** Mathematically, average velocity is the displacement undergone by an object in unit time.

In everyday situations, for example in a car, the units of velocity are kilometres per hour (kmh^{-1}). However, SI units for velocity are metres per second (ms^{-1}) as displacement has unit of measures in metres (m) and the time taken in seconds(s).

The table below shows the average velocity of some things you are familiar with in both ms^{-1} and kmh^{-1}. These velocities are useful for future comparison.

Movement	**m/s**	**Km/h**
Snail	0.003	0.01
Fast walker	2	7.2
Runner	5	18
Bicycling	14	50.4
Fast car	36	129
Sound in air	340	1224
Jet plane	600	2160
Earth around the Sun	30 000	108 000
Light and radio waves	300 000 000	1 080 000 000

Table 1 Object's movement and their speed

If a car takes 10 seconds to travel along a straight road for 300 metres, it means that the car travels at an average speed of 30 metres per second (30m/s or $30ms^{-1}$). When an object moves, it changes its position. The speed at which it moves depends on how far it moves and how long it takes.

The speed of a moving body is the distance travelled in unit time. When we talk about an object's speed, we have no idea about its direction (so speed is a scalar quantity).

$$\textbf{Speed} = \frac{\textbf{Distance}}{\textbf{Time}} \qquad s = \frac{d}{t}$$

Here, **v** is the speed, **s** is the distance and **t** is the time taken. Speed has a variety of units, for example, kilometres per hour (kmh^{-1}) or in millimetres per second (mms^{-1}). However, the correct S.I. unit for speed is the metre per second (ms^{-1}).

$$\textbf{Average speed} = \frac{\textbf{Total distance travelled}}{\textbf{Total time taken}}$$

$$\textbf{speed}_{av} = \frac{d}{t}$$

The average velocity of a moving object is found by dividing total displacement by the total time taken.

$$\textbf{Average velocity} = \frac{\textbf{Total displacement}}{\textbf{Total time taken}}$$

$$\textbf{velocity}_{av} = \frac{s}{t}$$

Definition of velocity is defined as speed in a particular direction. For example, $40kmh^{-1}$ East or $30kmh^{-1}$ West. Velocity is a vector quantity.

$$\textbf{Velocity} = \frac{\textbf{Distance travelled in a particular direction}}{\textbf{Time taken}}$$

$$\textbf{Velocity} = \frac{\textbf{Displacement}}{\textbf{Time}}$$

$$v = \frac{s}{t}$$

A car is travelling at a constant speed. Its velocity changes each second.

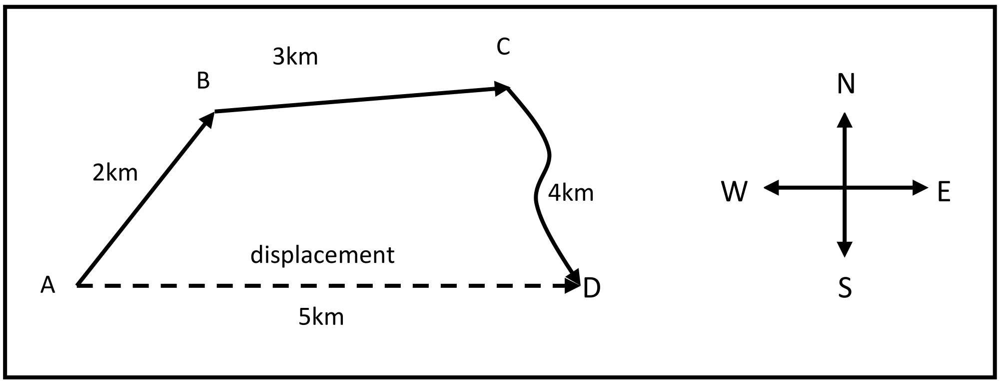


Figure 5 Car travelling at various velocities

As with speed, we can also use the term average velocity to describe the motion of an object like a car.

$$\textbf{Average velocity} = \frac{\textbf{Total displacement}}{\textbf{Total time}} = \frac{\Delta s}{\Delta t}$$

Where Δ(delta) represents 'change in', Δt means change in time. The speed of a car is sometimes given in kilometres per hour (km/h) or kmh^{-1}.

$$1km / h = \frac{1000}{60 \times 60} = \frac{1}{36} m/s$$

Example 2

A car travels from one city to another 96km away in 1.2 hours. What is the car's average speed in:

a) km/h?

b) m/s?

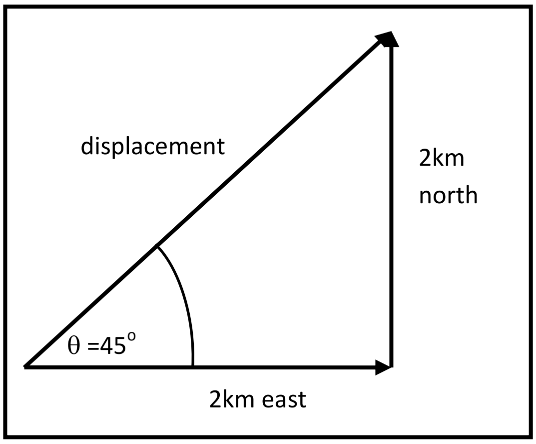
Solution

a) $$\text{Average speed} = \frac{\text{Distance covered}}{\text{Time taken}} = \frac{96}{1.2} = 80km/h$$

b) $$80 \text{ km/h} = \frac{80m/s}{3.6} = 22.2m/s$$

Example 3

A person rides a bicycle 2km east and then 2km north. The trip takes 2 hours.



Bicycle's route

Find the,

a) person's average speed and

b) average velocity

Solution

a) $$\text{Average speed} = \frac{\text{Total distance}}{\text{Time taken}} = \frac{4}{2} = 2\text{km/h}$$

b) $$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Time taken}} = \frac{2.82}{2} = 1.41\text{km/h North East}$$

The diagram below shows a car travelling at constant speed. Its velocity changes each time it turns a corner because the direction of its motion changes. The velocity of the car measured at a particular moment is called **the instantaneous velocity** of the car.

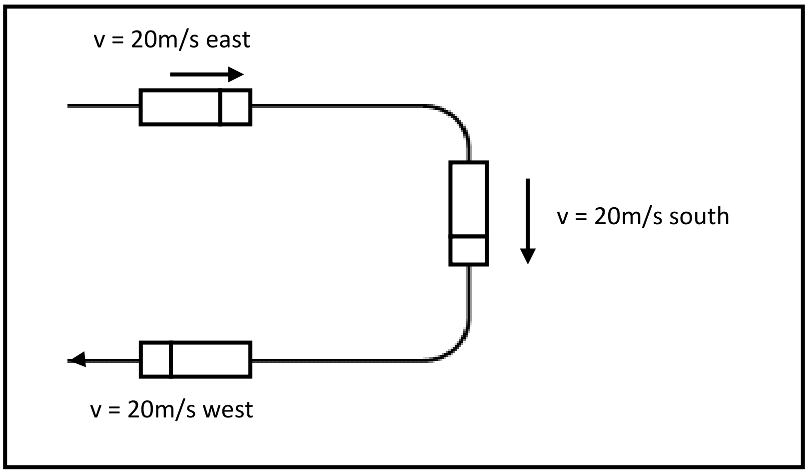


Figure 6 Car's route

$$\textbf{Instantaneous velocity} = \frac{\textbf{Small distance travelled in a stated direction}}{\textbf{Small time taken}}$$

$$\textbf{Instantaneous velocity (v)} = \frac{\textbf{Small distance travelled in a stated direction s (in metres)}}{\textbf{Small time taken t (in seconds)}}$$

Sometimes, when velocity is not uniform, we are concerned with the velocity of an object at a particular instant. This is called instantaneous velocity of an object at that instant. To find the instantaneous velocity of an object, we find its average velocity during a small period of time that we are interested in. The small period of time is written as **Δt**, and the small displacement is called **Δs**. We try to make Δt and Δs as small as possible.

Distance moved in a stated direction is often replaced by the term displacement. Thus, the instantaneous velocity is the rate of change of displacement.

In a modern car, a device within the transmission produces a series of electrical pulses, which are sent to a calibrated device that translates the pulses into the speed of the car. This information is then displayed on the car in the form of a deflected speedometer needle or a digital readout. In older cars, speedometers were linked mechanically to the transmission.

Here, a car's speedometer needle is pointing to zero, meaning that the car is at a standstill. The speedometer measures instantaneous speeds or speeds at that instant.

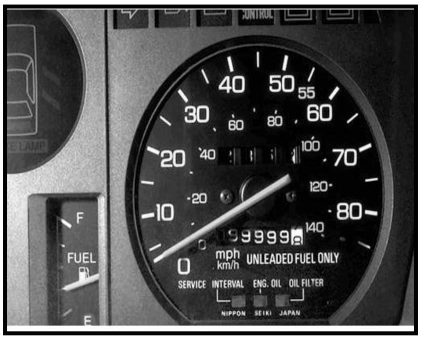
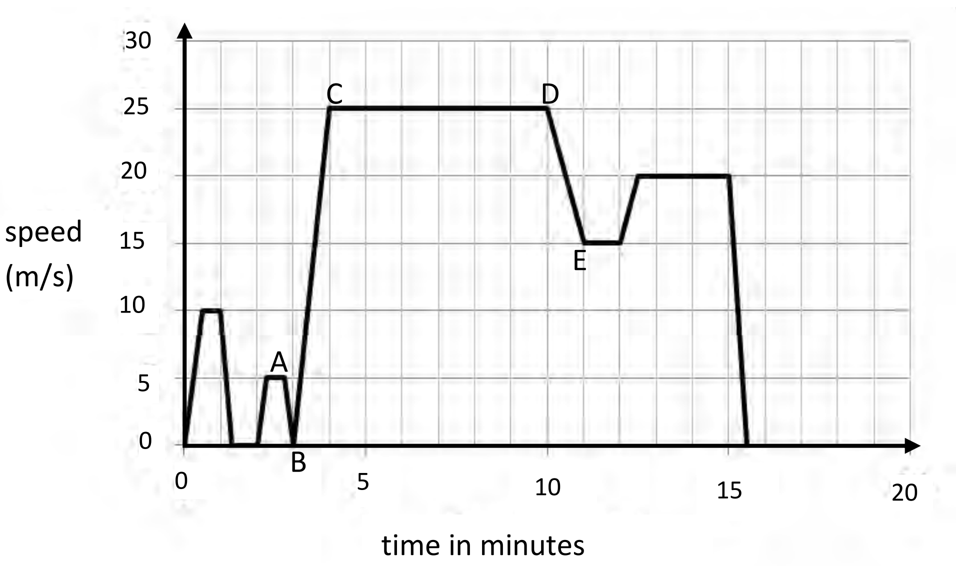


Figure 7 Speedometer of modern car

Instantaneous speed is speed in a small time interval and can be plotted on a graph such as the one given below.

SPEED TIME GRAPH



From the graph, the readings of the speeds at 2.5 seconds and 10.5 seconds are given below.

a) 2.5 seconds = 0

b) 10.5 seconds. Speed = 20m/s

Changing units
In many situations, the person solving the problem is required to change the units of a given quantity. The following conversions are commonly required.

1 kilometre = 1000 metres
1 minute = 60 seconds
1 hour = 60 minutes
1 hour = 60 x 60 = 3600 seconds

Imagine a velocity of $20kmh^{-1}$ is to be converted to ms^{-1}. Here, one unit should be converted at a time.

Now 1 hour = 3600s
So, $20\,000mh^{-1}$ = 20 000m in 3600 s
= 20 000/3600
= $5.55ms^{-1}$

In the laboratory you may require these conversions:
$1mm = 10^{-3}m$ ($1 \div 1000 = 0.001 = 10^{-3}m$)
$1cm = 10^{-2}m$ ($1 \div 100 = 0.01 = 10^{-2}m$)

Now check what you have just learnt by trying out the learning activity below!

Learning Activity 1

40 minutes

Answer the following questions on the spaces provided.

1 Suppose you walk a distance of 4m to the blackboard in your classroom and then back 3m.

(a) What distance have you covered?

(b) What is your displacement?

2. James walks 3km north, turns and walks 4km east, then walks 3km south. What;

a) will be the distance he has covered?

b) is his displacement?

3. A train covers 400m at a constant speed of $8ms^{-1}$. What is the time taken to travel that distance?

4. Charlie walks in a straight line at a steady speed of $2ms^{-1}$ for 15 seconds. Work out the distance Charlie travelled from the starting point after 10 seconds of walking.

5. A person rides a bicycle to a shop. The person travels 300m north along a straight road and then travels east for another 400m. The trip to the shop takes 2 minutes. Find the person's average

a) speed.

b) velocity.

6. A PMV travels two kilometres from one bus stop to another East of it in 20 minutes. Calculate the:

a) distance travelled.

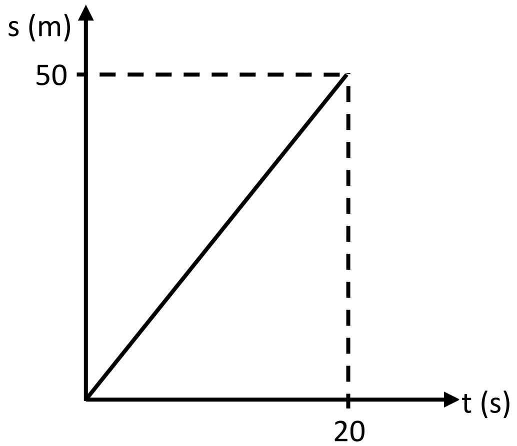
b) displacement.

c) speed.

d) velocity.

---

7. The graph shows a motion of a body.



What is the velocity of the body?

---

8. A boy walks 5km North East and then another 5km South East. It takes 2 hours. Calculate:

a) The distance he travels.

b) His displacement from his starting position.

c) His speed.

d) The velocity.

---

9. A runner races around a circular track of radius 80m.

a) Work out the distance the runner covers if the start and the finish line are the same? (use circumference = $2\pi r$)

b) Find the displacement as he crosses the finish line?

c) If the runner in the above question completes one circuit in 60 seconds, what is the average speed?

d) What is the average velocity if the runner races halfway around the field in 30 seconds?

10. An aeroplane flies North 200km for 2 hours and then East 200kmh^{-1} for 1 hour.

a) What is the total distance flown?

b) Find the total displacement.

c) Calculate the average speed.

d) Work out its average velocity.

Thank you for completing learning activity 1. Now check your work. Answers are at the end of the module.

Acceleration

Whenever the velocity of an object is changing, it has acceleration. The object could be speeding up, slowing down or changing direction. In each of this case, its velocity is changing. The equations of motion you shall come across later in the chapter describe the motion of objects that have a constant acceleration in a straight line.

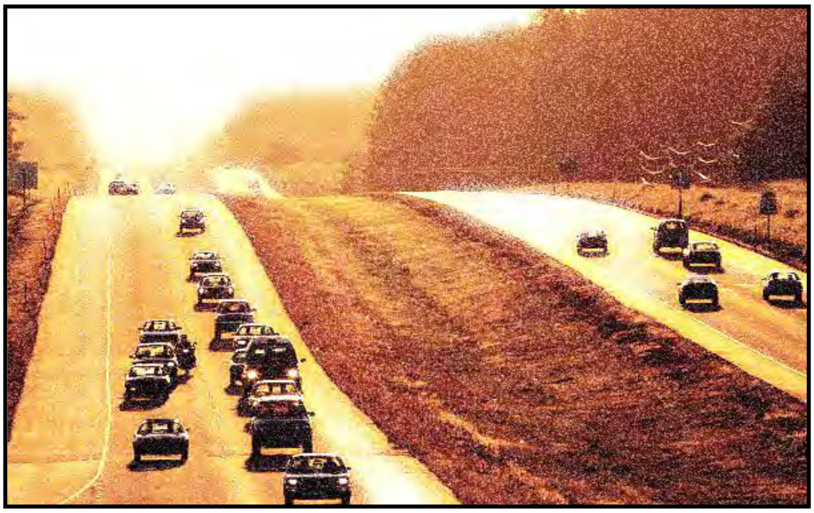


Figure 8 Cars travelling at different speeds

We have seen cars increasing their speed and slowing down on a busy road. We must know that the change of speed is called **acceleration**. When we want to measure acceleration we need to know how much time it takes for a certain change of speed or velocity, and we define acceleration as follows:

Acceleration, (a) is the change of velocity (v) over period of time (t).

So, we calculate the acceleration from the change of velocity which occurs in unit time.
Many objects move at a constant velocity. The velocity of a car increases when it starts moving from rest and decreases on an application of a brake. Cars can thus speed up (accelerate) or slow down (decelerate).

If the change in velocity is measured in metres per second and the time in seconds, then the acceleration is measured in metres per second squared that is ms^{-2}.

A car slowing down is said to decelerate. In other words, it will have a **negative acceleration.** Acceleration is a vector quantity. The term **retardation** is sometimes used instead of deceleration. You may at times encounter certain problems dealing with calculating acceleration of linear motion. For example, if the words deceleration and retardation are used in describing the motion, then the acceleration will have a minus sign in front of the value.

$$\text{Acceleration} = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Time}}$$

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time}} = \frac{\Delta v}{t}$$

$$a = \frac{\Delta v}{t} \text{ (measured in ms}^{-2}\text{)}$$

$$a = \frac{v - u}{t}$$

Example 1
A train increases speed from 4m/s to 10m/s in 3 seconds. Find the acceleration.

$$\text{Acceleration} = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Time}}$$

$$a = \frac{10 - 4}{3} = 2ms^{-2}$$

Example 2
A taxi travelling at 15m/s slows down to 5m/s in 2 seconds. Find its acceleration.

$$\text{Acceleration} = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Time}}$$

$$a = \frac{5 - 15}{2} = \frac{-10}{2} = -5ms^{-2}$$

Note: This worked example shows that the taxi is decelerating or slowing down.

Example 3
A large truck increases velocity from 6m/s to 10m/s in 2 seconds. Find its acceleration.

$$a = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Time}}$$

$$a = \frac{10 - 6}{2} = \frac{4}{2} = 2ms^{-2}$$

Example 4

A train traveling at 20m/s slows down to 10m/s in 2 seconds. Find its acceleration.

$$a = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Time}}$$

$$a = \frac{10 - 20}{2} = \frac{-10}{2} = -5ms^{-2}$$

Note: The negative sign here indicates that the car is slowing down. You can also describe it by saying that the car is decelerating at $5ms^{-2}$ or it has an acceleration of $-5ms^{-2}$.

Adding other vectors

Often an object can be given two velocities at once. An aircraft, for example, can be given a velocity by its engines and another by the wind. The two velocities then are added to give the aircraft its actual velocities. When adding a vector, the size and direction both affect the result. If the engine speed is 4m/s and the wind speed is 3m/s, the aircraft's actual speed can work out to be 1m/s or 7m/s or 5m/s.

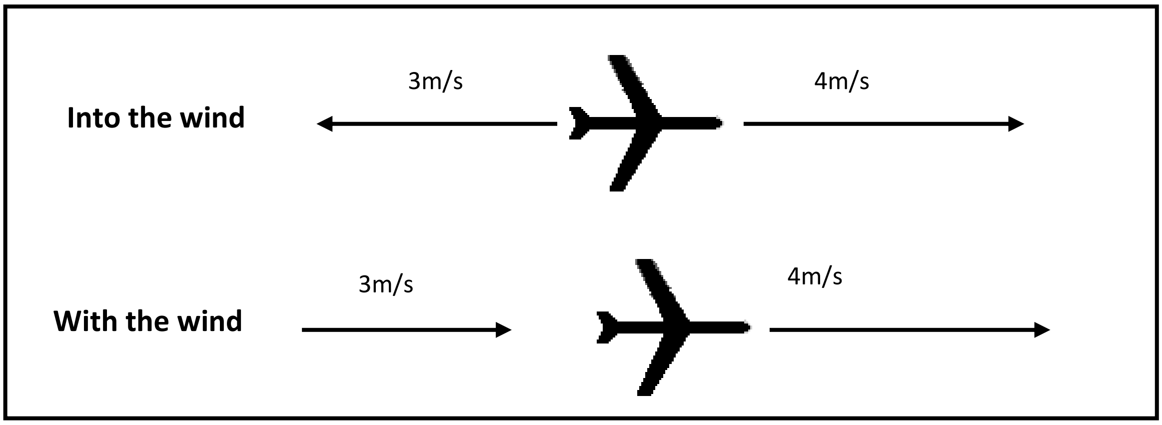


Figure 9 Adding vector into the wind and with the wind

Cross wind

Flying into the cross –wind

In the first second, the engine of the aircraft drives at 4m/s east and at the same time, the wind blows north at 3m/s. The aircraft finishes up at A, 4m/s east and 3m/s north from its starting point. So the diagonal line OA, as shown in the solution below is the aircraft's actual velocity vector.

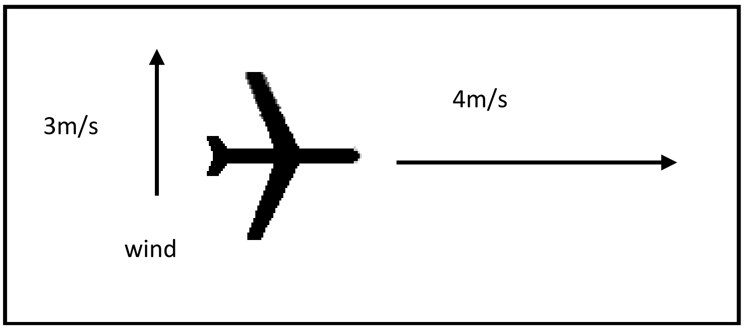


Figure 10 Resolving vector across the wind

R is the resultant vector you get if you add X and Y (see diagram in the solution below). To find the sum of the two velocity vectors we use the Pythagoras theorem. This is because the two vectors form a right angle triangle.

$$\mathbf{R^2 = X^2 + Y^2}$$

The diagram illustrates how Pythagoras theorem is used to calculate the resultant velocity of the plane when encountering a cross wind.

Solution

$OA^2 =-\sqrt{Y^2 + X^2}$

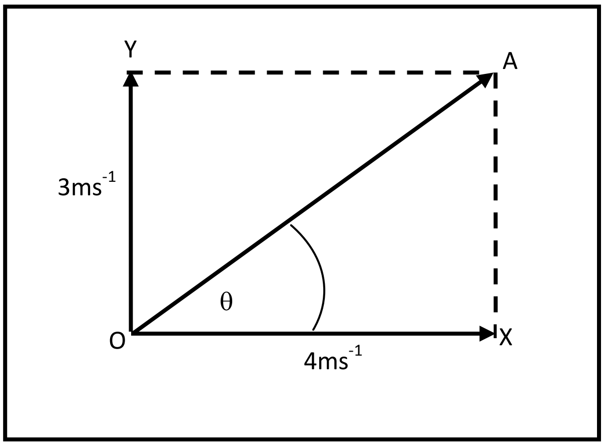
$OA^2 =-\sqrt{3^2 + 4^2}$

$OA^2 =-\sqrt{16 + 9}$

$OA^2 = -\sqrt{25}$

$OA = 5ms^{-1}$

$OA = 5ms^{-1}\ N\ 53.1^0\ E$



Solution

The angle R makes with X from

tan θ = Y/X = opposite /adjacent

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\theta = \frac{Y}{X}$$

$$\tan\theta = \frac{3}{4}$$

$$\tan\theta = 0.75$$

$$\theta = 36.9^0$$

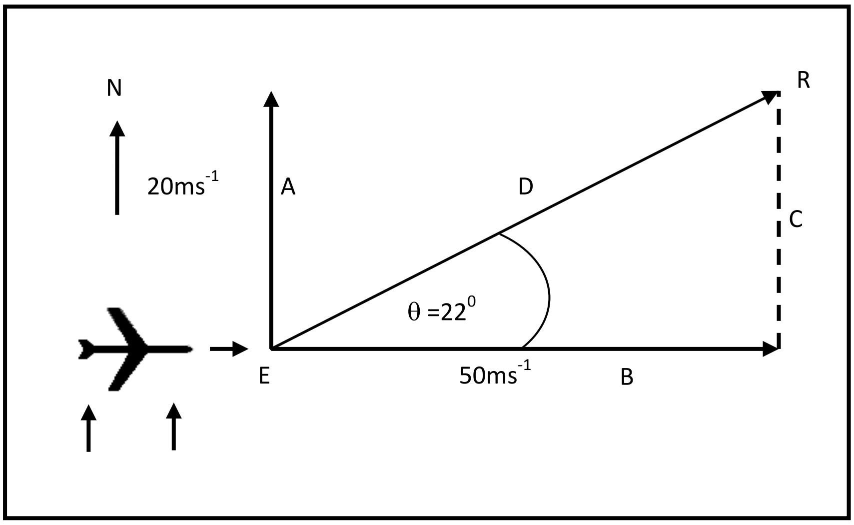
Now press the **shift** button followed by **tan** button on your calculator. You should arrive with the angle indicated above as the angle it makes with the horizontal component. However, your true bearing is calculated and written together with the resultant velocity like what is shown above.

The true bearing is: $90^0 - 36.9^0 = 53.1^0$

Adding other vectors is carried out in the same way as forces. These vectors include velocity and displacement. The example below can be resolved into components as well.

Example 1

A pilot aims his plane due east at 50m/s while the wind is blowing north 20m/s. The resultant speed and direction of the plane are found from the diagonal of the parallelogram.



The working out for this example is shown on the next page. It explains how you can find the resultant and the true bearing.

Solution

$$\text{Resultant} = \sqrt{20^2 + 50^2}$$

$$\text{Resultant} = \sqrt{400 + 2500}$$

$$\text{Resultant} = \sqrt{2900}$$

$$\text{Resultant} = 53.9\text{ms}^{-1}$$

$$\text{Resultant} = 53.9\text{ms}^{-1}\ \text{N } 68^0\text{E}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\theta = \frac{C}{B}$$

$$\tan\theta = \frac{20}{50}$$

$$\tan\theta = 0.4$$

$$\theta = 22^0$$

Resolving into two components

In some cases, it is necessary to split a single vector into two parts. It is called the **components of the vector**. The method is the reverse of an addition of forces called the resolution. Sometimes, it is useful to reverse the parallelogram method of adding vectors. In the diagram above, the resultant (54m/s) can be resolved into two components, (20m/s and 50m/s).

The diagram below shows how a single vector (**R)** has two components, horizontal (**H**) and vertical (**V**). The two components taken together have exactly the same effect as single vector(**R)**.

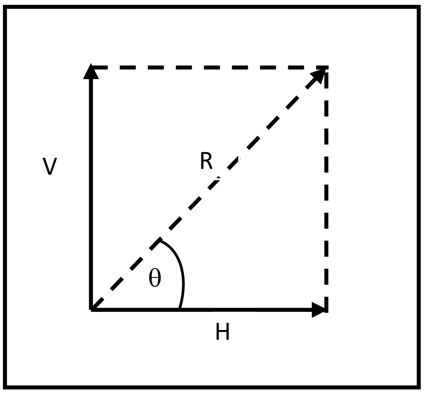


Figure 11 Resultant vector showing two components V and H

The magnitudes of the two components are given by:

Horizontal component of vector is H= R cos θ
Vertical component of vector is V = R sin θ

Using cosine ratio for horizontal component

$$\cos\theta = \frac{H}{R}$$

$$R\cos 22^0 = H$$

$$53.9 \times 0.927 = 50\text{ms}^{-1}$$

Using sine ratio for vertical component

$$\sin\theta = \frac{V}{R}$$

$$R \sin 22^0 = V$$

$$53.9 \times 0.374 = 20ms^{-1}$$

Note: the calculated answers are rounded to the nearest whole number.

Adding velocities

(a)

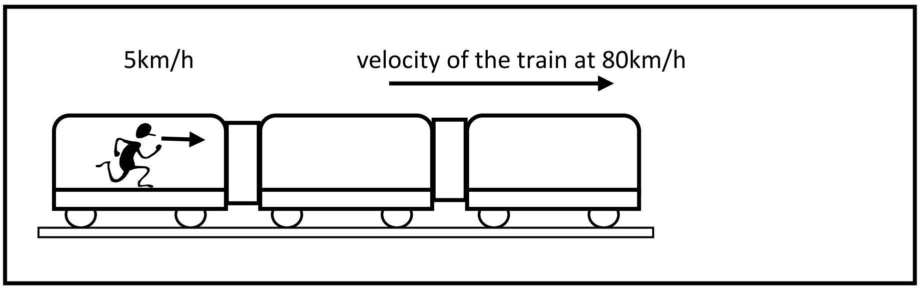


Figure 12 Illustration of adding velocities at same direction

The diagram above shows a train moving at a velocity of 80km/h in a direction along the railway track to the right. This is the velocity of the train relative to the track and how an observer would see as the train went past.

A man inside the train can walk along the train either to the right or to the left at a velocity of 5km/h. How fast is the man on the train moving about the railway track?

Clearly the direction in which he walks makes a difference because if he walks in the same direction with the train, his velocity adds to the velocity of the train. On the other hand, if he walks opposite the train's direction, his velocity is subtracted from the train's velocity.

We usually take the velocity to the right as positive and velocity to the left as negative.

b)

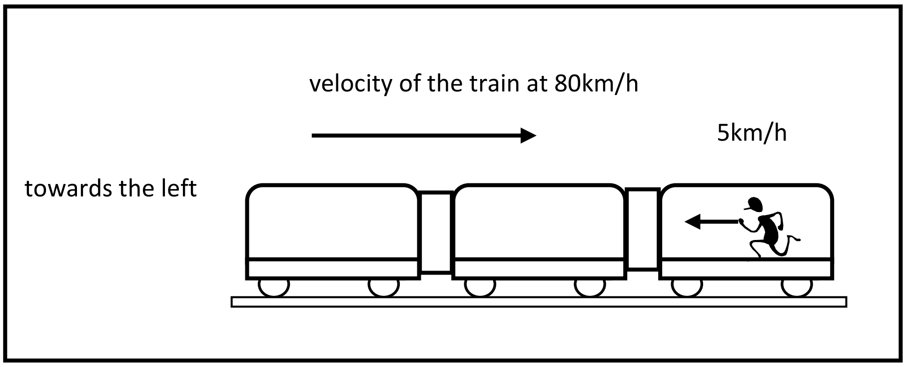


Figure 13 Illustration of subtracting velocities, at opposite direction

It gives the train a velocity of +80km/h in two cases. The man has a velocity of +5km/h if he went towards the right and he has -5km/h if he went towards the left.
Therefore, we add the velocities of the train and the man in both cases to find their resultant or combine the velocity relative to the track.

a) resultant, v = 80km/h + 5km/h = 85km/h to the right.

b) resultant, v = 80km/h – 5km/h = 75km/h to the right.

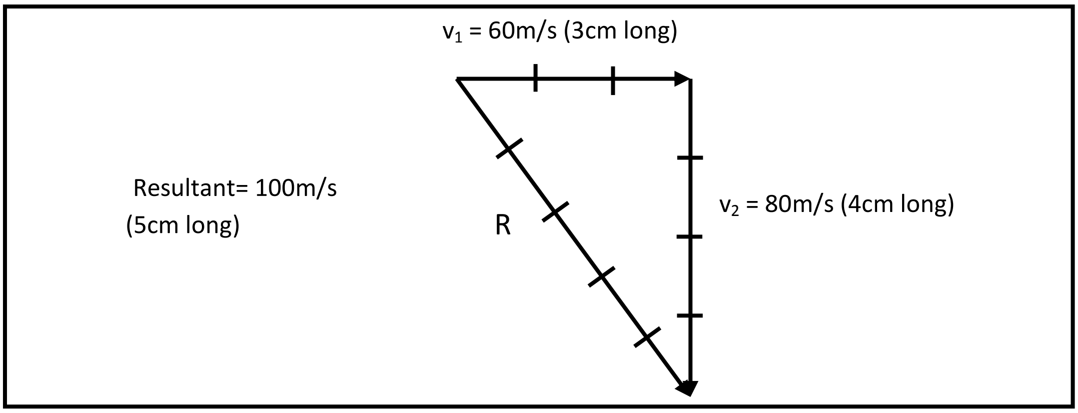
Arrows always represent the direction of vectors.

Adding velocities by parallelogram law
Two velocities that are not in the same straight line can be added using the same parallelogram law that we use for adding forces.

Example 2
A car travels 60ms^{-1} East and then 80ms^{-1} South. What is the resultant velocity of the car?

Suppose two velocities are represented in size and direction by the sides of a parallelogram (drawn to scale). The resultant velocity is represented in size and direction by a diagonal drawn from where the two velocities act.

Use a scale of 1cm = 20m/s



Resultant using Pythagoras Theorem

Using Pythagoras theorem for velocity at right angles (90^o), we can solve for the resultant, which is;

$$\text{Resultant} = \sqrt{80^2 + 60^2}$$

$$\text{Resultant} = \sqrt{6400 + 3600}$$

$$\text{Resultant} = \sqrt{10\,000}$$

$$\text{Resultant} = 100\text{ms}^{-1}$$

Resultant =100ms^{-1} at 53^o to the East.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\theta = \frac{V_2}{V_1}$$

$$\tan\theta = \frac{80}{60}$$

$$\tan\theta = 1.33$$

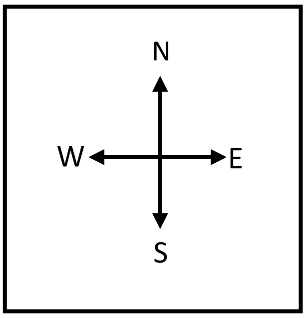
$$\theta = 53^0$$

The Pythagoras theorem is the **analytical method** of finding the resultant of the force. An arrow represents a vector. The length of an arrow represents a magnitude and a direction of the vector shown by the direction of the arrow. The size of the arrow measures from the arrow head to the tail of the arrow. This method will be discussed next.

Combining vectors

Rules for adding vectors

Step 1. Set up a direction reference, e.g., cardinal points.



Direction reference

Step 2. Calculate a suitable scale. Vector diagrams should not be small because this will reduce its accuracy.

Step 3. Draw the first vector keeping its length and direction accurate. Show the direction of the vector with an arrow head in the centre or at the end.

Step 4. From the head of the first vector, draw the second vector. You must be accurate with its length and its direction.

Step 5. The resultant or the vector sum is the line joining the tail of the first to the head of the second vector. The magnitude of the sum is the length of the vector, and the direction is the direction of the vector.

This process can be used in adding two or more vectors.

Arrow represents two or more vectors can be combined to produce a single resultant vector.

Now let us consider an aeroplane flying East at a velocity of 120km/h. The wind also blows east at 20km/h. The resultant velocity is found by drawing the two vectors to the same scale and adding head to tail. The resultant is 140km/h and has the same direction as the two velocities.

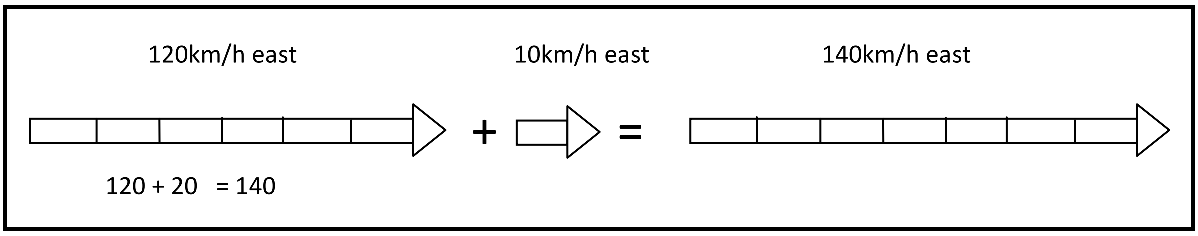


Figure 14 Direction of aeroplane flying East

The second case is where an aeroplane is flying eastward at 120km/h with the wind blowing towards the west at 20km/h. The head wind will slow down the plane. The resultant velocity is found by drawing the two vectors at the same scale and adding head to tail. The resultant velocity of the plane will be 100km/h east. The resultant simply has the same direction as the greater velocity.

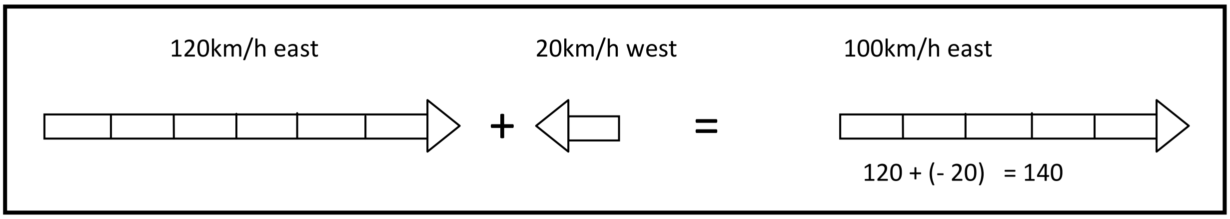


Figure 15 Direction of aeroplane flying East with Wind flowing West

An aeroplane flying eastward at 120km/h encounters a strong wind blowing to the north at 90km/h. To find the resultant we draw a scaled diagram and use trigonometry involving a triangle.

In this case, the vectors are added head to tail and the resultant is found by joining the starting point to the finishing point as shown in the diagram on the next page.

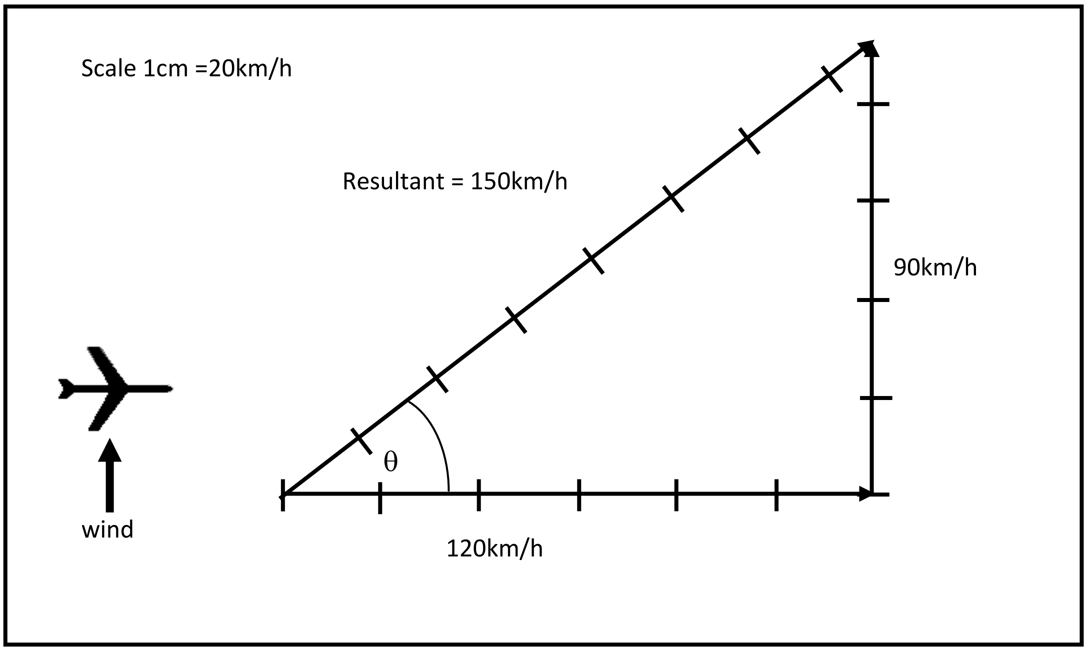


Figure 16 Direction of aeroplane flying East with Wind blowing North

The resultant is the diagonal to the parallelogram. This is constructed by using the component velocity vectors as sides. In a scaled diagram such as this one, you can use a ruler to measure your resultant velocity. The resultant is 150km/h in the East direction. Now to find the angle, you can still measure it with a protractor. It should approximately measure up to 36.9^0.

It can also be calculated by using Pythagoras theorem as examples shown earlier. If the vectors are drawn to scale 1cm=20km/h, then the resultant will be 150km/h.

To work out the angle using a calculator, 90/120 = 0.75. Press **shift button** on your calculator, then press **tan button**. You should get the answer 36.9 on your calculator. Remember, it is the angle and should be written in degrees.

$$\tan\theta = \frac{90}{120} = 0.75 = 36.9^0$$

The velocity and direction written in true bearing is $150kmh^{-1}$ $N53.1^0E$

The parallelogram method to add vectors acting at the same point but not in a straight line is used. The order in adding vectors does not affect the magnitude and the direction of the resultant. There are four different vectors shown in the diagram on the next page. If you add the four vector head to tail in different ways but keeping its direction, you will have the same effect for their resultant.

Two possible ways of joining your vectors are shown in the diagram below.

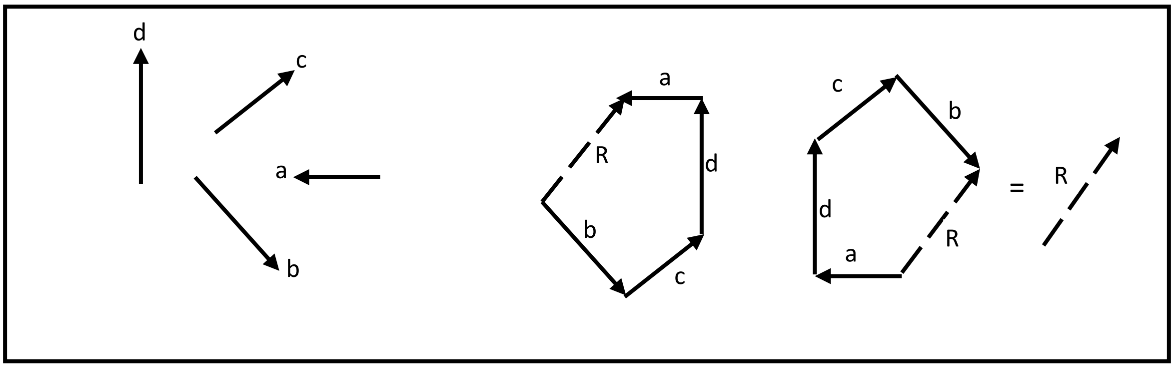


Figure 17 Resultant of a number of vectors

The resultant of many vectors is a single vector that would have the same effect as all the single vectors added head to tail.

Subtracting vectors

In certain problems, it is necessary to subtract vectors. For instance, when subtracting vector **B** from vector **A**, the direction of vector **B** is reversed and added to **A** (head to tail).

A - (-B)

Here, vector **B** has its direction reversed, and when added to a vector **A**, it falls under the subtraction process for an unlike sign. The determination of resultant is in the same way.

For example, if you want to subtract a displacement of 12m south which is **B** and a displacement of 18m north which is **A.** Then we add the vectors and obtain an answer 30m north. Other vectors can be subtracted in the same way as the displacement.

A = 18m North = +18

B=12m South = -12

18m - (-12)m = 30m North

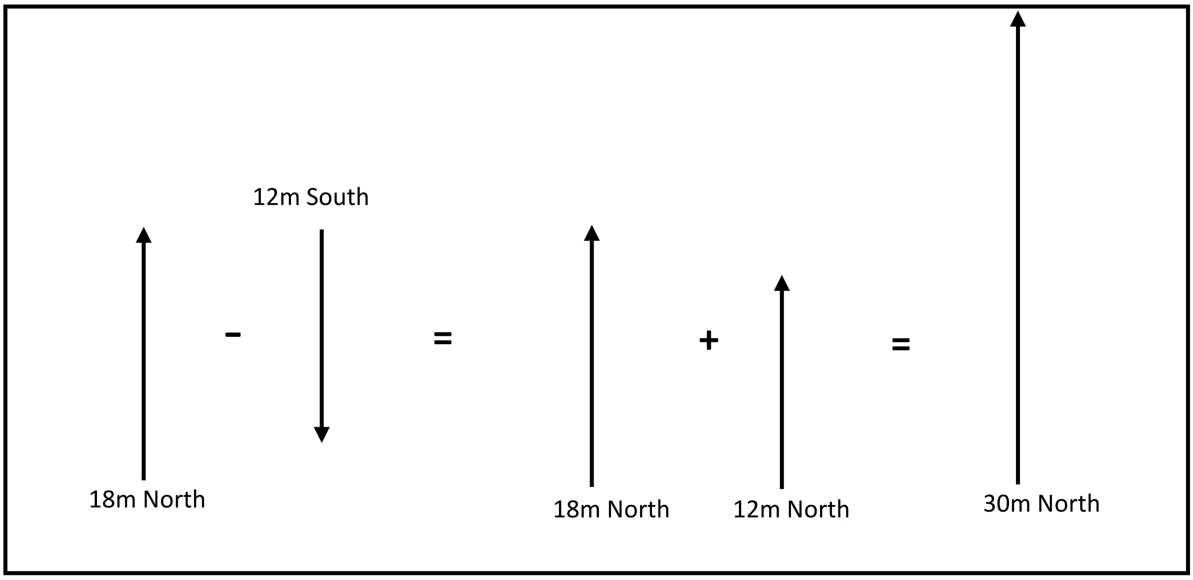


Figure 18 Vector illustration on subtracting opposing direction

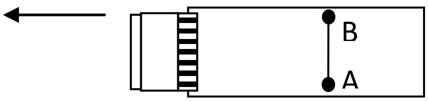
Now check what you have just learnt by trying out the learning activity below!

Learning Activity 2

20 minutes

Answer the following questions by writing all your answers on the spaces provided.

1. A flat top truck is moving west at $12ms^{-1}$. A person on the back of the truck runs from A to B at $3ms^{-1}$. Calculate the velocity of the person with respect to the road.



---

2. An aeroplane is flying due North at 300m/s and a Westerly wind of 25m/s is blowing it off course. Calculate the velocity and direction of the plane**.**

3. A car travelling at 50km/h enters a restricted speed zone and reduces speed to 35km/h. Calculate the change in velocity.

---

4. A person rows a boat across the river at 4km/h. The boat is kept headed at right angles to the banks. The river is flowing at 6km/h and is 200m across.

   a) In what direction from the bank does the person's boat go about the shore?

   b) How long does it take the person to cross the river?

   c) How far from the person's landing point downstream is the starting point?

   d) How long would it take the person to cross the river if there were no current?

---

5. A pilot aims his plane due east at 100m/s while the wind is blowing north at 2m/s. What is his actual speed and direction?

6. A car travels 80m East and 60m South in 10s.

a) What is the change in position?

b) What distance has it travelled?

c) What is the speed of the car?

d) What is the car's velocity?

---

7. A plane is flying at 300kmh^{-1} at 30^{o} east of north. What is the northerly and easterly component of the velocity?

---

8. A power boat which can travel at 10kmh^{-1} in still water heads due south across a river flowing from the west to east at 4kmh^{-1}. What is the boats velocity relative to the river bed?

Thank you for completing learning activity 2. Now check your work. Answers are at the end of the module.

11.2.2 Graphs of Motion

The equations that we studied earlier can be used when acceleration and velocity are constant. However, the graphical analysis of motion has the advantage that it can be used for both constant and variable acceleration because the concept is the same in each case. Moreover, graphs are often useful as they give a continuous picture of the motion from data which is taken at a regular and very small time interval. For example, the gradient of a velocity time graph gives the acceleration as:

$$a = \frac{\Delta v}{\Delta t}$$

If the acceleration for each of the different sections of a velocity – time graph is calculated, the results can be used to plot acceleration - time graph. The time scales for the two graphs are often same. In the same way, the gradient of the displacement - time graph gives the velocity. We can also work backwards using the area under the velocity – time graph. As velocity is calculated from the displacement divided by time, displacement equals velocity multiplied by time.

It is where the area under the velocity versus time graph gives us the size of the displacement. We cannot calculate without additional information, therefore, we find out where an object is at a particular time from a velocity –time graph. It is simply because the graph only gives us the displacement or change in position. To find out the object's final position, we need to know its initial position to which the displacement can be added. The area under the acceleration-time graph has the units of acceleration multiplied by time (a x t) which gives the units of velocity $ms^{-2} \times s = ms^{-1}$. In this case, a velocity-time graph can be created by calculating the area under the acceleration-time graph.

An acceleration–time graph comes from a velocity–time graph after calculating the velocity. The area under the acceleration-time graph measures these changes but give no information about the initial velocity, so you cannot calculate the final velocity from the graph.

We have seen that velocity is almost the same thing as speed, but it is a vector quantity. In a similar way, displacement is almost the same as distance, but it is a vector quantity. Displacement has a size (called distance) and direction.

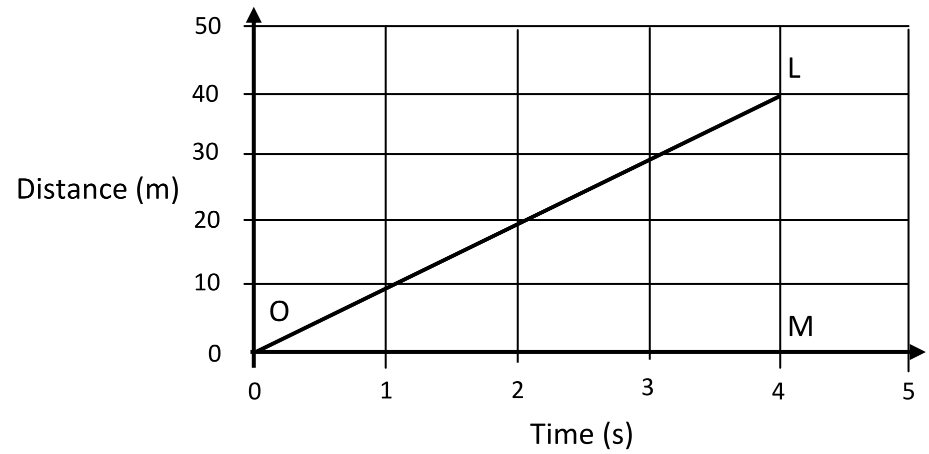
If you move to a different part of the room, you might have a displacement of 2 metres in an Easterly direction. The direction is important. The ticker timer measurements are sometimes used to draw displacement-time graphs, although they are not as useful as velocity time–graphs.

The techniques of graphing are very useful in kinematics. Drawing graphs can simplify calculation of velocity, displacement and acceleration.

Distance –Time Graph

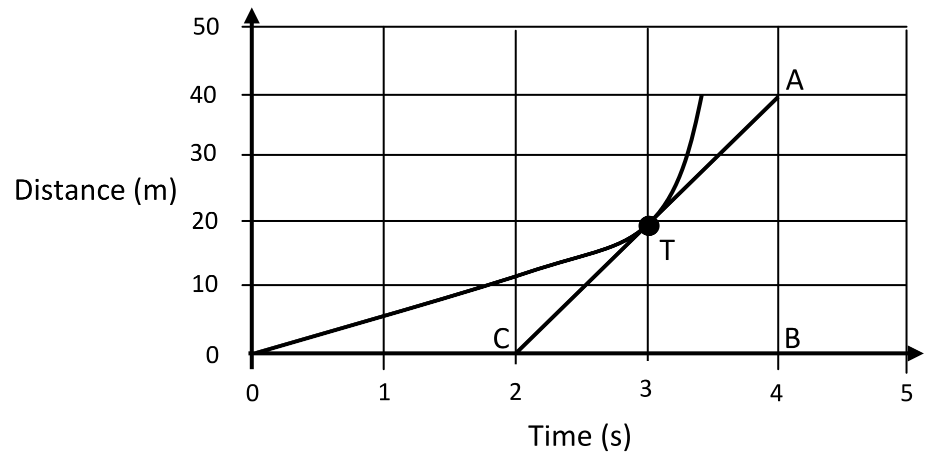
A body travelling with constant speed covers equal distances in equal time. If distance –time graph is a straight line, like OL in the diagram below, it shows a velocity of $10ms^{-1}$. The slope of the graph is OL/OM = 40m/4s = 10m/s

DISPLACEMENT-TIME GRAPH FOR CONSTANT SPEED



When the speed of the body is changing, the slope of the distance-time graph varies and at any point, equals the slope of the tangent. For example, the slope of the tangent is AB/BC = $40ms^{-1}/2s$ = 20m/s. The velocity at that instant corresponding to T is therefore $20ms^{-1}$.

DISPLACEMENT-TIME GRAPH FOR NON CONSTANT SPEED



The two graphs shown above are distance–time graph for a constant speed and a non-constant speed. We can now conclude about the slope of the two graphs.

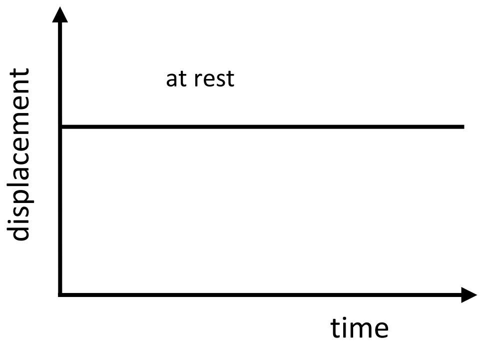
The slope of a distance -time graph gives the speed.

Displacement-time graph

If the car moves away at constant velocity, then the displacement will increase. If the car moves away at a higher velocity, then the line will be steeper. The velocity is shown by the slope (or gradient) of the displacement–time graph. The diagram shows a displacement–time graph for a car at rest (stationary, not moving).

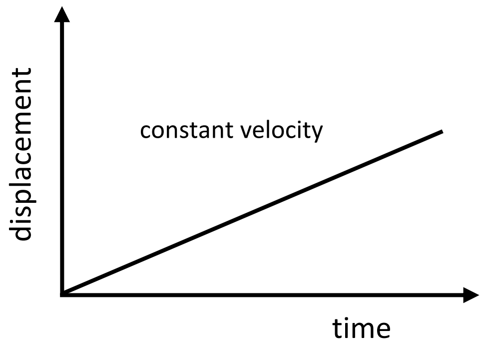
A

DISPLACEMENT-TIME GRAPH FOR AT REST



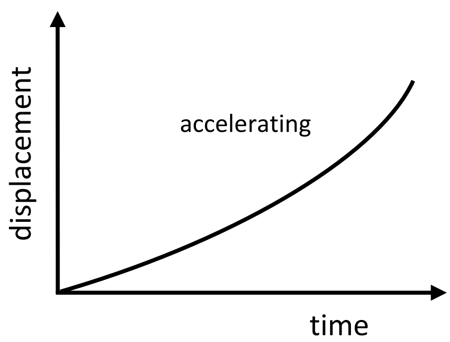
B

DISPLACEMENT-TIME GRAPH FOR CONSTANT VELOCITY



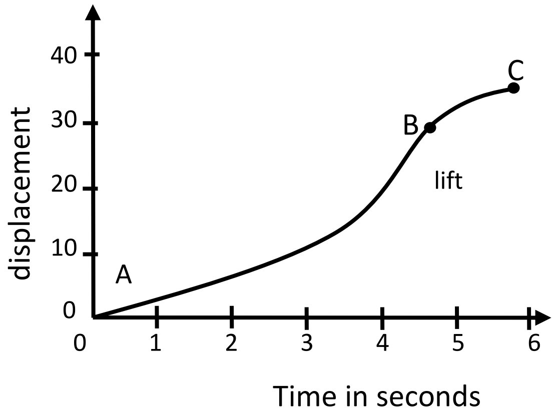
C

DISPLACEMENT-TIME GRAPH FOR ACCELERATING



D

DISPLACEMENT-TIME GRAPH FOR LIFT



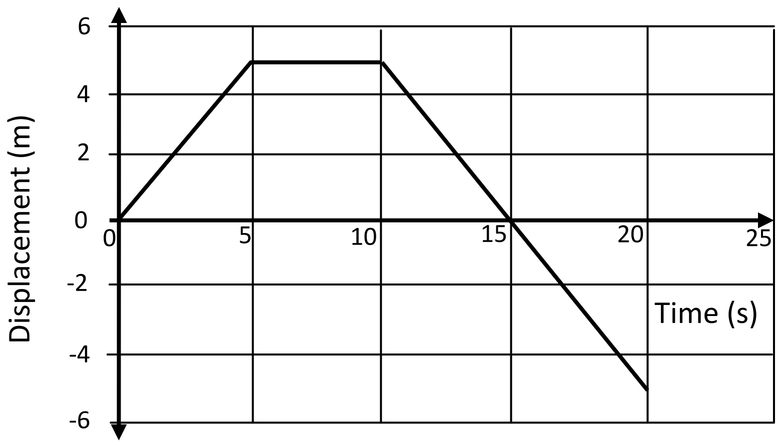
The graph shown above as well as the three on the previous page is displacement-time graph for four different bodies. We can now conclude about the slope of the four graphs.

The slope of a displacement – time graph gives the velocity.

You can still convert a displacement-time graph into a distance time graph. Consider the displacement time graph given below. We may extract following information from it. The initial displacement is zero. This means that a body started from rest and moves with a positive velocity of $1ms^{-1}$ for 5 seconds. For the next 5 seconds, the body was resting. It then moves with a negative velocity of $-1ms^{-1}$ for the next 20 seconds. This means that the body is now travelling in the opposite direction between 10 seconds and 20 seconds.

- Velocity at 4s $=1ms^{-1}$
- Velocity at 15s $= -1ms^{-1}$

DISPLACEMENT-TIME GRAPH



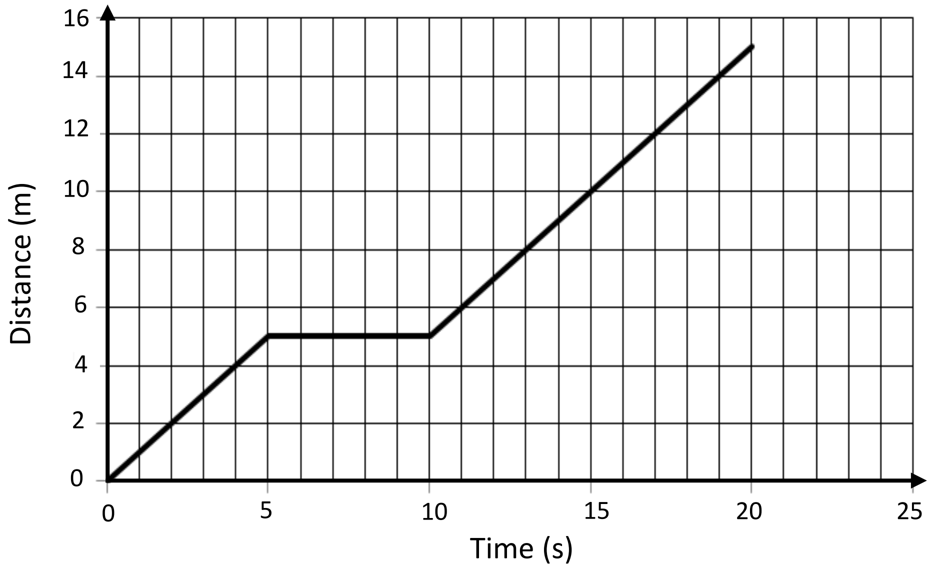
If you convert the graph into a distance–time graph, it will be like the one shown on the next page. There is no negative value of distance travelled. Distance time graph cannot be changed to a displacement-time graph as there is not enough information available.

Speed is the gradient of the distance–time graph at the time indicated.

Speed at time 4s = $1ms^{-1}$.

Speed at time 15s = $1ms^{-1}$

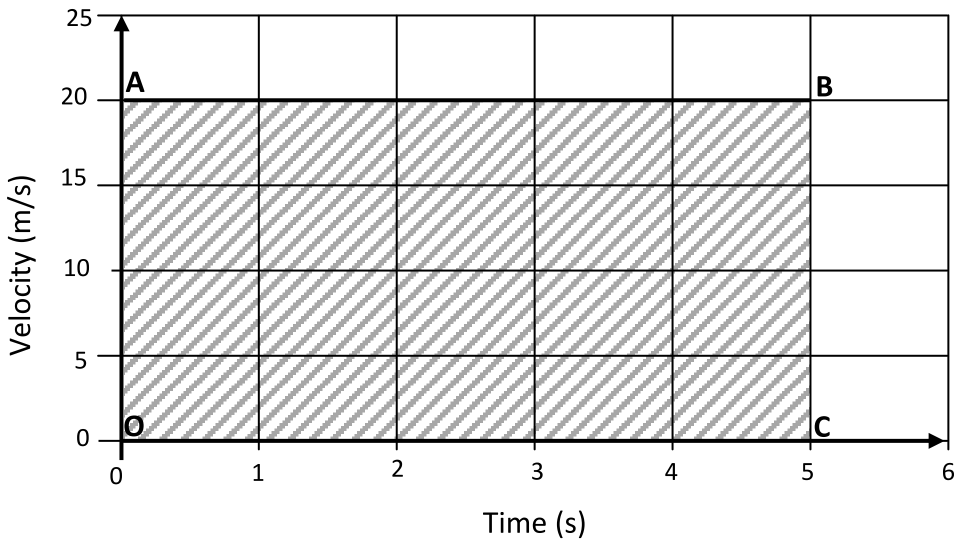
DISTANCE –TIME GRAPH



Velocity–Time Graph

When the velocity of a body is plotted against time, the graph obtained will be a velocity – time graph. It provides a way of solving motion problems. Tape charts are crude velocity-time graphs which show the velocity in jumps rather than smoothly, as occurs in practice. A motion sensor gives a smoother plot.

CONSTANT VELOCITY TIME GRAPH



In the graph, AB is the velocity-time graph for a body moving with a constant velocity of 20m/s. Since distance = average velocity x time, after 5 seconds it will have moved 20m/s x 5s = 100m. It is the shaded area under the graph, the rectangle OABC.

The diagram below shows another graph with constant velocity. The PQ is the velocity–time graph for a body moving with uniform acceleration. At the start of the timing, the velocity is 20m/s but increase steadily to 40m/s after 5 seconds. If the distance covered equals the area under the PQ, that is the shaded area OPQRS, then

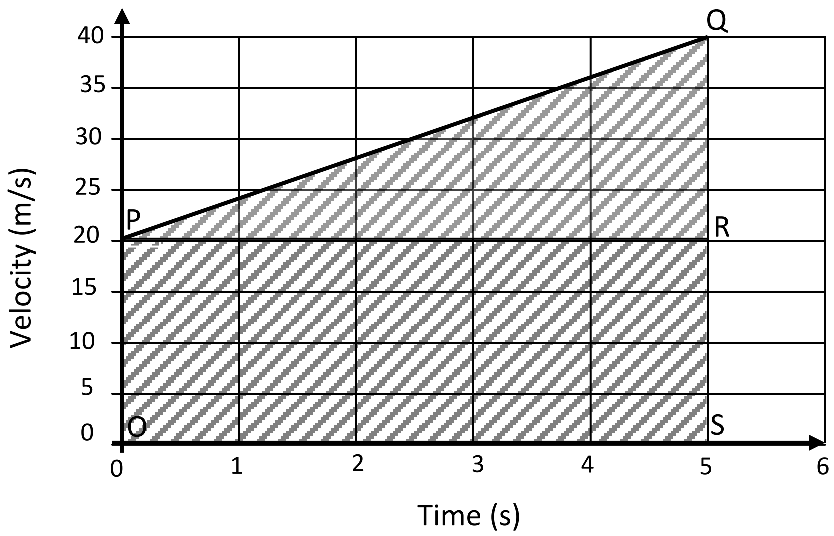
Distance = area of rectangle OPRS + area of triangle PQR

Distance = OP x OS + ½ x PR x QR **(area of triangle = ½ base x height)**

Distance = 20m/s x 5s + ½ x 5s x 20m/s

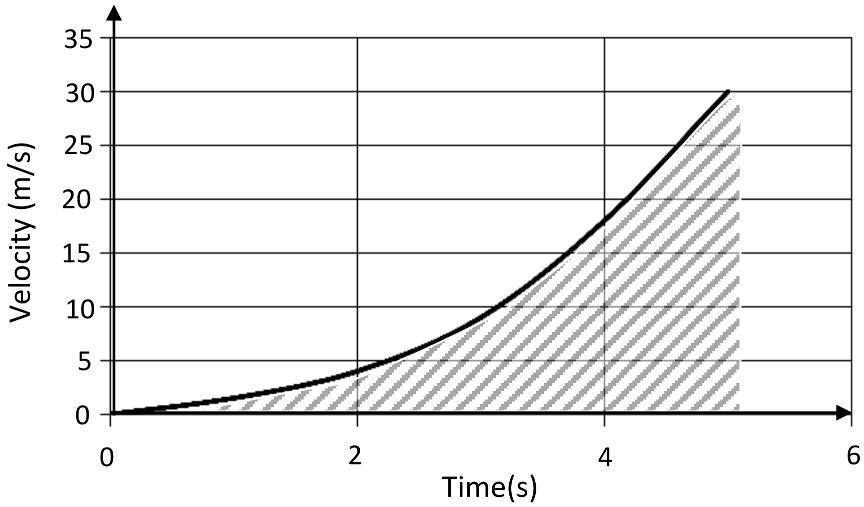
= 100m+ 50m = 150m

VELOCITY TIME GRAPH FOR CONSTANT ACCELERATION



The diagram shows a velocity – time graph for a car travelling at different velocities. It shows that the car is accelerating or increasing its acceleration.

VELOCITY TIME GRAPH FOR INCREASING ACCELERATION

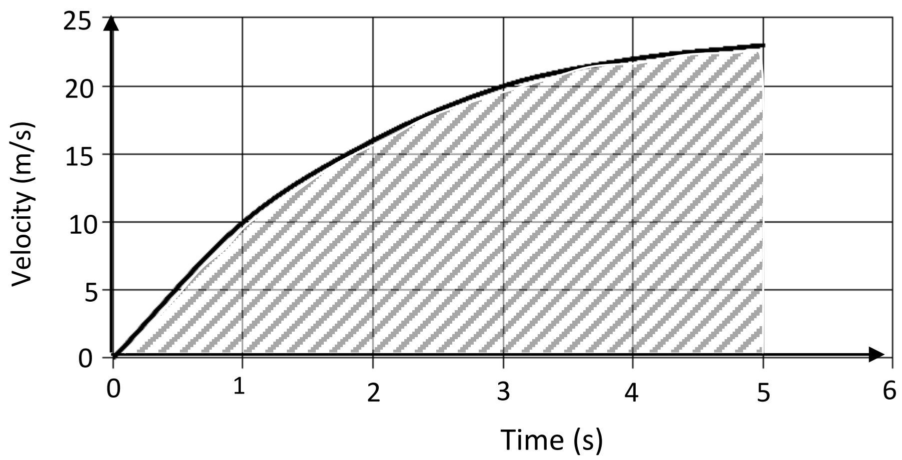


What would the graph look like if the car accelerated gently?

Because its velocity would increase the graph would slope upwards. In this graph, the car started from a rest where the velocity is zero and accelerated uniformly (steadily). A straight line graph means a uniform constant acceleration.

The graph below shows the car decreasing its velocity.

VELOCITY TIME GRAPH FOR DECREASING ACCELERATION

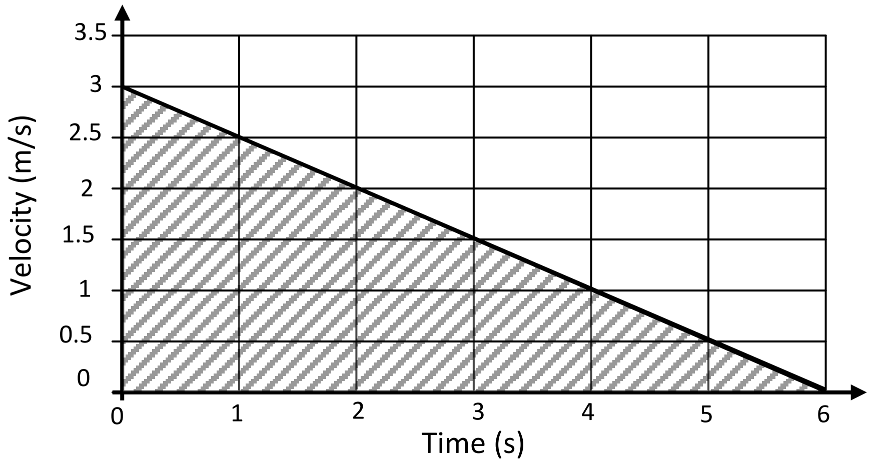


Remember: If an object starts from rest, the velocity is zero.

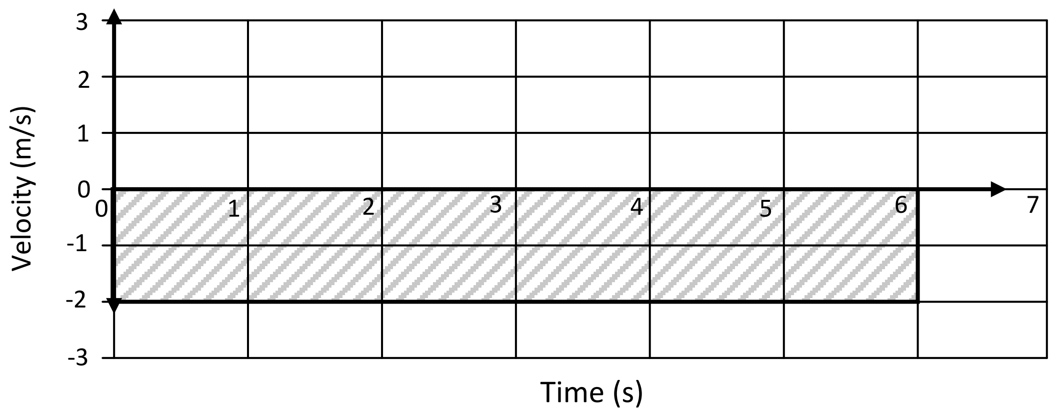
A straight line graph means uniform, constant acceleration.

What would the graph look like if the car accelerates more rapidly?
Since its velocity increased more rapidly, the graph would be steeper.

VELOCITY TIME GRAPH FOR CONSTANT DECELERATION



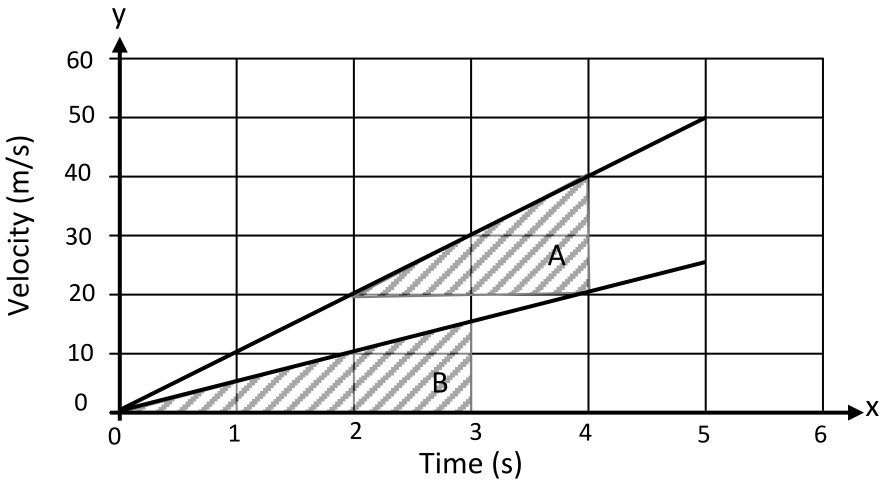
VELOCITY TIME GRAPH FOR CONSTANT NEGATIVE ACCELERATION



The slope of a velocity- time graph shows how fast a body accelerates. The steeper the slope, the acceleration will be greater. This simply means the body is moving faster in a time of one second. If the slope of the graph is less steep, then it has a gentle acceleration. Hence, the body is moving slowly in a time of one second.

The velocity-time graph below will help compare the motion of two bodies moving with constant velocities.

VELOCITY TIME GRAPH COMPARING THE MOTION OF TWO BODIES MOVING WITH CONSTANT VELOCITY



To calculate the acceleration of the two bodies, you can use the formula below. The answer, you obtain from calculating the slope should give some idea about the motion of the two bodies. Unless the bodies are accelerating, then you must plot a line of best fit or tangent as shown earlier. This will make it easy for you to calculate the slope. Regardless of where you want to work out the slope, the value for each will be same anywhere on the line graph. This is because both motions are constant.

Now let us try and calculate our slope.

The acceleration is shown by the slope (gradient) of the velocity-time graph).

$$\textbf{Slope} = \frac{\textbf{rise}}{\textbf{run}}$$

$$\textbf{Slope} = \frac{\Delta y}{\Delta x}$$

$$\textbf{Acceleration} = \frac{v_2 - v_1}{t_2 - t_1}$$

Slope A

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{40 - 20}{4 - 2}$$

$$\text{Acceleration} = \frac{20}{2}$$

$$= 10\text{ms}^{-2}$$

Slope B

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{Slope} = \frac{15 - 0}{3 - 0}$$

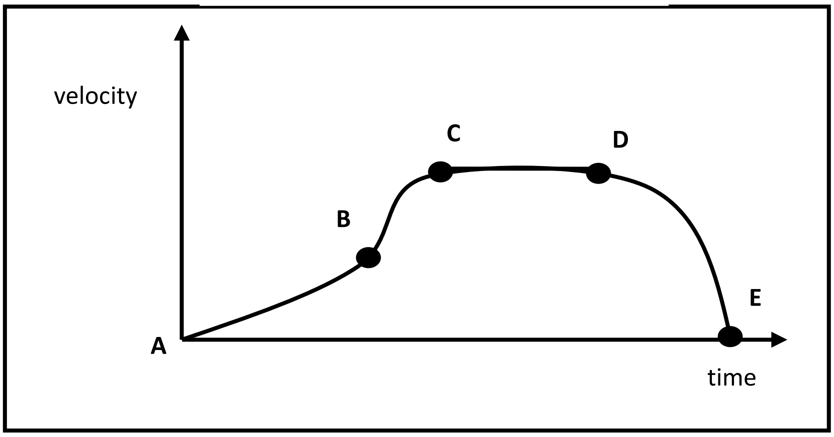
$$\text{Acceleration} = \frac{15}{3}$$

$$= 5\text{ms}^{-2}$$

You will now see the acceleration is much higher for line graph A and is less for line graph B.

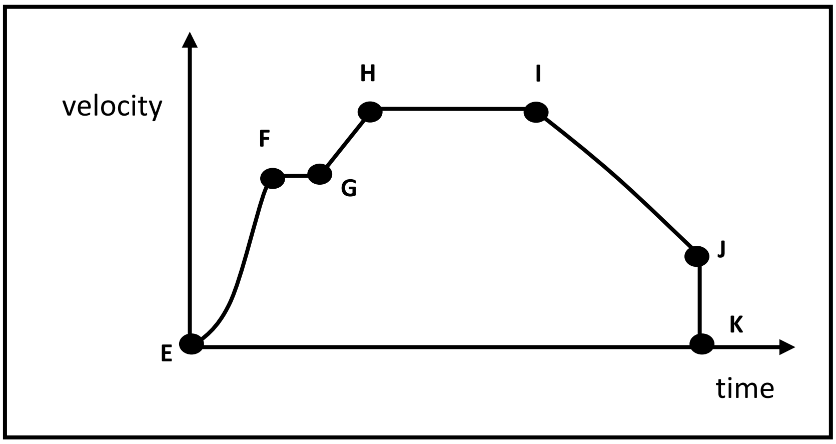
The diagram shows a velocity-time graph for a car starting off from one set of traffic light and stopping at the next set of lights.

VELOCITY- TIME GRAPH FOR A CAR



In this graph, you will see that the car accelerates from **A** to **C,** than travels at a constant velocity from **C** to **D** and then decelerates (brakes) rapidly to stop at point **E**. The car is accelerating most at point **B** and **C.**

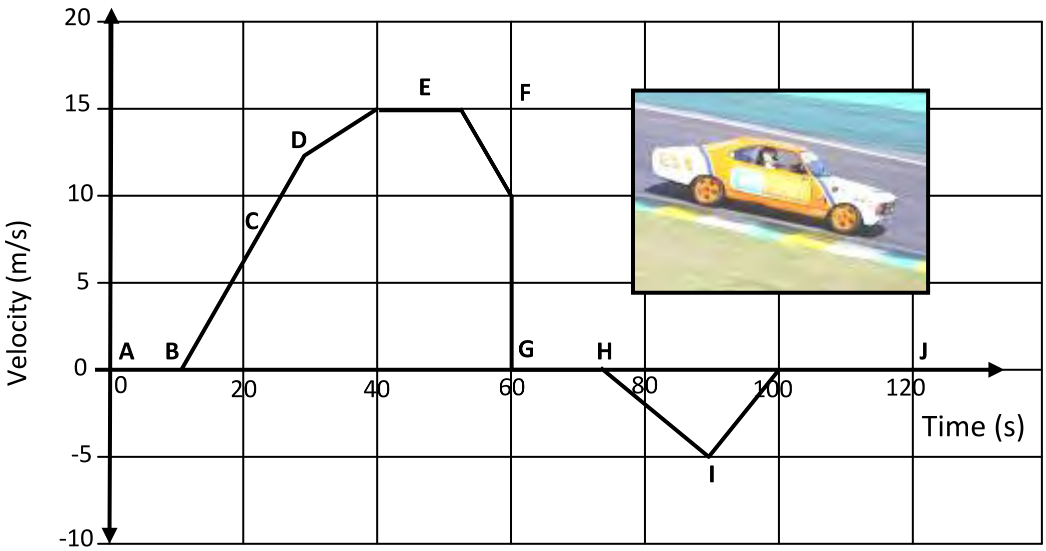
VELOCITY- TIME GRAPH FOR A MOTORBIKE



From the diagram, you will see that the motorbike is accelerating more rapidly at **E** and **F**. The motorbike will shift gear at point **G** and **H** where the speed changes. The fastest speed will be achieved between **F** and **G**. The rider will start to use the brake to stop the car at point **I**. At point **J** and **K,** the graph shows the rider hitting a solid brick wall.

Distance travelled is shown by the area under the velocity – time graph.

VELOCITY TIME GRAPH FOR CONSTANT NEGATIVE ACCELERATION



The diagram above shows the velocity of a stock car (insert) when starting a race crashing into another car and then reversing.

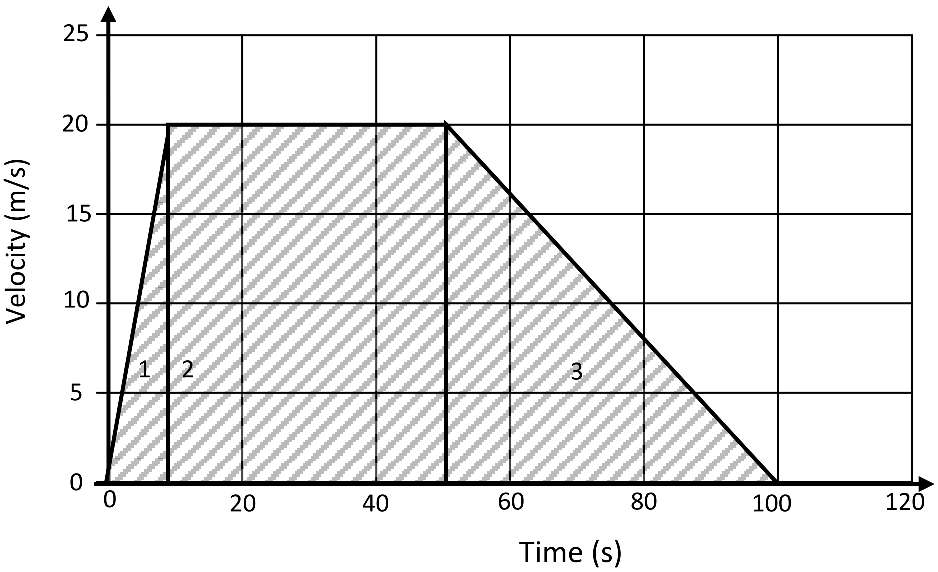
From the diagram, we can see that the driver has to wait 10 seconds before the starter waves his flag to start the race. The car acceleration is greatest at point **C** because of the steepness of the graph. The fastest speed will be achieved between point **D** and **E** where the velocity will be 15m/s. The driver will start to stop his car by pressing the brake at point E and it will come to a stop at point **G** after hitting another car. The time for the car to travel this far is 60 seconds. The driver rested for 10 seconds before reversing his car at point **H**. When the car is reversing the velocity is negative. The car's maximum velocity at reverse will be negative 5m/s (-5m/s) at point **I**. The driver took 30 seconds to reverse the car and finally came to a stop at point **J.**

Area under the velocity-time graph

The graph below shows a body that starts from rest and accelerates uniformly for 10s reaching a velocity of 20ms^{-1}. It then travels at this velocity for 40s, before deceleration to rest in 50s.

Example 1

VELOCITY TIME GRAPH



To calculate the total displacement you can use two methods.

a) Calculate the displacement for each part of the graph and then add them together.

Displacement for segment 1 = average velocity x time
= (0 + 20) x 10 /2 = 100m

Displacement for segment 2 = velocity x time
= 20 x 40 = 800m

Displacement for segment 3 = average velocity x time

(20 + 0 x 50 /2 = 500m

Therefore, total displacement = 100 + 800 + 500 = 1400m

You will see from the graph that two segments are triangles and the other is a rectangle. You can use area, A = ½ base x height for triangles or area, A = length x width for rectangles.

b) Use the fact that the area under the velocity vs time graph is the displacement.
Now the shape of the area between the graph and the 'x' axis is a trapezium.
The formula for the area of the trapezium is.

Area = half the sum of the two parallel sides multiplied by the height.
Where, Area = (40 + 100)/2 x 20 = 140 x 10 = 1400m.

You will see that the answers obtained either way are same but method B is much quicker.

The acceleration of the velocity- time graph is the slope of the graph.

Segment 1. Slope = $\frac{\text{rise}}{\text{run}} = \frac{v_2 - v_1}{t} \quad \frac{20 - 0}{10} = 2ms^{-2}$

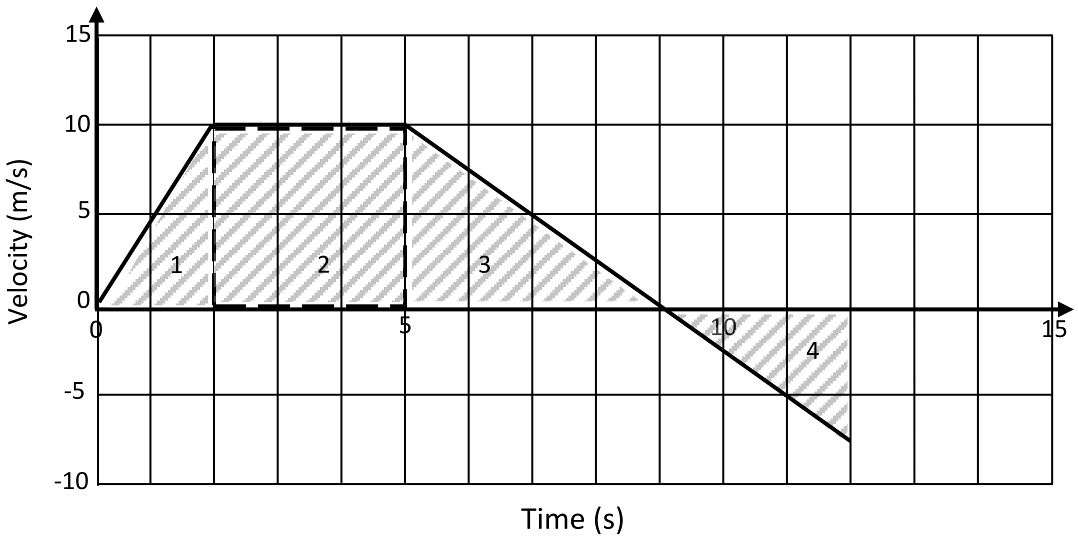
Segment 2. The acceleration = 0 (since there is no change in velocity)

Segment 3 . The acceleration = Slope = $\frac{\text{rise}}{\text{run}} = \frac{v_2 - v_1}{t} = \frac{0 - 20}{50} = -0.4ms^{-2}$

Since the acceleration has a negative value, we conclude that the object's motion is slowing down or decelerating.

Example 2

VELOCITY TIME GRAPH FOR A CAR TRAVELLING ALONG A STRAIGHT ROAD



Given is a velocity–time graph for a car travelling along a straight road. Find the:

a) acceleration at 1s, 3s and 6s.

b) displacement after 12s. You must know that the acceleration is the gradient of the graph.

Between t =1 and t = 2, the gradient of the graph is 10/2 = 5. Therefore, the acceleration is $5ms^{-2}$. Even if you calculate the gradient at t = 1s, it will still be $5ms^{-2}$.
Between t = 5 and 12, the gradient is -17.5/7 = -2.5, so the acceleration is $-2.5ms^{-2}$.

The displacement is the area under the graph. You divide the total area into four parts and add your answer.

Total area = area of triangle 1 + area of rectangle + area of triangle 2 + area of triangle 3

$$= {}^{1}/_{2} \times (10 \times 2) + 10\,(5-2) + {}^{1}/_{2} \times (10 \times 4) + {}^{1}/_{2}\,(-7.5 \times 3)$$
$$= 60 - 11.25$$
$$= 48.75m$$

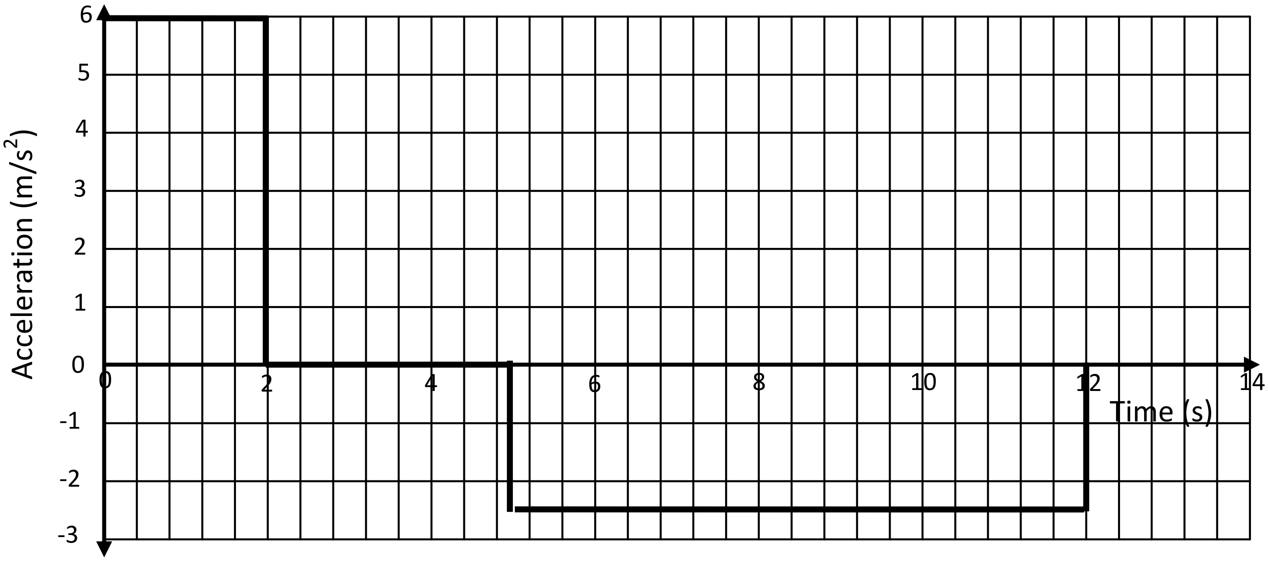
The area under the time axis is a negative displacement. It means that the moving object is travelling in the opposite direction. The distance covered in the above example would be, 60 + 11.25 = 71.25.

The acceleration for the velocity time graph is represented in the acceleration time graph below.

Acceleration – time graphs

It is also possible to draw acceleration–time graphs from velocity–time graphs. The following example is based on the above velocity-time graph.

ACCELERATION –TIME GRAPH



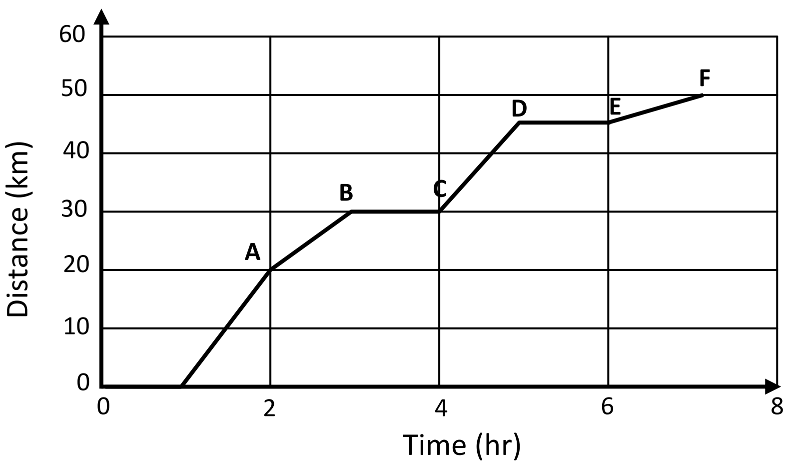
Now check what you have just learnt by trying out the learning activity below!

Learning Activity 3

60 minutes

Answer the following questions on the spaces provided.

1. The graph below shows a distance-time graph for a girl on a cycle ride.



a) How far did she travel?

b) How long did she take to travel?

c) What was her average speed in km/h?

d) How many stops did she make?

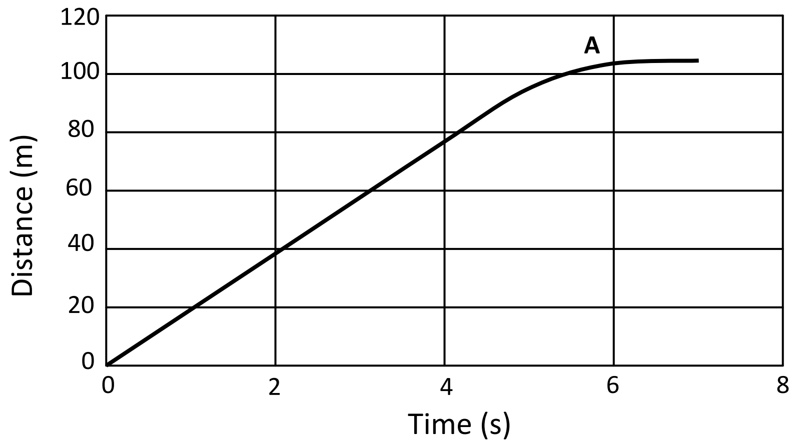
e) How long did she stop for altogether?

f) What was her average speed, excluding stops?

g) How can you tell from the shape of the graph, when she travelled fastest? Over which stage did this happen?

2. The graph below shows the distance travelled by a car plotted against time.

DISTANCE AGAINST TIME GRAPH OF A CAR

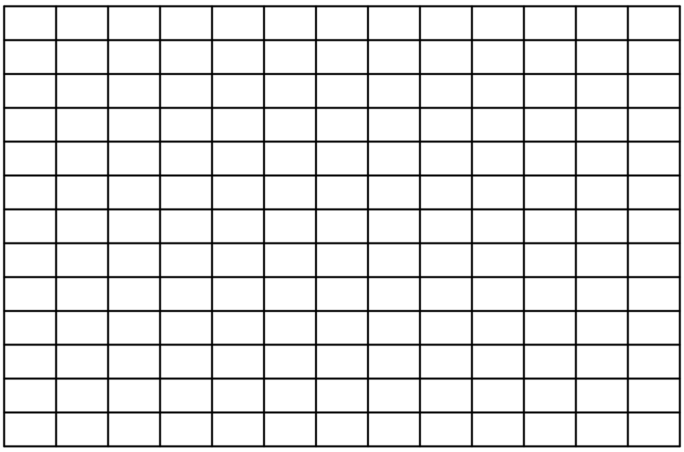


a) How far has the car travelled at the end of 5s?

b) What is the speed of the car during the 5s?

c) What happened to the car after A?

d) Draw a graph showing the speed of the car during the 5s.



3. A car traveling at 20m/s slows down to 10m/s in 2 seconds. Find its

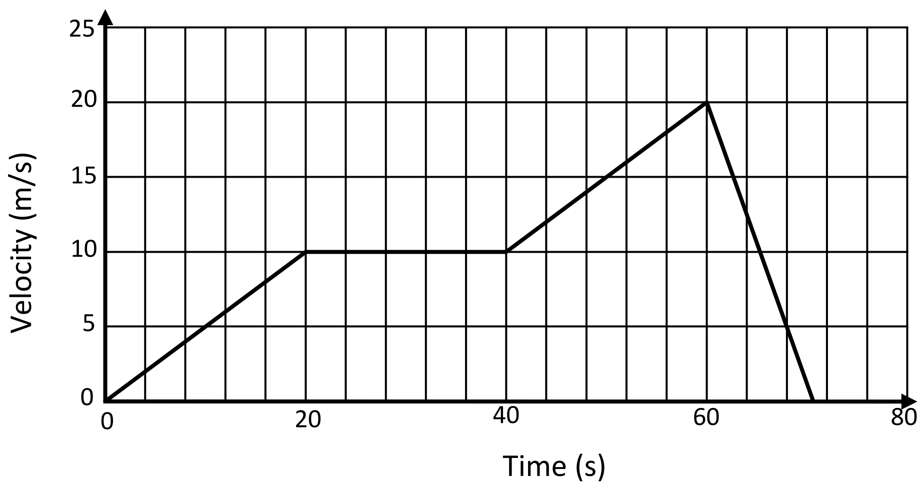
a) acceleration.

b) deceleration.

4. A truck is travelling at 12m/s and increases its speed to 28m/s in 4 seconds. What is its acceleration?

5. The velocity–time graph below show the motion of a bus.

VELOCITY TIME GRAPH OF A BUS



a) Find the acceleration at t= 10s and t= 30s.

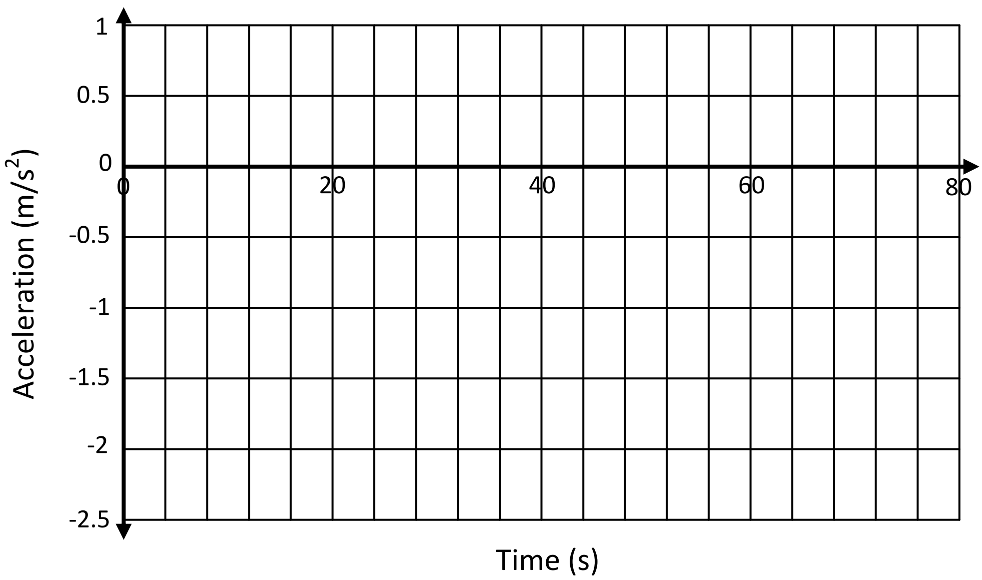
b) Find the velocity at time = 60s.

c) Find the distance covered in first 40s.

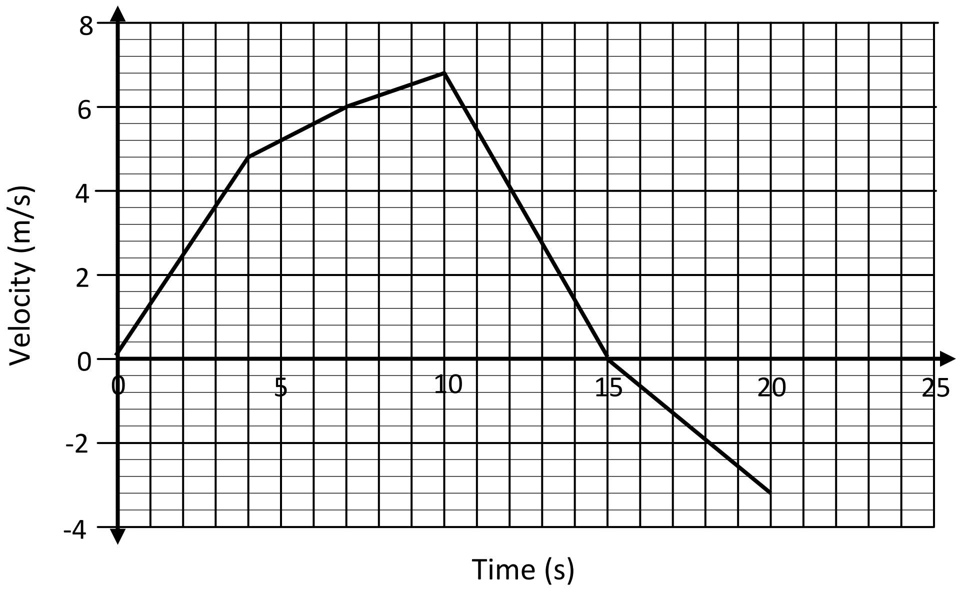
d) Find the acceleration at t= 70s.

e) Draw an acceleration-time graph for the above motion.

ACCELERATION TIME GRAPH



6. The graph below shows a velocity–time graph for an object moving in a time interval of 20s.



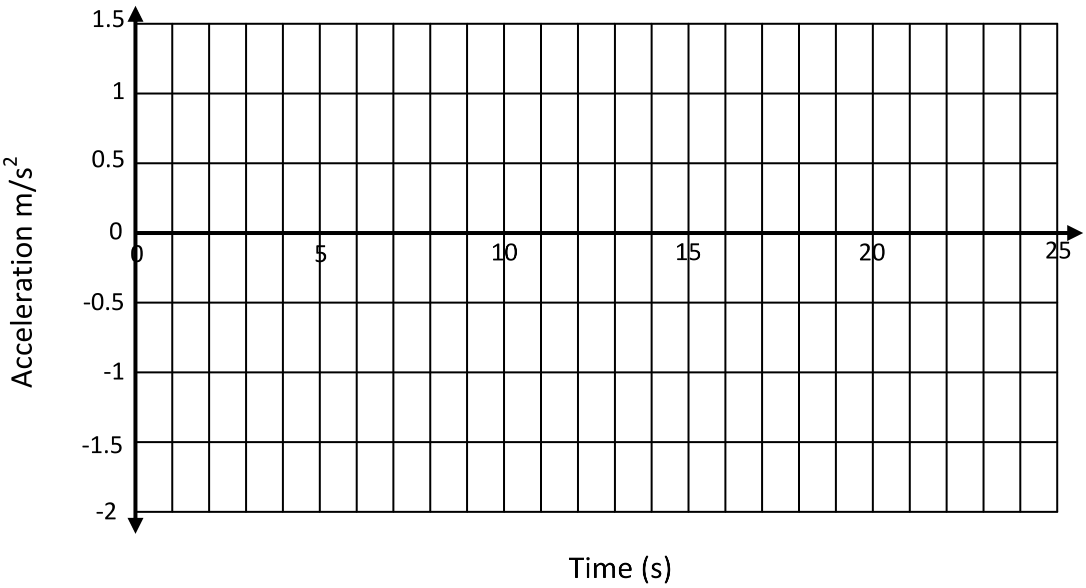
a) Find the distance travelled.

b) Find the displacement.

c) Work out the average velocity.

d) Determine the acceleration during each part of the motion and draw an acceleration–time graph.

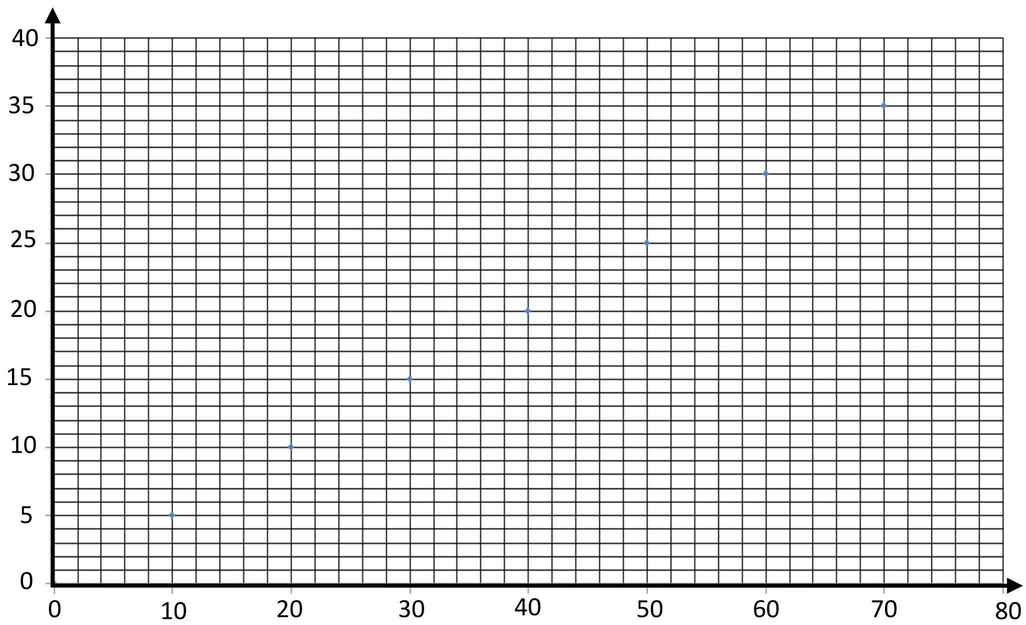
ACCELERATION TIME GRAPH



7. The velocity time graph varies with time as shown in the table below.

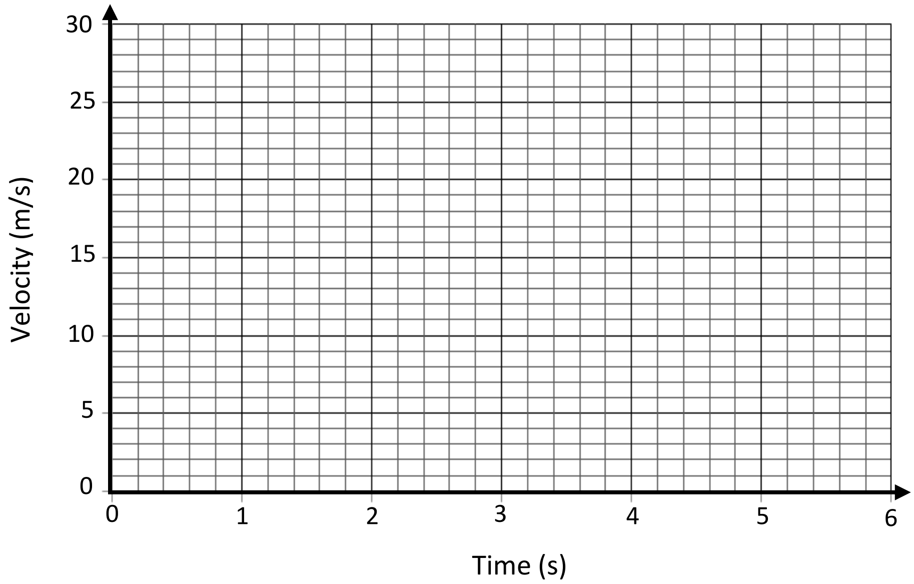
0	5	10	15	15	15	15	12	9	6	3	3
0	5	10	15	20	25	30	35	40	45	50	55

On the graph paper below, draw a graph of the velocity vs. time for this car.



8. Plot the following data

s (metres)	0	1	4	9	16	25
t (seconds)	0	1	2	3	4	5



Work out your answer from the graph.

a) Is this uniform velocity?

b) What is the average velocity during the 5 seconds?

c) What is the average velocity during the first second?

d) What is the average velocity during the first two seconds?

e) What is the average velocity during the time between t=2s and t=4s?

f) What is the average velocity during the time between t=2.5s and t=3.5s?

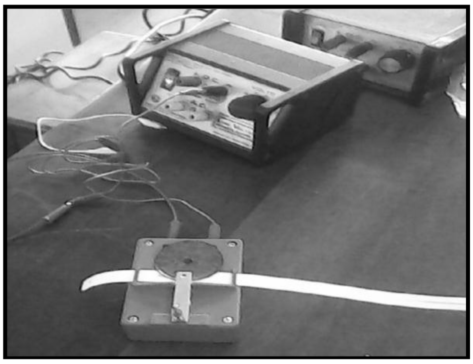
g) Estimate the instantaneous velocity at t= 3s.

h) Draw a tangent to the graph. Find the slope of the tangent. How does this slope compare with your answer?

Thank you for completing learning activity 3. Now check your work. Answers are at the end of the module.

Ticker - Tape Timer

A ticker–tape timer is a device that can be used to record the motion of an object. In the laboratory, a convenient method of measuring speed over a short time interval is the ticker timer as shown in the diagram below.



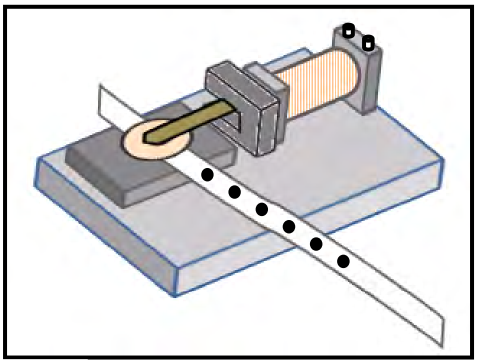


Figure 20 Ticker timer with tape attached

Here, a length of the ticker tape is attached to a moving object. The tape passes through the ticker timer under a piece of carbon paper and a hammer. The hammer strikes the carbon paper and the tape at a regular time interval leaving series of dots on the tape. These dots provide information about the motion of the object.

When a paper tape is pulled through the timer that is operated at main frequency (50Hz), a dot is marked on the tape every 0.02s (i.e. the time between the dots is a twentieth of a second). This constant time interval between successive dots is called one 'tick'. The distance travelled by a moving object to which the tape is attached can be measured from the distance between the first and the last dots. It is a method of measuring the average speed of a moving object over a particular time interval. The greater is the separation of the dots, the greater the velocity of the object. Close dots represent a low velocity while a high velocity is represented by dots being far apart. Now look at the diagram below. It shows how the ticker timer should be set up in the laboratory.

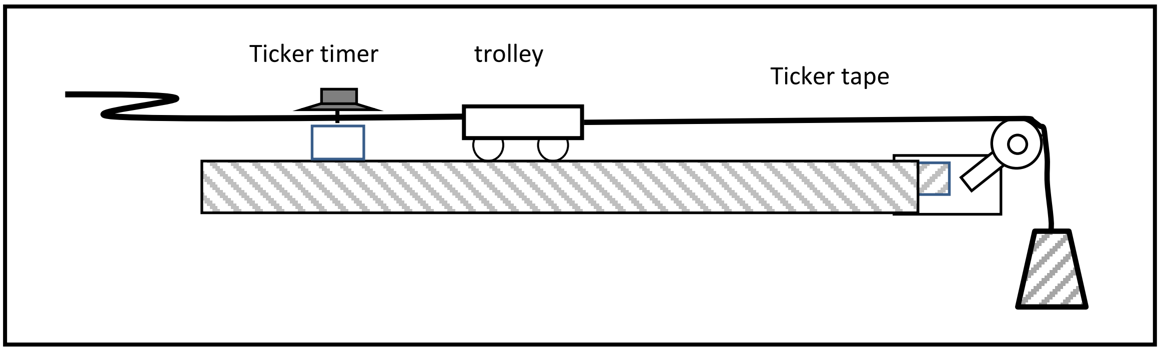


Figure 21 Ticker tape being pulled by a trolley.

Average speed in each 0.02s interval v = s/t (cm/s)	100	200	300	400	500

Table 2 Average speed table.

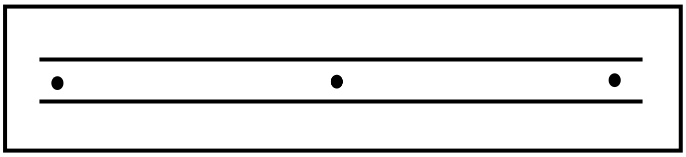


Figure 22 Ticker tape showing a higher velocity.

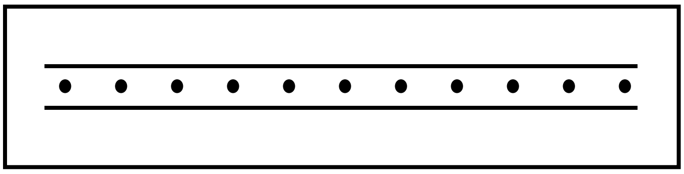
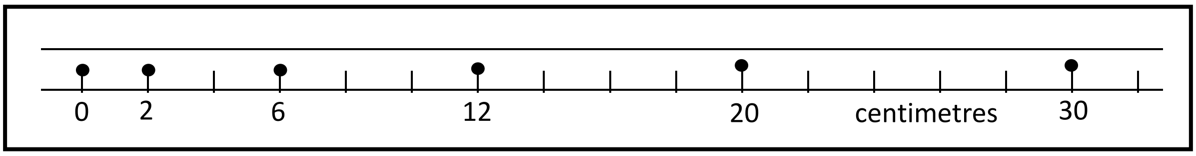


Figure 23 Ticker tape showing a low velocity.

Example



Ticker tape of 30m length

The total distance travelled = total tape length =30cm

The total time taken is 5 ticks = 5 x 0.02s = 0.1s

Therefore the average speed = 30cm /0.1s = 300cm/s

This is the average speed over the whole length of the tape.

The increase in speed between successive 0.02s time intervals is 100cm/s. Thus, the average acceleration is:

$$100\text{cms}^{-1}/0.02\text{s} = 5000\text{cm/s}^2.$$

Another method

For a uniform accelerating body, we can calculate the acceleration between any two tape lengths another way.

For example, consider the first and the fifth lengths:

The 1st length represents the initial speed, u = 100cm/s

The 5th length represents the final speed, v = 500cm/s

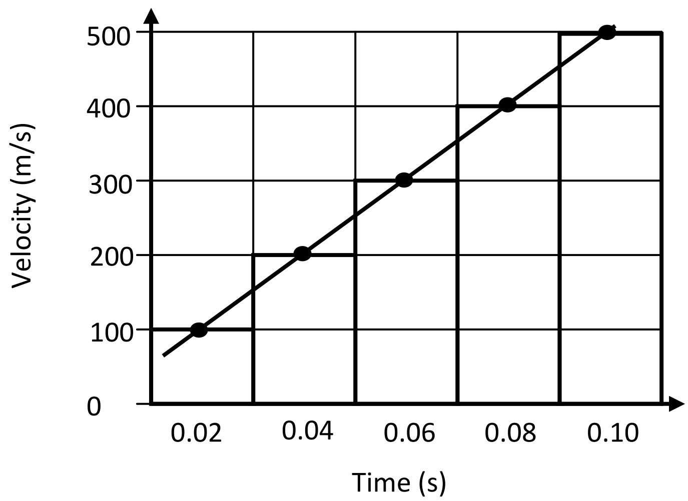
The time interval between the 1st and the 5th strips = 0.02 x 4 = 0.08s

$$\text{Acceleration} = \frac{v - u}{t}$$

$$\text{Acceleration} = \frac{500 - 100}{0.08} = 5000\text{ms}^{-2}$$

If this is a ticker tape strip chart, you could find the total distance travelled by measuring the total amount of tape under the graph. However, from the graph like this, the area under the graph is the distance travelled.

VELOCITY-TIME GRAPH FOR TICKER TAPE



Using what you have just learnt about working out time intervals with the ticker timer, you can use this to help you calculate the velocity at which the tape is travelling.

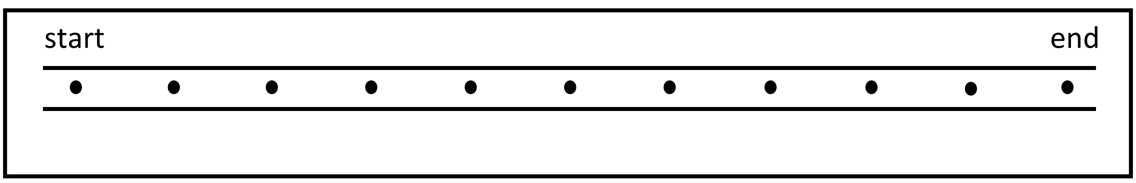


Figure 24 Ticker tape dot interval

Looking at the tape on the previous page, you will see that there are 11 dots on it. This means that there are 10 time intervals between these 11 dots. The time interval between two dots is 0.02s or 0.2 seconds.

An easier way of saying this is:

1 space = 0.02s therefore 10 spaces = 0.2s

If you measure the length of the tape, the distance between the first and the eleventh dots, you will find that it is 10cm long. It means that the 10cm of the tape has passed through the ticker timer in 0.2 seconds and from this you should be able to calculate the velocity.

$$v = s/t$$
$$= 10cm/0.2s$$
$$= 50cms^{-1}$$

Consider a piece of ticker tape as shown in the diagram below. From this, the displacement and the average velocity can be calculated. By treating each piece separately, much more details about the motion can be determined.

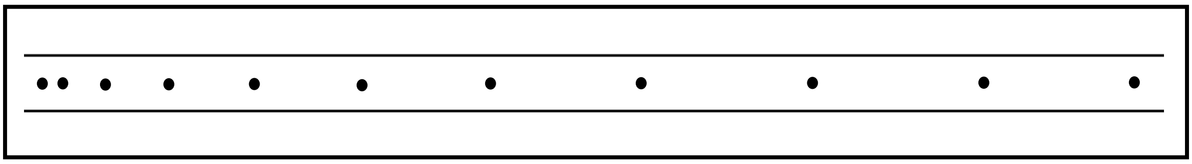


Figure 25 Ticker tape dot interval

The following table shows the ticker tape tick interval, displacement, time taken and average velocity.

Tick interval	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}	6^{th}	7^{th}	8^{th}	9^{th}	10^{th}
Displacement (cm)	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0	2.0	2.0
Time taken (s)	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Average velocity	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0	20.0	20.0

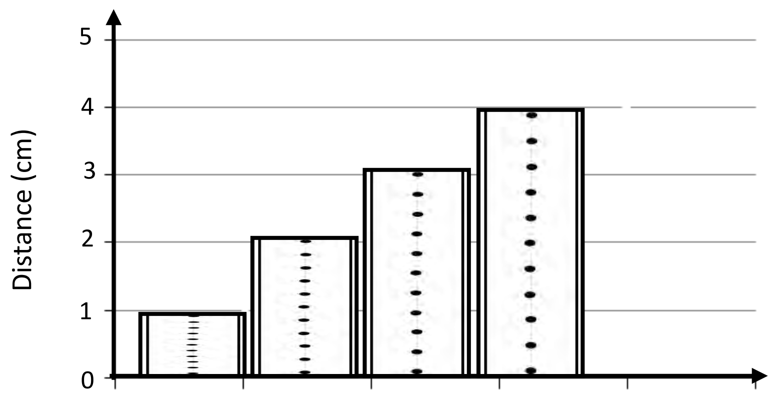
Table 3 Ticker tape tick interval, displacement, time taken and average velocity

For this ticker tape, the tick interval was 0.1s. Upon measuring the total displacement, 13cm, the average velocity, $v_{av} = s/t = 13.0/1.0 = 13.0cm/s$. The velocity did change over the 13cm tape. If you want to find the details about the velocity of smaller time interval, then you can cut up the tape and measure the length of each piece. Then you can calculate the velocity of smaller time intervals.

From what you have seen from the ticker tape, the velocity has increased over the length of the tape to 20cm/s and has remained constant at that value. The changes are easily shown in graphs. Two types of graphs can be drawn easily using pieces of ticker tape. They are displacement time graph and the velocity time graph.

Another way of measuring velocity and acceleration of a ticker timer experiment is to cut up the tape into ten (10) space strips and make a ticker timer graph by gluing the strips vertically. You glue the tapes side by side on a set of axes.
You can now calculate the acceleration shown on your ticker timer graph in many ways. See one example below.

TICKER TAPE STRIP CHART



On the ticker timer graph shown above there are four (4) strips. The first strip is 1cm long and the fourth strip is 4cm long. Each strip is a 10 space strip and therefore represents a time interval of 0.2s.

To work out the acceleration, you must first choose two strips to work with. To begin with, we will work with the first two strips.

Average velocity of the first strip = length of strip/time = 1cm/0.2s = 5cm/s
Average velocity of the second strip is 2cm/0.2s = 10cm/s

Therefore the acceleration is;

$$\text{Acceleration} = \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Time taken}}$$

$$a = \frac{v - u}{t} = \frac{10 - 5}{0.2} = \frac{5}{0.2} = 25\text{cms}^{-2}$$

Another way of working out the acceleration could be to use two strips that are further apart. We will now work out the acceleration using the 1st and the 4th strips.

Velocity for the first strip is 5cm/s

Velocity for the fourth strip = 4/0.2 = 20cm/s

The time interval between the 1st and the 4th strip is 3 x 0.2 = 0.6s

Therefore the acceleration is:

$$a = \frac{v - u}{t} = \frac{20 - 5}{0.6} = \frac{15}{0.6} = 25cms^{-2}$$

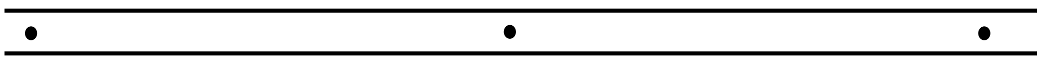
Now check what you have just learnt by trying out the learning activity below!

Learning Activity 4

20 minutes

Answer the following questions on the spaces provided.

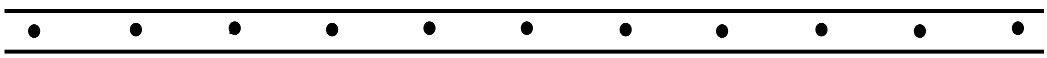
1. Work out the time taken to make the following tapes, assuming the ticker timer that made the dots was marking the tape at a rate of 50 dots per second. The diagrams are life–size.

a) 

____

b) 

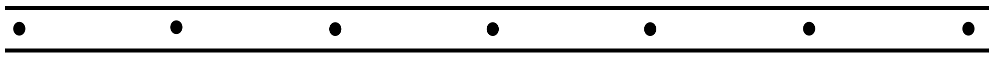
____

c) 

____

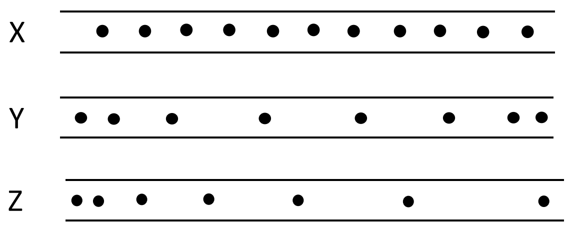
2. Find the velocities of the strips below, assuming that the ticker timer that made them was vibrating at 50 dots per seconds. The lengths of the tapes are 20cm long.

a) 

b) 

c) 

Refer to the diagrams of the ticker tapes below to answer Question 3.



3. Which of the following ticker tape represent an object that is;

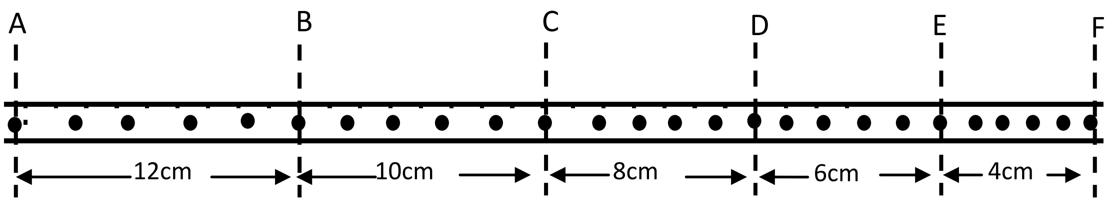
   a) slowing down? ____________________

   b) accelerating? ____________________

   c) travelling at constant speed? ____________________

---

4. A ticker tape timer was used to study the motion of a trolley that travels in a straight line. The tape is shown below.



a) In the experiment, the trolley was accelerating. Which end, A or F of the tape represents the beginning of the motion?

b) The ticker timer was operating at 5 hertz (50 cycles per second). How much time elapsed between A and B?

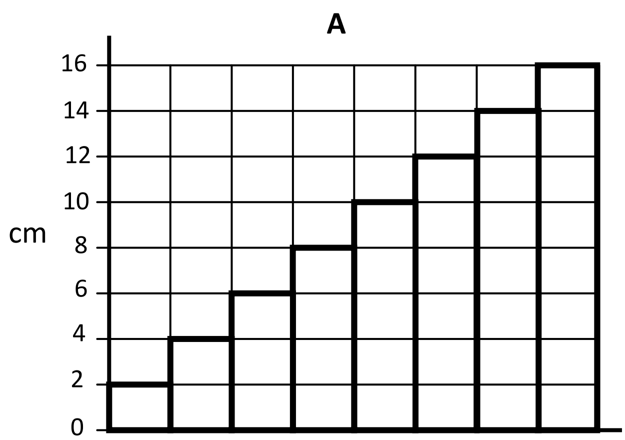
c) Find the average velocities for the parts AB, BC, CD, DE, EF.

d) What is the acceleration of the trolley?

---

5. The ticker timer graphs shown on this page were all made using a ticker timer that makes 100 dots per second. Each strip is a ten (10) space strip.

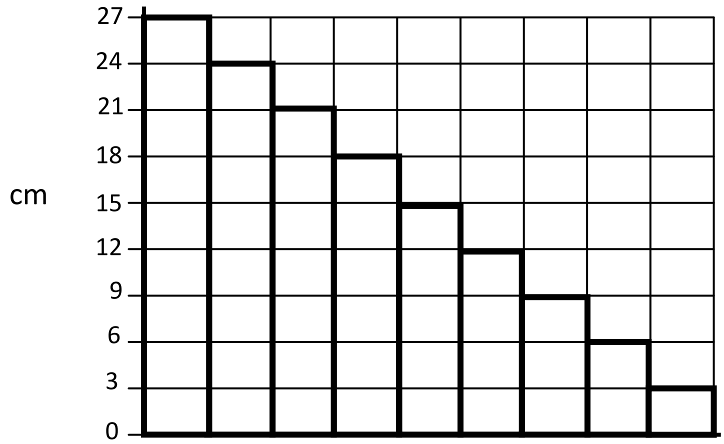
a) Describe the motion of the body that produced graph A.



b) Calculate the value of the acceleration of the body shown in graph A.

c) Describe the motion of the body that produced graph B.

B

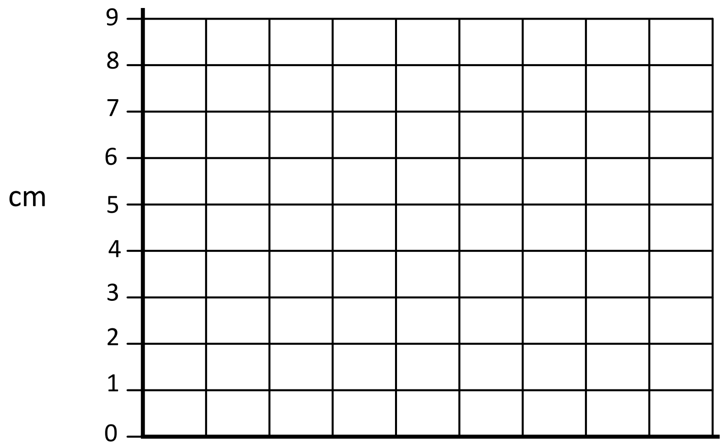


____
____
____
____

d) Calculate the value of the acceleration of the body shown in graph B.

e) Describe the motion of the body that produced graph C.

C



____
____
____

f) Calculate the value of the acceleration of the body shown in graph C.

6. Draw dots on a ticker tape to represent the following:

a) A fast speed

b) A slow speed

c) An acceleration

d) Explain how to determine a speed of an object from a ticker tape.

7. The time interval between each dot is 1s. Calculate the time elapsed for the motion recorded on a ticker tape below. (Measure using a ruler)

Find the average speed over the

a) first second of 5mm/s.

b) last second of 37mm/s.

c) whole tape of 20mm/s.

Thank you for completing learning activity 4. Now check your work. Answers are at the end of the module.

11.2.3. Linear Motion

Linear motion refers to motion in a straight line. The motion of an object has been described using the different quantities. You have studied the connections between position, displacement, velocity, speed, acceleration and time. If we use the symbols to represent the size of each quantity, you would come up with mathematical links between these quantities.

Equations of motion are used to study the motion of an object travelling with a constant acceleration in a straight line. In this section, you will look at how these quantities are related to each other. Each of the four equations for motion with constant acceleration connects four of the five variables, s, u, v, a and t. Problems can be solved by selecting the suitable equation or combination of an equation that involves those variables that are known and those that are not.

Equations of Linear Motion

There are four equations that link together the important factors required to define any particular motion. The quantities with their symbols and the units are given in the table below.

Quantity	Quantity symbol	Unit	Unit symbol
Displacement	s	metres	m
Initial velocity	u	metres per second	m/s or ms^{-1}
Final velocity	v	metres per second	m/s or ms^{-1}
Acceleration	a	metres per second per second	m/s/s or ms^{-2}
Time	t	second	s

Table 4 Table of quantities and symbols

You may use different equations to solve word problems when dealing with motion in straight line. The four equations, which can easily be rearranged into different forms, are given below.

First equation
If a body is moving with a uniform acceleration **a** and its velocity increases from **u** to **v** in time **t** then;

$$a = \frac{\text{change in velocity}}{\text{time taken}} = \frac{v - u}{t}$$

$$at = v - u$$

$$v = u + at \qquad (1)$$

Note that the initial velocity **u** and the final velocity **v** refer to the start and finish of the timing and do not necessarily mean the start and finish of the motion.

Second equation

The velocity of the body moving with uniform acceleration increases steadily. Its average velocity therefore, equals the sum of its initial and final velocities, that is;

$$\textbf{average velocity} = \frac{u+v}{2}$$

If **s** is the distance moved in time **t**, since average **velocity = distance /time = s/t, then**

$$\frac{s}{t} = \frac{u+v}{2}$$

$$s = \frac{u+v}{2}t \qquad \textbf{(2)}$$

Third equation

If you substitute equation **(1)** into equation (**2)**

$$s = \frac{u+v}{2}t$$

$$s = \frac{(u+u+at)\,t}{2}$$

$$s = \frac{2\,ut + at^2}{2}$$

This gives,
$$s = ut + \frac{1}{2}at^2 \qquad \textbf{(3)}$$

Fourth equation

From equation (**1**) we have:

$$v = u + at$$

$$t = \frac{v-u}{a}$$

Substitute the above into equation (**2**) by replacing (**t**) and cross multiply.

$$s = \frac{u+v}{2}t$$

$$s = \frac{u+v}{2} \times \frac{v-u}{a}$$

$$s = \frac{v^2 - u^2}{2a}$$

$$2as = v^2 - u^2$$

Therefore
$$v^2 = u^2 + 2as \qquad \textbf{(4)}$$

Steps in solving word problems

1. Read the problem very carefully and make sure you understand it.
2. List the quantities given.
3. Identify the unknown quantity that is to be determined.
4. Choose the appropriate formulae to solve the problem. In choosing the formula you must ensure that all the quantities are given except one.
5. Make the unknown become the subject of the formula.
6. Substitute the known quantities and simplify.
7. Write down the correct answer with the units.

It is necessary to know how to derive the equations of motion shown. The text does give you some idea of how to derive the equation. It is also necessary to learn the equations by heart. The important thing is to be able to solve problems using these equations. If you know any of the three of **suvat,** then you can work out the other two easily**.**

> Note: These formulae apply when the acceleration is constant and the motion is in a straight line. Velocity, acceleration and displacement are vector quantities and therefore may be positive or negative.

Example 1

A car starting from rest reaches a velocity of $20ms^{-1}$ in 10 seconds. Assuming that the acceleration is constant, calculate the:

(a) acceleration and

(b) distance travelled in this time interval.

Solution

$u = 0$ (at rest), $v = 20ms^{-1}$, $t = 10s$, $a = ?$

$$v = u + at$$

$$a = \frac{20 - 0}{10}$$

$$a = 2ms^{-2}$$

Example 2

"A car starts from rest, and accelerates steadily at $3m/s^2$. How far must it travel before it is moving at 30m/s?"

In this example, $u = 0m/s$; $v = 30m/s$, and $a = 3m/s^2$, so the fourth equation is suitable.

Substituting the correct values into the equation again:

$$v^2 = u^2 + {}^2as$$

$$30^2 = 0 + 2 \times 3 \times s$$

Therefore s = $30^2/6$ = 150m, so the car must travel a distance of 150 metres before it is moving at 30m/s.

Example 3

A motorbike starting from rest reaches a velocity of 20m/s in 10 seconds. Assuming that acceleration is constant, calculate the

a) acceleration and

b) distance travelled in this time interval.

Solution

u = 0 (starting from rest), v = 20m/s ,t = 10 s ,a = ?, s = ?

a) $a = \frac{v - u}{t} = \frac{20 - 0}{10} = \frac{20}{10} = 2ms^{-2}$

b) $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} 2 \times 10 \times 10 = 100m$

Now check what you have just learnt by trying out the learning activity below!

Learning Activity 5

20 minutes

Answer the following questions on the spaces provided.

1. A body starts from rest and reaches a velocity of $5ms^{-1}$ after travelling with uniform acceleration for 2s. Calculate its acceleration.

2. A body starts rest and moves with a uniform acceleration of $2ms^{-2}$ in straight line.

   a) What is its velocity after 5s?

b) How far has it travelled in this time?

c) When will it be 100m from the starting point?

---

3. A car accelerates from $4ms^{-1}$ to $20ms^{-1}$ in 8s. How far does it travel in time?

---

4. A motor cyclist travelling at $12ms^{-1}$ decelerates at $3ms^{-2}$.

a) How long does it take?

b) How far does he travel in coming to rest?

---

5. A sports car accelerates from rest at $4ms^{-2}$ for 10s. Calculate;

a) its final velocity.

b) the distance travelled.

6. A girl on a bicycle accelerates uniformly from rest to $10ms^{-1}$ in a distance of 50m. Find:

   a) her acceleration.

   b) the time taken.

---

7. A fighter plane lands on the deck of an aircraft carrier at a velocity of $60ms^{-1}$ and is brought to rest in 2 seconds by an arrester wire.

   a) What is the acceleration?

   b) How far does it travel in this time?

---

8. A train increases speed steadily from 10m/s to 20m/s in 1 minute.

   a) What is its average speed during this time in m/s?

   b) How far does it travel while increasing its speed?

---

9. A boat starts from rest and has a uniform acceleration of $2ms^{-2}$. Calculate the time taken and the distance travelled to attain a velocity of $20ms^{-1}$.

10. A car with a velocity of $12ms^{-1}$ takes 6s to come to rest when the brakes are applied.

Find its:

a) its acceleration.

b) its deceleration.

c) the distance it covers in coming to rest.

Thank you for completing learning activity 5. Now check your work. Answers are at the end of the module.

Reaction time and stopping distance

When the driver of a car sees that an accident is about to happen, he or she must react by stepping on the brakes. It may take a while for the driver to react. The time between seeing that accident about to happen and braking the car is called the **reaction time**. This time may depend on the driver's age, experience, alertness and physical fitness. The reaction time may range from 0.2 seconds to 0.8 seconds. The reaction distance is the distance travelled during reaction time until the car comes to a complete stop.

Reaction distance = reaction time x the average speed during this time

Braking distance is the distance travelled by a vehicle with the brakes application (the car does not stop suddenly) with the wheels locked. Such braking produces uniform deceleration.

Stopping distance = reaction distance + braking distance

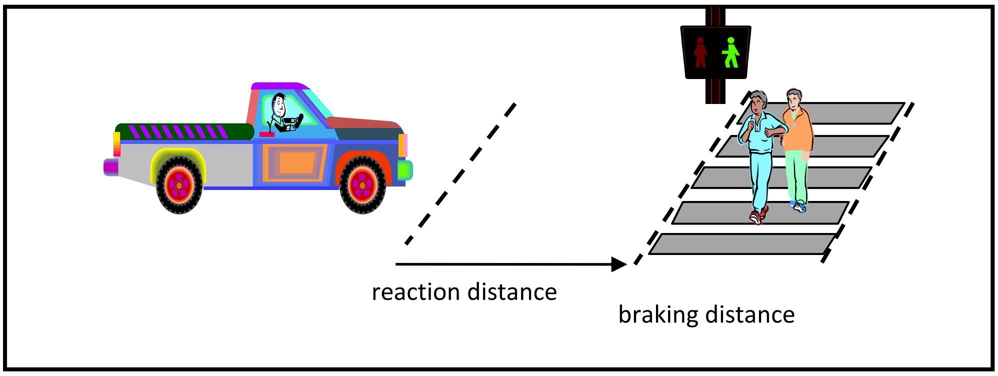


Figure 26 Illustration of stopping distance

Example
When the driver of a car travelling at 36km/h suddenly sees a pedestrian in front of a car, he applies the brakes. The reaction time of the driver is 0.2s. If the maximum deceleration that the brakes can produce is 20ms^{-2}, find the maximum stopping distance for the car.

Solution

36km/h = 10m/s
Reaction distance s = ut = 10 x 0.2 = 2m
Braking distance;

$$s = \frac{v^2 - u^2}{2a} = \frac{0 - (10)^2}{2 \times 20} = \frac{100}{40} = 2.5\text{m}$$

Stopping distance = reaction distance + braking distance
2 + 2.5 = 4.5m

The car would have travelled 4.5m after the driver reacted by applying the brakes and the car coming to a stop completely.
The table below shows the braking distance increasing with speed.

Speed of the car (m/s)	Reaction distance(s)	Braking distance (m)	Stopping distance(m)
100	20	60	80
90	18	48	66
80	16	38	54
70	14	29	43
60	12	22	34

50	10	15	25
40	8	10	18
30	6	6	12
20	4	3	7
10	2	2	4

Table 5 Table showing that the braking distance increases with speed

The above example shows that the braking distance increases with the speed. This braking distance also depends on the road surface, which is the friction between the tyres and the road surface. It may also depend on the conditions of the weather. When a vehicle is travelling downhill, the braking distance also depends on the slope of the hill. Smooth tyres and brakes that are not working properly may also increase the braking distance.

Therefore, the above factors must be considered when determining a safe distance when travelling behind another car, coming to stop at a crossing or a traffic light.

Now check what you have just learnt by trying out the learning activity below!

Learning Activity 6

20 minutes

Answer the following questions on the spaces provided.

1. When the driver of a car travelling at 72km/h suddenly sees a pedestrian in front of the car, he applies the brakes. Consider the reaction time of the driver is 0.2s. If the maximum deceleration that the brakes can produce is 20 ms^{-2}, find the maximum stopping distance for the car.

2. A person driving a car at 108km/h sees a fallen tree on the road. He braked immediately upon seeing the obstacle and managed to stop the car in 5 seconds.

   a) What was the acceleration of the car, assuming it to be uniform?

b) What distance did the car travel before it stopped?

---

3. Do you consider the reaction time of the driver to be the important factor in determining the braking distance of the car? Explain.
____
____
____
____

---

4. A car travelling at 72km/h is brought to a stop near a traffic light in 4 seconds. Find the:

a) average deceleration of the car.

b) distance moved during deceleration.

---

5. List the factors that determine the stopping distance of a vehicle when emergency rises.
____
____
____
____

Thank you for completing learning activity 6. Now check your work. Answers are at the end of the module.

Free Fall Motion

Consider an object dropped from a cliff. It accelerates downwards under gravity. We can apply the equations of motion for constant acceleration to this motion. We let **u** and **v** to be the initial and the final velocities, **a** as the acceleration, **t** as the time and **s** as displacement. If an object is thrown vertically upwards and returns to the same position, we can use the equations studied earlier. However, we need to be careful in assigning negative or positive to indicate its direction. For situations that only involve downward motion, we choose a positive sign for all quantities.

Acceleration due to gravity
The gravitational force in downward direction holds the motor vehicles to the ground. Motor vehicles pushed over a cliff fall downwards because of the force of gravity. As objects fall from the height, they accelerate downwards. If air resistance or friction is ignored, then all objects (regardless of their masses) will accelerate at the same rate when close to the earth's surface. This rate called the acceleration due to gravity has the symbol, g, and has a value of $9.8ms^{-2}$. For easier calculation, the value is rounded off to $10ms^{-2}$. It means that, objects falling freely under gravity increases their speed by about 10m/s in every second.

When we are working with calculations involving the acceleration of gravity, we need to assign a positive and negative direction of the motion. There are different forms of motion under gravity depending on how the motion starts. If an object is released from rest and allowed to fall, we call it a free-falling object. If an object is thrown or projected with an initial velocity, we call this a projectile. The size of the Earth's gravitational pull on an object is proportional to its mass.

The acceleration of free fall, g = $9.8m/s^2$ is also known as acceleration due to gravity.

Free fall motion can also be solved using the two equations given below. The acceleration (**a**) can be replaced by **(g)** to represent acceleration due to gravity. The distance (s) can be replaced by (**h**) for height. Any object that is dropped from an initial height will be considered as zero.

$s = ut + ½ at^2$
$h = ut + ½ gt^2$
$v^2 = u^2 + 2gh$

Two examples of the sorts of problems that can be solved using the above equations are as follows:

Example 1
A stone is dropped down a well 50 metres deep. How long will it take to reach the bottom?

It is dropped from rest, so u = 0m/s. It is falling under gravity, and therefore, the acceleration, a, is $9.8m/s^2$. The third equation is suitable, and the relevant values can be substituted into it:

$$50 = 0 + (1/2 \times 9.8 \times t^2)$$

Therefore, $t^2 = 50 \times 2/9.8 = 10.2$ and t = 3.19s, so the stone will take 3.19 seconds to reach the bottom of the well.

Example 2

A bomb is dropped from a helicopter 500m above ground. Ignoring the air resistance, calculate the:

a) time that it takes before hitting the ground,

b) velocity of the impact. (Consider $g = 10ms^{-2}$)

Solution:

a) Take the downward direction as a plus (+).

$a=10ms^{-2}$, u = 0, s = 500m, v =?

$$h = ut + \frac{1}{2} gt^2$$

$$500 = 0 + \frac{1}{2} 10 \times t^2$$

$$\frac{500 \times 2}{10} = t^2$$

$$t = \sqrt{100} = 10s$$

b) $v^2=u^2+2gh$

$$v = \sqrt{2x10x500} = 100ms^{-1}$$

The formula applies when the acceleration is constant and the motion is in straight line. Also remember that velocity, acceleration and displacement are vector quantities and can be positive or negative.

In reality, air resistance opposes motion, which makes the time taken for the object to fall much longer than 9 seconds. This causes the velocity of the impact to be much less.

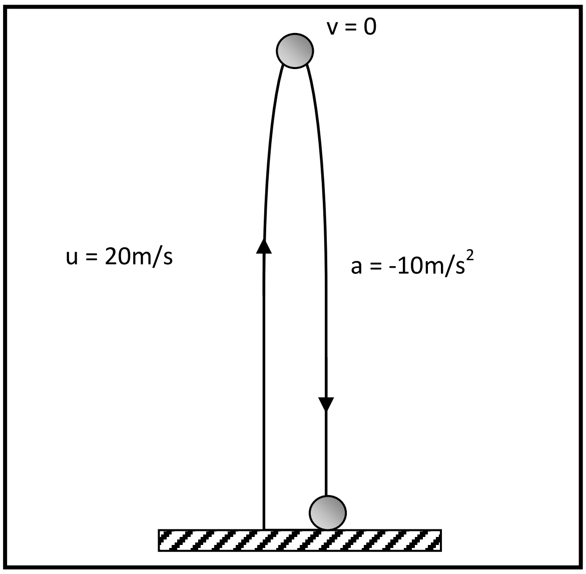
Example 3

A stone is thrown vertically upwards at an initial velocity of 20m/s. Ignore air resistance and calculate:

a) how high the ball goes.

b) the time to reach that height.

c) the total time taken for the flight. ($g = 10ms^{-2}$)



Ball thrown and free fall after

When the ball is going upwards, it is decelerating because the acceleration due to gravity is always downwards. The acceleration will have a negative value. Since we ignore air resistance, we assume that it has a value of $-10ms^{-2}$.

a) The height of the ball

$g = 10ms^{-2}$, $u = 20ms^{-1}$, $v = 0$ (at the maximum height the velocity will be zero).

$v^2 = u^2 + 2gh$

$0 = 20^2 + 2 - 10 \times s$

$20 \times s = 20 \times 20$

$h = 20m$

b) The time to reach the height

$v = u + gt$

$0 = 20 + -10t$

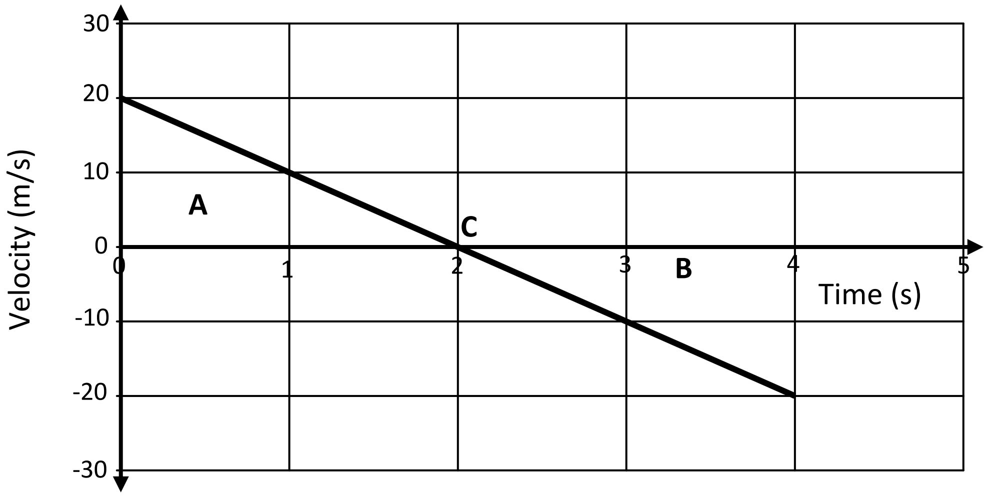
$10t = 20$

$t = 2s$

Since it took 2s to go up, it will take another 2s to come down. The total time of the flight is 4 seconds.

You can also present the information in the previous page on a graph as shown below. It is a velocity-time graph for a stone thrown vertically into the air.

VELOCITY TIME GRAPH FOR A STONE THROWN VERTICALLY INTO AIR

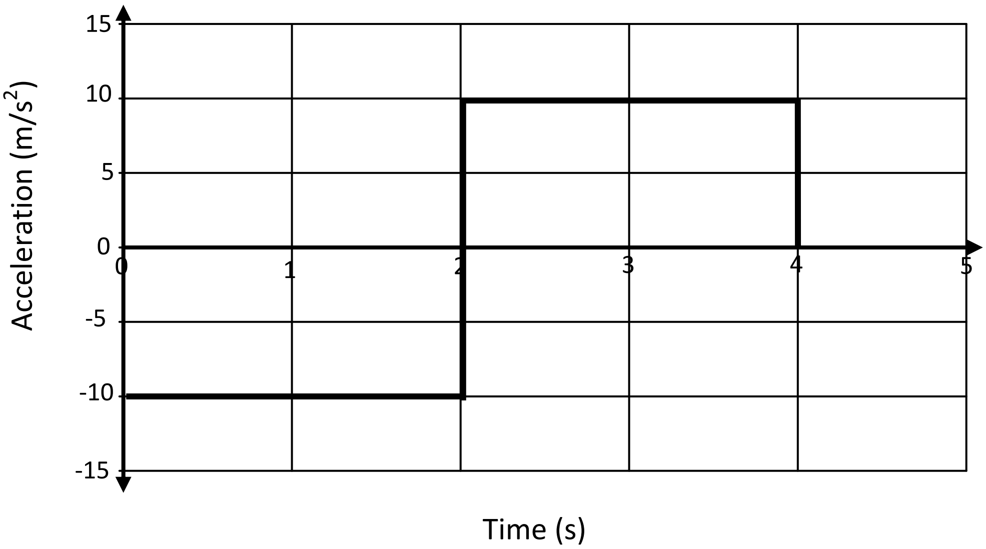


Area A is the maximum height in which the stone went up.

Point C shows the time in which the stone was at maximum height.

If you calculate the gradient at B, it will give you the acceleration due to gravity.

ACCELERATION TIME GRAPH FOR A STONE THROWN VERTICALLY INTO AIR



Terminal velocity

When a raindrop starts to fall, it will accelerate under gravity. As it gains speed, air resistance opposes its motion.

When the speed is high enough, the air resistance is equal to the gravitational force on the raindrop. This makes the forces balance out. The water droplet continues to fall at constant velocity.

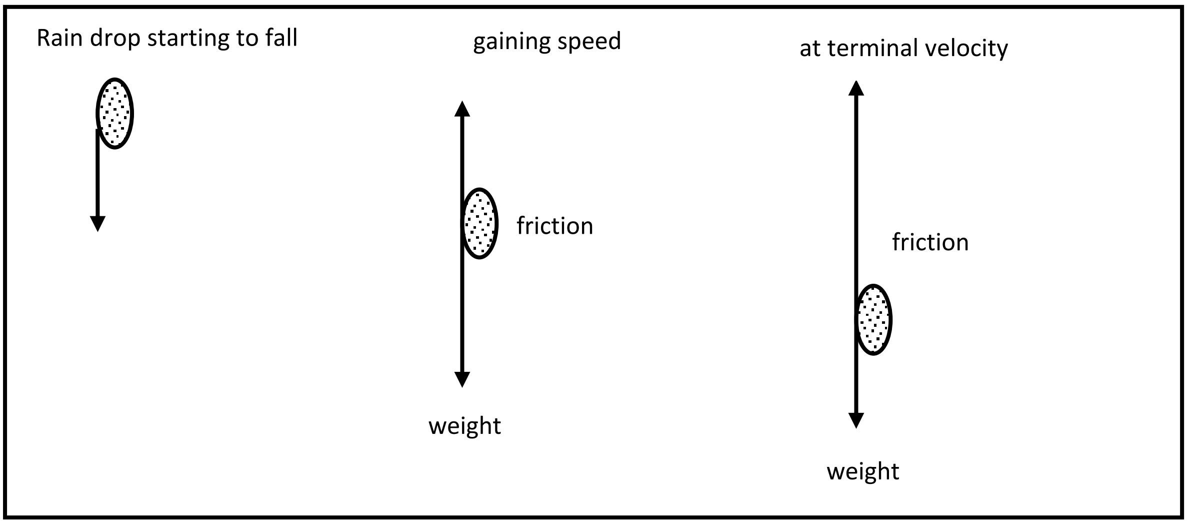
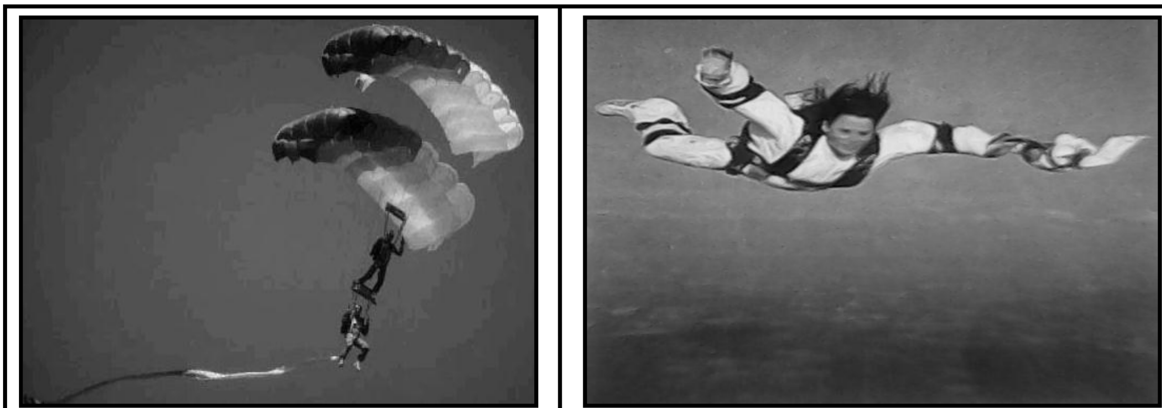


Figure 27 Factors affecting terminal velocity

The velocity of a constant motion is called terminal velocity.

Sky divers often change the shape of their bodies to speed up and slow down. Rolling up into a ball while falling will increase their terminal velocity because of the decrease in surface area. To decrease speed, it will require spreading out their arms. The change simply increases their surface area. When the parachute opens up, it increases the surface area that will increase air resistance. It will make the terminal velocity of the parachutist small enough for him to land safely.



An opened parachute increases air resistance | Less air resistance when parachute is not opened

Figure 28 Terminal velocity of a diver

A human falling freely (with parachutes not opened) from an aeroplane will gain speed at a rate of 10m/s each second until he or she reaches a velocity of 50m/s or 80km/h. The person will then keep falling at this rate which is the terminal velocity. Different falling objects under gravity have different terminal velocity.

Factors that determine terminal velocity are weight, size and the surface area of falling objects.

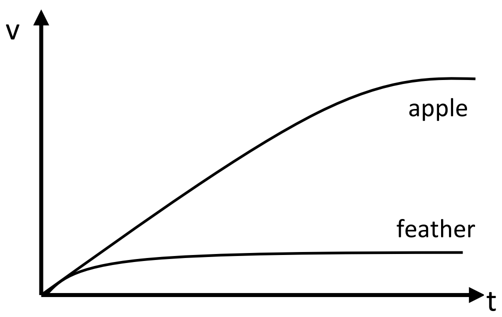
A small dense object like a steel ball–bearing has a high terminal velocity and fall a longer distance with an acceleration of $9.8m/s^2$ before air resistance equals its weight. A light object, like a raindrop or an object with a large surface area accelerates for a short distance before air resistance equals its weight. The large surface areas of the parachute increase resistance that will reduce terminal velocity. Terminal velocities of this substance may also depend on mass and their surface area.

The table below shows the terminal velocities of some bodies.

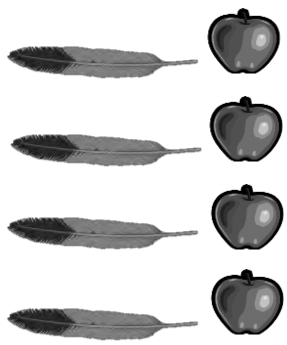
1mm insect = 6m/s
1 cm round stone = 24m/s
4 mm raindrop = 8m/s
Human = 50m/s
Human with parachute = 10m/s

Table 6 Terminal velocities of some bodies

Through this module, it has been assumed that all bodies fall at a rate determined by g that is $9.8m/s^2$. **It is true only for bodies falling in a vacuum**. The fact is that most objects fall in air, where the continual acceleration of gravity is counter balanced by air pushing upwards.



The terminal velocity of feather is lower than that of apple in air.



An apple and feather falling at the same time in vacuum.

Figure 29 Terminal velocities of apple versus feather in air and vacuum.

Galileo showed us in his experiment that all bodies fall in a vacuum with the same acceleration. You have seen that a piece of paper is reaching the ground much less rapidly than a stone dropped from the same height. After a short time, the velocity of the paper reaches a constant value which is maintained during the rest of the fall.

The stone continues to accelerate past the paper. This constant velocity is what you have seen earlier as terminal velocity.

Now check what you have just learnt by trying out the learning activity below!

Learning Activity 7

20 minutes

Answer the following questions on the spaces provided.

1. A stone takes 3s to reach the ground. What is the height above the ground?

---

2. A stone is dropped from a bridge. It takes 4 seconds to reach the water below. How high is the bridge above the water?

---

3. A ball falls 20 metres from rest to the ground below.

   a) What is the velocity as it strikes the ground?

b) How long does it take to fall?

---

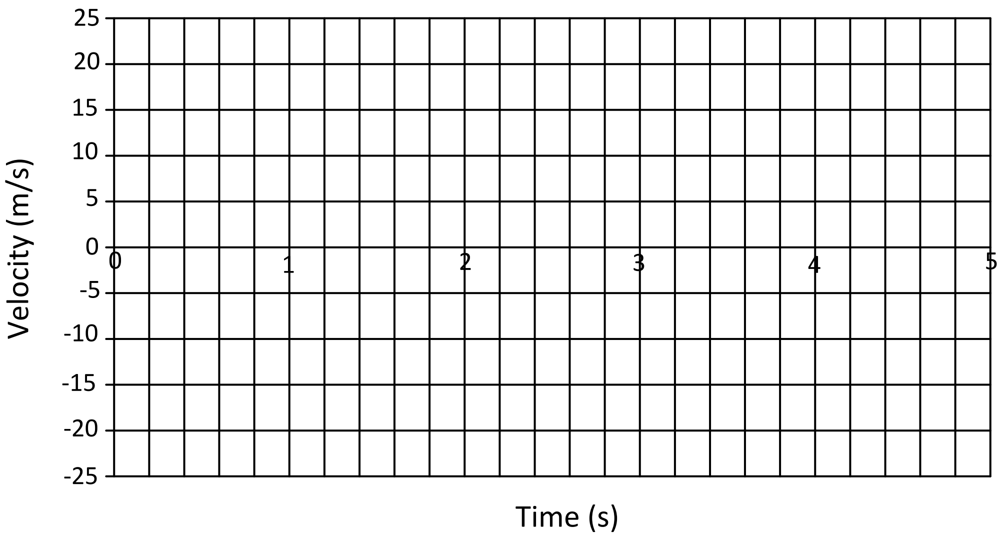
4. A stone is thrown vertically upwards from the ground with an initial velocity of $20ms^{-1}$.

a) What is the maximum height reached?

b) How long does the ball take to reach this maximum height?

c) How long does it take for the ball to fall to the ground again from the maximum height?

c) Sketch a velocity time graph for this motion.



5. A stone is thrown upwards from a cliff top with a velocity of $40ms^{-1}$.

a) What is the height above the cliff top after

i) 3s? ____
ii) 4s? ____
iii) 5s? ____
iv) 8s? ____

b) If the stone strikes the sea after 10s, what is the height of the cliff top above the sea?

6. On the moon an object is dropped from the height 3.2m. It takes 2 seconds to reach the ground. What is the acceleration of the free fall on the moon?

7. A water fall is 300m high. Assuming that there is no air resistance, how long does it take water to fall from the top to the bottom?

8. An object is dropped from a height of 20m.

a) How long will it take to fall to the ground?

b) What will be its velocity as it strikes the ground?

---

9. An object of 1kg mass is dropped and reaches the ground in 2 seconds.

   a) Calculate the velocity of the object just before it hits the ground.

   b) Calculate the height from which the object was dropped.

   c) What would be the difference in your answers to parts (a) and (b) if the mass of the object was 2kg, instead of 1kg?

---

10. Explain why a parachutist does not fall with constant velocity.

    ____
    ____
    ____
    ____

Thank you for completing learning activity 7. Now check your work. Answers are at the end of the module.

11.2.4 Two-Dimensional Motion

You have seen and experience the manner in which a ball thrown forward through the air will follow a curved path. Other objects thrown through the air will travel in a similar manner. Let us say a ball of any kind, an arrow, a bullet, a missile, a javelin or discus is just a few of objects that follow a curved path through the air. The object that is launched is called the **projectile** and its path is determined by the speed, the angle of projection, the shape of the object, its air resistance and whether or not it is spinning. **The path followed by a projectile is called its trajectory.**

For simplicity of calculation, air resistance and the spin are usually ignored when studying these curved motions, however, both of these effects can be useful. There are many examples of objects that rotate or travel long circular, or close to circular paths. Wheels, gears, engines and generators, all rotate as their primary motion. The planets, their moons, satellites, some fun park rides and athletics hammer follow a circular path. Vehicles that go around a corner or over an arched bridge, roller coasters and aircraft travel along circles or parts of circles, as part of their motion at a different time.

The rate of rotation of an object is measured by counting the number of rotations made per unit time measured in **revolution per minute (rpm**). Things like the clothes driers operate at 400 -800 revolution per minute. For objects that move in circular paths, we could count the number of orbits completed per unit time. For example, a satellite may complete several orbits per day. The time of the orbit is often called the **period** when the object travels the entire circumference of the circle. The average speed is the circumference divided by the period.

Many motions are a complex combination of simple motions. For example, a projectile motion often includes a rotation or a spin. Circular motion can be a spin or be a part of a projectile motion. Apart from motions that are vertically upwards or downwards, projectiles follow curved paths. The velocity of the projectile at any point in the path is in a particular direction to the tangent to the path. At each stage of the projectile motion, the velocities can be different and at the highest point, the velocity is **not zero but horizontal**. That is what you will notice in some of the diagrams as you go through the topic.

Projectile motion

There are two important types of motion. In **rectilinear** motion, objects move in straight lines. This motion normally occurs when the objects are moving freely, such as a stone falling from the top of a building. In **curvilinear** motion, objects move in a curved path because they receive a sideways push, such as a cyclist moving around a circular bend or an aeroplane doing a “loop the loop” flight exercise.

In the last topic, you saw that there are different forms of motion under gravity, depending on how the motion is started and often called free fall motion.

When an object is released and allowed to fall, we call it **free falling** object. However, if the object projects with an initial velocity, we call this **projectile motion**.

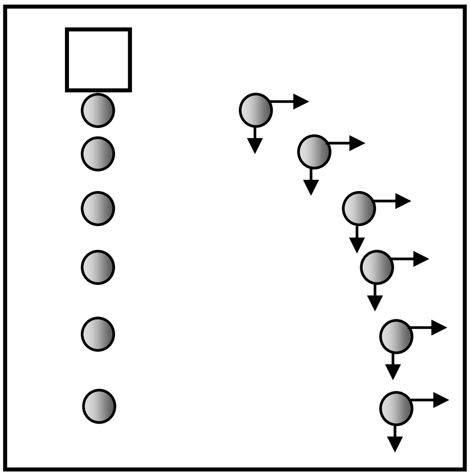


Figure 30 Projectile motion

In a projectile motion, the vertical and horizontal motions are independent and can be treated separately.

You will see that the diagram above is showing a motion of two balls. One is projected horizontally while the other drops vertically. Notice that the two balls are at the same level at any instant. Their initial vertical velocities are zero and their accelerations due to gravity are equal. Thus, a projectile falls like a body dropped from rest. Its horizontal velocity does not affect its vertical motion.

For example, if a ball is thrown horizontally from a cliff and takes three seconds to reach below, we can calculate the height of the cliff by considering the vertical motion only. We have u = 0 (since the ball has no vertical velocity initially), a= g = +10m/s^{-2} and t = 3s. The height of the cliff is given by,

$$h = 0 \times 3 + \frac{1}{2}(+10 \times (3^2)$$

$$h = 45m$$

So far, we have only considered the vertical motion of an object under the influence of gravity. Often, we need to study the motion of objects that have a horizontal component to their velocity as well as a vertical component, for example, a bullet or cannon shell. We will in this case ignore air resistance.

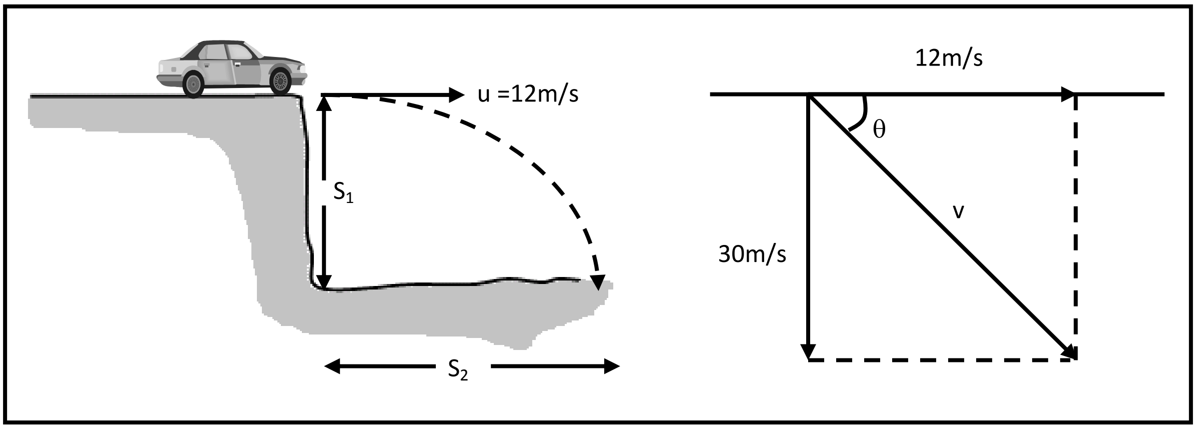
Suppose we have a projectile launched at velocity, v. At first it has a horizontal component of its velocity v, and a vertical component to its velocity of zero. Since there is no horizontal force acting, the horizontal component of its velocity remains v. However, if we consider the vertical component of its velocity, it is changing at a rate of 9.8ms^{-1} downwards.

Horizontal projection

Apart from the special situation of an object being thrown vertically upwards or downwards, projectiles follow curved paths. The velocity of the projectile at any point in its path is in a particular direction along a tangent to the path. The object's velocity changes along the part both in size and direction but due to gravity acting vertically downwards. At the highest point, the velocity is horizontal and not zero. If it were zero, the object would fall vertically downwards. Beyond the highest point, as the objects fall, the size of the velocity increases and the path becomes steeper.

Example 1

A car is driven horizontally off a cliff at a velocity of $12ms^{-1}$ and takes 3s to reach the ground below.



Projectile motion of a car from off the cliff

Calculate the,

a) height of the cliff,

b) distance from the cliff the car lands and

c) velocity with which the car strikes the ground.(Use $g=10ms^{-1}$)

Solution

a) $u = 0$, $a = ms^{-2}$, $t = 3s$, $s = ?$

$$s = ut \ \frac{1}{2} gt^2$$

$$s = 0 + \frac{1}{2} \times 10 \times 3^2$$

$$s = 5 \times 9$$

$$s = 45m$$

b) Horizontal motion only

$u = 12ms^{-1}$, $a = 0\ ms^{-2}$, $t = 3\ s$, $s = ?$

$$s = ut + \frac{1}{2} gt^2$$

$$s = 12 \times 3 + 0$$

$$s = 36m$$

c) The velocity of the car has not changed throughout the motion which is $12ms^{-1}$. The vertical velocity increase as the car falls. Vertical velocity at impact. ($v = gt = 10 \times 3 = 30ms^{-1}$.)

The two velocity components form the sides of a rectangle of which the actual velocity of the impact is given by the diagonal.

$$v^2 = 12^2 + 30^2$$

$$v = \sqrt{12^2 + 30^2}$$

$$v = \sqrt{144 + 900}$$

$$v = \sqrt{1044}$$

$$= 32.3ms^{-1}$$

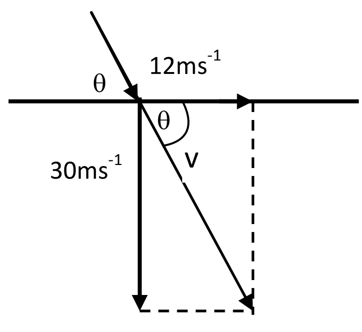


Figure 31 Velocity – impact components

The angle of impact, θ, can be found from $\tan \theta = 30 / 12 = 2.5$

So, $\theta = 68.19^o$

Example 2

A stone of mass 0.5kg is propelled horizontally from the top of a tower of height 40 metres. The initial speed of the stone is $15ms^{-1}$. Calculate the, (use $g=9.8ms^{-2}$)

a) time taken to hit the ground.

b) horizontal distance of the impact point from the base of the cliff.

Solution

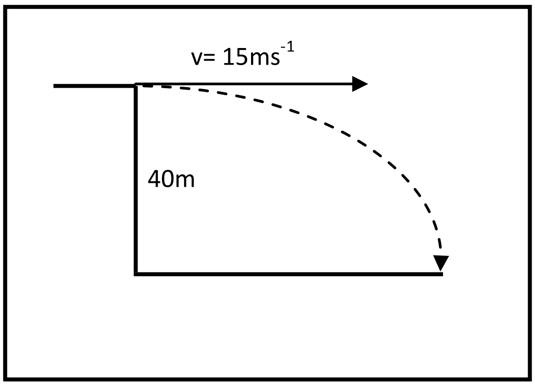
a) Consider the vertical motion, u =0m/s, s =50m, $a=9.8m/s^2$, t=?

$$s = ut \quad \frac{1}{2} gt^2$$

$$40 = 0 \quad \frac{1}{2} x\ 9.8\ x\ t^2$$

$$t^2 = \frac{80}{9.8}$$

$$t = 2.9s$$



b) Consider the horizontal motion u =15m/s, t =3.2s, a = 0, s =?

$$s = ut + \frac{1}{2} gt^2$$

$$s = 15\ x\ 2.9 + 0$$

$$s = 43.5m$$

Projection at an angle

Not all projectiles are launched horizontally, but sometimes at an angle, θ, to the horizontal. In this case, the horizontal component of the projectile has not changed since no horizontal force is acting (neglect air resistance) while the vertical component of the velocity is subject to acceleration, g, $9.8ms^{-2}$ downwards. If the initial velocity of the projectile is v ms^{-1} at an angle θ to the horizontal then;

- The horizontal component of velocity initially is v cos θ.
- The vertical component of the velocity is v sin θ.

Projectiles in sports often follow similar paths, but the requirements may vary in different type of sports. In sports such as hammer, discus or javelin throwing, the object has to be propelled far at a horizontal direction. In the case of bowling in cricket and pitching in softball, the distance the ball travels before being hit by the opponent is constant.

Projectile such as basketball balls and explosives shells are projected from the ground level and at an angle.

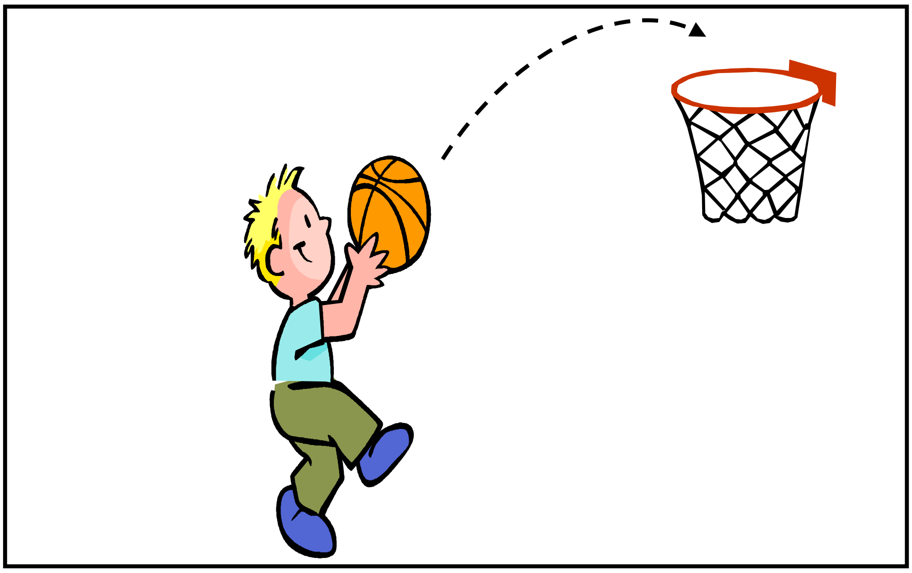


Figure 32 projectile motion from ground at a certain angle

The horizontal distance that the object travels and its range depends on

- The speed of the projection. The greater the speed, the greater the range.
- The angle of projection. The range is maximum when the angle is 45^o.

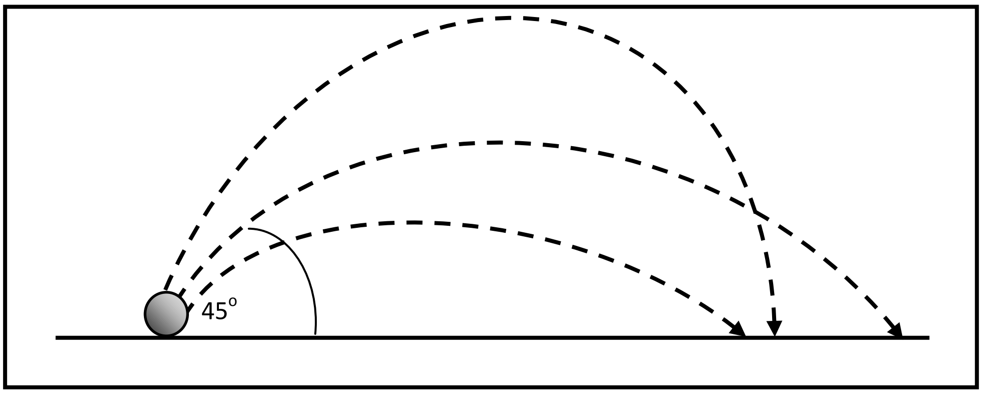


Figure 33 Projectile motion from ground at a certain angle

To study this kind of motion, we have to know the angle θ, that is the angle between the direction of the initial velocity and the ground. The initial velocity has a vertical and horizontal component. Because of gravity, such projectile travels in a parabolic path (with no air resistance)

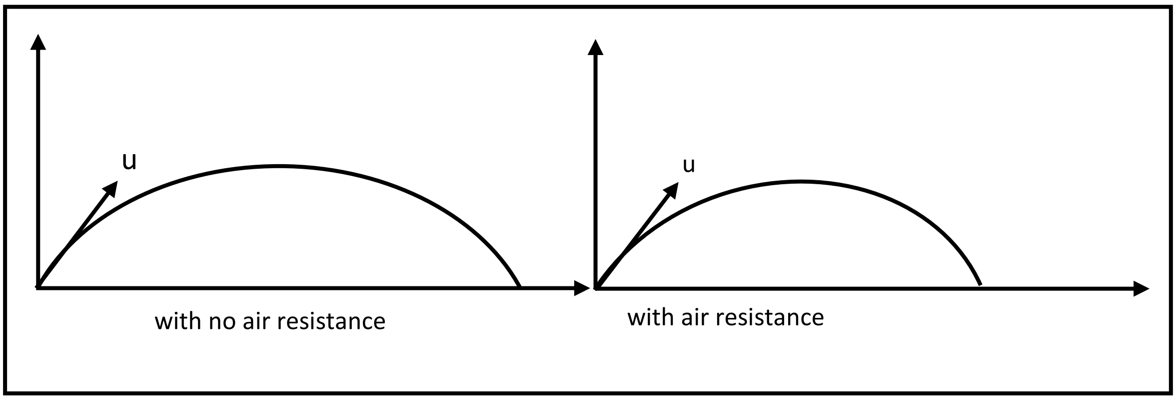


Figure 34 Velocity – impact with and without air resistance

Example 1

A mortar shell is fired at $100ms^{-1}$ at an angle 30^{o} above the ground. Assume that $g = 10ms^{-2}$.

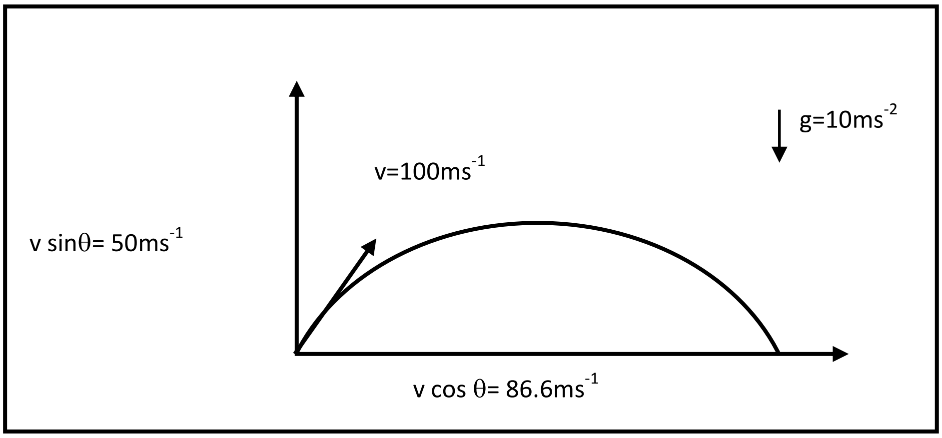
Find the:

a) initial horizontal component of the shell's velocity.

b) maximum height reached.

c) time the shell spends in the air.

d) distance travelled horizontally (the range of the shell).



Velocity – impact components

Solution:

a) $u = 100 \cos 30^{o} = 86.6ms^{-1}$ (horizontally)

$u = 100 \sin 30^{o} = 50ms^{-1}$ (vertically)

b) At the highest point reached, the velocity will be zero. The horizontal component is still 86.6m/s.

Vertically, u= $50ms^{-1}$, v = $0ms^{-1}$, a = $-10ms^{-2}$ (since up is taken as a positive direction) s=?

$v^2 = u^2 + 2as$
0= 50 X 50 – 2 x 10 x s
20s = 2500
s = 125m (the maximum height reached by the shell)

c) The time projectile in the air equals the time it takes to reach its maximum height plus the time it takes to fall from its height. This is equal to twice the time it takes to reach maximum height. (Note: Treat the vertical and the horizontal components of the motion separately.)

Vertically
v= $0ms^{-1}$ (at maximum height), a = $-10ms^{-2}$, u= $50ms^{-1}$, t=?

t = v – u /a
= 0 – 50 /-10 = 5 seconds
The total time of flight is 2 x 5 = 10s

d) To calculate the range of the shell, we use the horizontal component

Horizontally
v = 86.6ms-1, a =0ms-1, t = 10s from c , s =?
s = ut + ½ at^2
86.6 x 10 + 0 = 866m

Example 2
A ball is kicked at an angle of 45° to the horizontal with a velocity 0f $20ms^{-1}$. Ignore air resistance and take g to be $10ms^{-2}$. Calculate the:

a) initial horizontal component of the ball's velocity.

b) maximum height reached.

c) time the ball spends in the air.

d) distance travelled horizontally (the range of the ball).

This problem is similar to the example above.

Solution:
a) u = 36 cos 45° =$18.91ms^{-1}$(horizontally)
u = 36 sin 45° = $30.63ms^{-1}$ (vertically)

At the highest point reached, the velocity will be zero. The horizontal component is still $18.91ms^{-1}$.

Vertically $u = 30.63ms^{-1}$, $v = 0ms^{-1}$, $a = -10ms^{-2}$ (since up is taken as a positive direction) s=?

$v^2 = u^2 + 2as$
$0= 30.63 \times 30.63 – 2 \times 10 \times s$
$20s = 938.19$
$s = 46.9m$ (the maximum height reached by the ball)

b) The time projectile is in the air equals the time it takes to reach its maximum height plus the time it takes to fall from its height. This is equal to twice the time it takes to reach maximum height. (Note that the vertical and the horizontal components of the motion can be treated separately)

Vertically
$v= 0ms^{-1}$ (at maximum height), $a = -10ms^{-2}$, $u= 50ms^{-1}$, t=?

$t = v – u /a$
$= 0 – 30.63 /-10 = 3$ seconds
The total time of flight is $2 \times 3 = 6s$

d) To calculate the range of the shell we use the horizontal component

Horizontally
$v = 18.91ms^{-1}$, $a =0ms^{-1}$, t = 10s from c, s =?
$s = ut + ½ at^2$
$18.91 \times 10 + 0 = 189m$

When a motor vehicle is brought to rest in a collision, loose material from the vehicle is often projected forward. It is easier to establish the speed before impact from the distance travelled by the projected material. Such information is useful in accident reports.

Example 3
A car runs off the road and collides with a tree. Glass particles from the windscreen are projected forward, and are found at an average distance of 12m from the car. The average height of the windscreen is 1.2m. Find the speed of the car at the time of impact. Assume that acceleration due to gravity is $10ms^{-2}$.

Solution
Vertically
$s = 1.2m$, $u = 0ms^{-1}$, $a = 10ms^{-2}$, t=?

$s= ut + ½ at^2$

$1.2 = 0 + 1/2 \times 10 \times t^2$

$2.1 / 10 = t^2$

$t = 0.49s$

Horizontally

$S = 12.0m$, $t = 0.49s$ (from above), $a = 0ms^{-2}$, $v = ?$

$v = s/t = 12 / 0.49s = 24.49ms^{-1}$

Now check what you have just learnt by trying out the learning activity below!

Learning Activity 8

20 minutes

Answer the following questions on the spaces provided.

1. A car is driven off a cliff with a horizontal velocity of $20ms^{-1}$ and took 5s to reach the ground below. Calculate

   a) the height of the cliff;

   b) how far from the base of the cliff the car lands?

---

2. An object is released from an aeroplane travelling horizontally with a constant velocity of $200ms^{-1}$ at a height of 500m. Ignoring air resistance and assuming $g=10ms^{-1}$, find:

   a) how long it takes the object to reach the ground?

b) horizontal distance covered by the object between leaving the aeroplane and reaching the ground.

---

3. A motorcycle is driven at a speed of $20ms^{-1}$ horizontally off a coastal cliff 50m above sea level. Find the:

a) time of the flight.

b) velocity of the cycle when it hits the water.

c) distance of the motorcycle from the cliff when it hits the water.

---

4. A tennis ball is hit with a racket. It travelled with a velocity of $8ms^{-1}$ at an angle of 60^o to the horizontal. Find the:

a) horizontal and vertical components of the ball's velocity.

b) maximum height reached by the ball.

c) time the ball spends in the air.

d) The range of the ball.

e) The acceleration at maximum height.

---

5. Sketch the shape of the path of a projectile, without considering air resistance. How would you expect the

   a) maximum height?

   b) range of the projectile?

6. A gun fires a bullet at an angle of 30^o to the ground, with a muzzle velocity of $500ms^{-1}$. Calculate the;

   a) maximum height reached by the bullet.

   b) range of the bullet.

   c) time of flight.

---

7. A stunt woman on a motorcycle traveling at $30ms^{-1}$ speed jumps from a ramp inclined at 45^o to the ground. Find:

   a) the horizontal and the vertical components of the motorcycle velocity on leaving the ramp.

   b) the maximum distance reached horizontally by the cyclist.

c) whether the cyclist would clear a river 90m wide at this speed and in the angle of projection. (If not, what change could she make?)

---

8. A car left the road and collided with a tree. Glass from the windscreen was projected forward and landed in the grass 5.0m away. The average height of the windscreen was 1.1m above the ground. Determine the speed of the car at the time of the impact.

---

9. A bowler releases a cricket ball horizontally from a height 2.5m towards a batsman who is 20m away (ignore air resistance and assume that g= $10m/s^2$).

   a) What time will the ball bounce upon its release?

   b) What initial velocity is given to the ball if it were to bounce 18m from the bowler?

10. A small truck runs off a road and over a 100m high cliff. It lands 50m away from the base of the cliff. ($g = 9.8 m/s^2$)

    a) How long did it take the truck to hit the ground?

    b) What was the velocity of the truck as it went over the cliff?

    c) What was the velocity of the truck on impact?

---

11. A stuntman's motorcycle leaves the ramp inclined at 45^o to the ground. If the reading on the speedometer is 108km/h at the point where he leaves the ramp, calculate the components of the bikes velocity.

Thank you for completing learning activity 8. Now check your work. Answers are at the end of the module.

Circular Motion

There are many everyday situations where objects travel in circular paths. The Earth spins on its axis and orbits the sun. A compact disc spins on disc players when the music is being played. Electrons orbit the nucleus of an atom. The hands of a mechanical clock follow a circular path as the time passes. Communicating satellites orbit the earth many times a day and winds within tropical cyclones move in circular paths.

Car accidents often occur when the drivers of motor vehicles are trying to travel around a bend in a road at high speed. A vehicle that exceeds the safe speed limit can slip off the road and usually crash into anything along its path such as a tree or a fence.

A vehicle travelling around a bend on a level road is moving along a circular path. A vehicle travelling along a curved path is continually changing its velocity (its speed may be constant) because the direction of the motion is always changing. So it has acceleration.

Consider a particle moving around a circle of radius r in the diagram below. At point A, the velocity of the particle is at a tangent to the circular path. When the particle reaches point B, its velocity still has the same magnitude, but its direction has changed. To find the acceleration, we need to determine the average change in velocity.

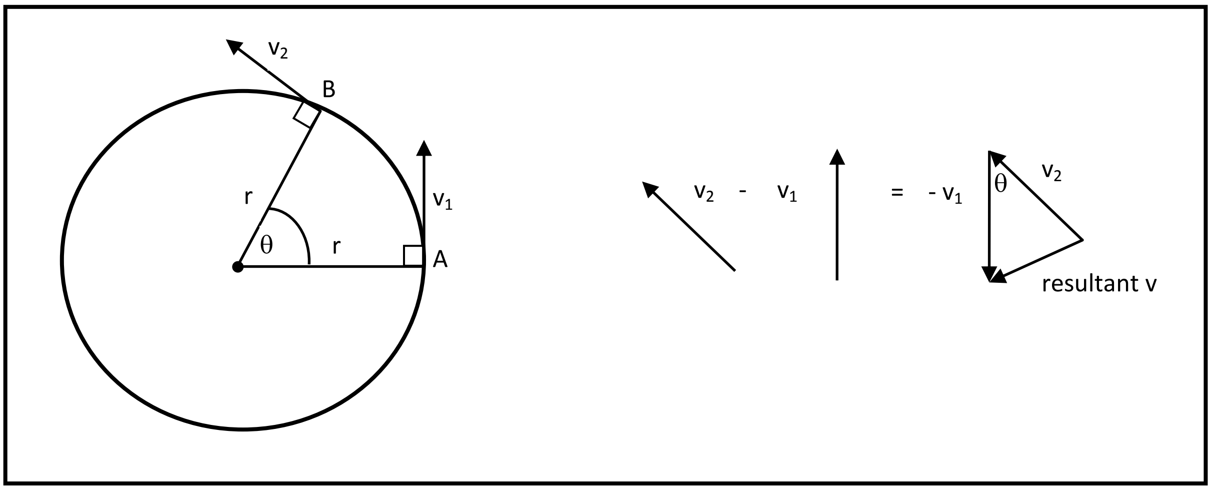


Figure 35 Centripetal acceleration

The change in velocity = final velocity (v_2) – initial (v_1)

This is where we use rules of subtraction of vectors. We reverse the direction of the subtracted vector and add head to tail to get the resultant vector.

You will observe that the resultant's direction is towards the centre of the circle. We can also say that, the acceleration experienced by the particle directs towards the centre of the circle.

The acceleration of this type is called **centripetal acceleration** or in other words centre seeking acceleration.
A formula to calculate centripetal acceleration is:

$$a = \frac{v^2}{r}$$

where, r**,** is the radius of the circular path in metres. Because it has a centripetal acceleration, a car of mass m travelling in a circular path also experiences associated force, which is called the **centripetal force**. Therefore, by using Newton's law of motion **(F = ma)** we get the equation for uniform circular motion. From the equation given above, we substitute centripetal acceleration (**a**) into the equation below.

$$F = \frac{mv^2}{r}$$

This centripetal force points towards the centre of the circle. If you swing a ball at the end of a string, a horizontal circular path above your head is formed, and you can feel yourself pulling continually on the string to keep the ball in orbit. The string applies a centripetal force to the ball. If the string breaks, the ball will fly off at a tangent to the curved path at the point where the string broke. This behaviour is according to Newton's first law of motion.

When a ball is whirled in a circle, it is accelerating inwards. This inward acceleration is caused by a centripetal (centre-seeking) force — the tension in the string. The required force is equal to mv^2/r, where m is the mass of the ball, v is its speed, and r is its distance from the centre of revolution. The hand pulling the string experiences an outward or centrifugal reaction force.

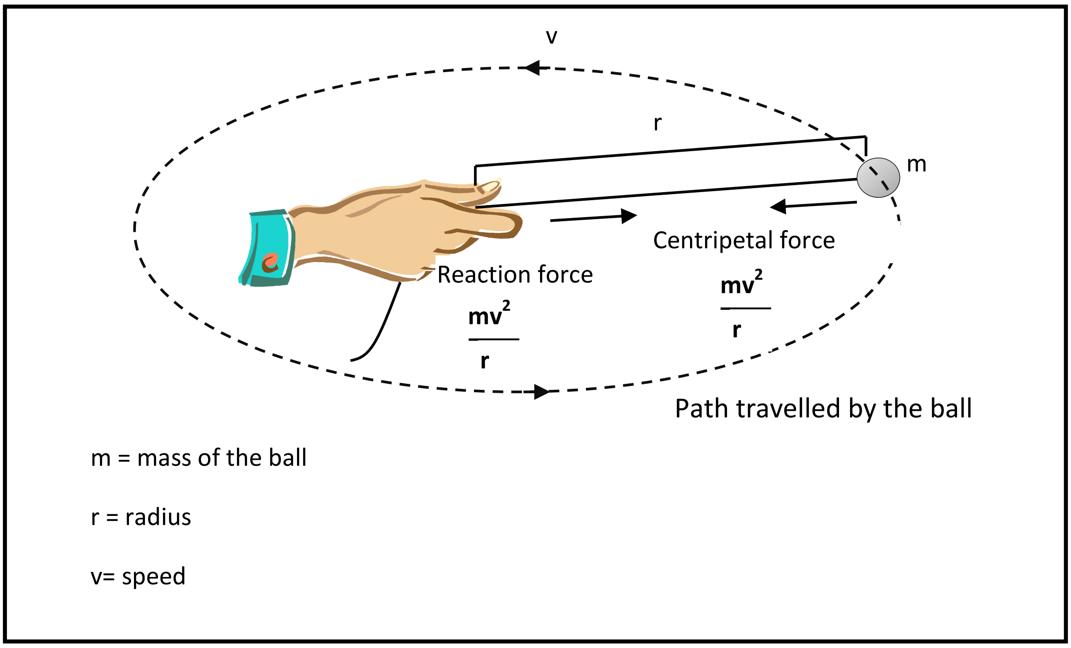


Figure 36 Centrifugal reaction force

In a car going around the bend, the centripetal force is provided by the sideways friction between the tyres and the road surface. The centripetal force does not work in the motion; it is only one of the many forces at work.

For an object travelling in a full circle at uniform speed, v, the distance covered in one complete circuit is **s = 2πr**. If the time for one complete circuit is **T**, then;

$$v = \frac{2\pi r}{T}$$

Since $a = \frac{v^2}{r}$, then $a = \frac{4\pi^2 r}{T^2}$

On a level surface, friction **F**, is proportional to the weight **W** of an object. The ratio of friction to weight is equal to a constant called the coefficient of friction (**μ**). The coefficient of friction depends on the nature of the surface.

$$\mu = \frac{F}{W}$$

Where F_r, and **W** are measured in Newtons. **W** is the force of gravity acting on the object.

$$W = mg$$

Where **m** is the mass and **g** is the acceleration due to gravity. As mentioned above, for a vehicle moving around the curve, the centripetal force is provided by the sliding friction between the tyres and the road surface.

Combining $F = \frac{mv^2}{r}$ and $F = \mu W$ we get $\frac{mv^2}{r} = \mu W = \mu mg$

where $v = \sqrt{\mu g r}$

The speed **v** in the last equation is taken as the maximum safe speed for rounding a curve. It depends on the coefficient of friction between the road and the tyres and the radius of the bend since g is constant.

A vehicle can safely travel if driven at a speed that is lower than the maximum safe speed. If it is driven faster than the safe speed, the vehicle will slip sideways because the friction force is not large enough to provide the centripetal effect. If the road around a bend tilts at an angle, then the vehicle can travel at higher speed.

Centripetal acceleration and force

In the last discussion, we saw that an object travelling along a circular path at a constant velocity has acceleration towards the centre of the circular path. Such acceleration is called the **centripetal acceleration (a_c)**.

$$a_c \quad \frac{v^2}{r}$$

Where v = linear velocity (m/s) and r = radius of the orbit (m)

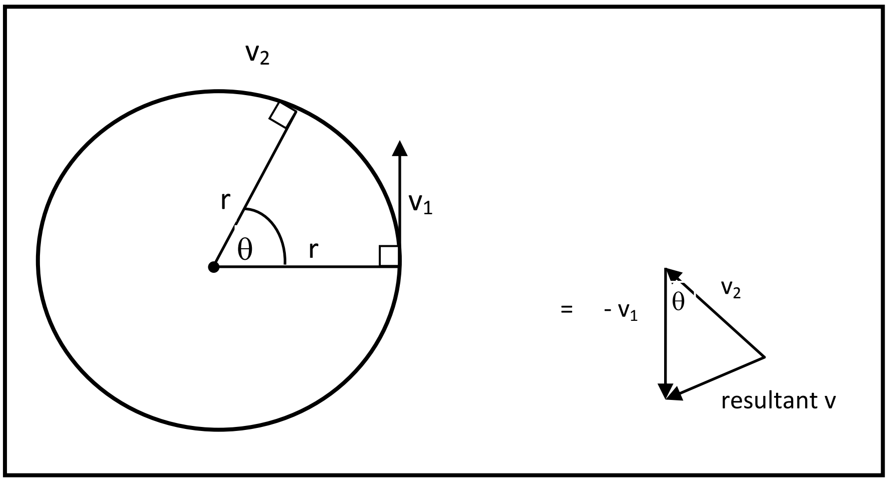


Figure 37 Centripetal acceleration and force

Because of the centripetal acceleration, an object of mass **m** experiences a **centripetal force (F_c)**.
For an object travelling at a uniform velocity **v** in a circle of radius **r**, the distance travelled during a complete circuit is $\mathbf{s = 2\pi r}$.

The time taken to complete this circuit is the period of the motion.

$$\mathbf{T = \frac{2\pi r}{v}} \quad \text{or} \quad \mathbf{v = \frac{2\pi r}{T}}$$

Since $\mathbf{a_c = \frac{2\pi r}{v}}$ we obtain $\mathbf{a = \frac{4\pi^2 r}{T^2}}$ and $\mathbf{F_c = \frac{4\pi^2 mr}{T^2}}$

Example 1
An aeroplane is travelling in a circle of radius 2500 metres at a speed of 200m/s. Calculate the radial acceleration of the plane and the time to complete the revolution.

$$a = \frac{v^2}{r} =$$

Time for the revolution = period of motion = T

$$T = \frac{2\pi r}{v} = \frac{2 \times 3.14 \times 2500}{2500} = 78.5\text{s}$$

Example 2
A bus weighing 60 000N is driven at a speed of 20m/s around a horizontal bend that has a radius of 250m. What is the centripetal force of the road on the bus?

W = mg, so m = W/g = 60 000/10 = 6000kg

$$F = \frac{mv^2}{r} = \frac{6000 \times 20 \times 20}{250} = 9600N$$

Note: Friction force will be studied in the next module.

Circular motion at non- constant speed

A ball swinging on the end of a string in a vertical circle has a non-uniform velocity around the circle because of the influence of gravity (the weight of the ball). The speed of the ball is highest at the bottom of the circle and lowest at the top of the circle. The centripetal acceleration is at a minimum at the top and maximum at the bottom. (In the figure below, **T** is used to represent the tension in the string, while **W** is the weight of the ball). The centripetal force is the net force directed towards the centre of the circle.

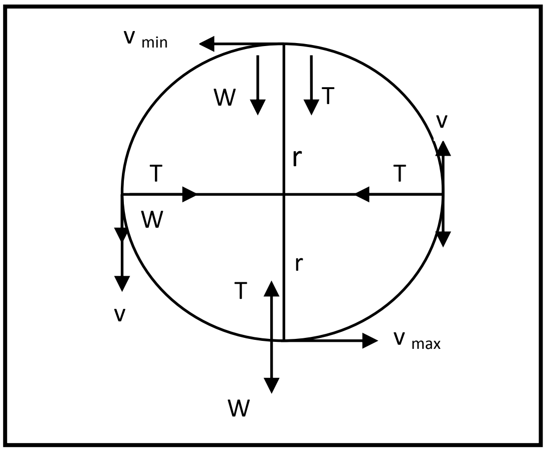


Figure 38 circular motions at non-constant speed

At the top, $\frac{mv^2}{r} = T + W$

At the side, $\frac{mv^2}{r} = T$

At the bottom, $\frac{mv^2}{r} = T - W$

The minimum velocity needed to keep the ball in the orbit is found to be the velocity at the instant when the string begins to slacken (when T = 0 at the top). This is when,

Minimum velocity (min) $\frac{mv^2}{r} = W = mg$

$v^2 = gr$ so $v = \sqrt{gr}$

Maximum tension happens at the bottom of the path.

Example 3

A breaking strain of a string is 196N. A length of a string is used to swing a ball in a vertical circle of radius 1.0m. Calculate the maximum velocity the ball can have before the string breaks.

maximum velocity (at the bottom)

$$\frac{mv^2}{r} = T - mg = 196 - 1 \times 10 = 186$$

$$v^2 = \frac{186 \times 1}{1}$$

$$v = \sqrt{186} = 13.6\text{m/s}$$

Now check what you have just learnt by trying out the learning activity below!

Learning Activity 9

20 minutes

Answer the following questions on the spaces provided.

1. A motorcycle is travelling at a constant speed of 72km/h around a circular track of radius 150m.

   a) What is the centripetal acceleration?

   b) How long does the cyclist take to complete one full circuit of the track?

2. A car of mass 1200kg is rounding a bend of radius 40m at a speed of $20ms^{-1}$. The coefficient of friction between the road and the tyres is 0.71.

   a) Is the car travelling above or below safe speed?

b) What will happen to the car's motion?

___

3. A circus stunt motorcyclist drives around a vertical loop of radius 5m. What is the minimum velocity at the top of the loop?
(Assume that g = $9.8ms^{-2}$)

___

4. A car travels over a hump in the road of radius 10m. What is the velocity at which the wheels of the car leave the ground?

___

5. Find the lift force on a 400kg aeroplane pulling out of an arc – shape dive 500m, at a velocity of 900km/h (assume g =$10ms^{-2}$).

___

6. An aeroplane flies in a vertical circular path of radius 200m at a constant speed. The passengers feel weightless just above the floor when the plane is at the top of a circular path.

a) At what speed must the plane be travelling?

___

b) Explain why the passengers feel weightless.

___

c) If the plane travelled along the same path with a higher velocity, what would happen to the passengers?

____

____

____

Thank you for completing learning activity 9. Now check your work. Answers are at the end of the module.

Rotary Motion

Everything in the universe is in motion. Even as you sit, you are moving very rapidly because the Earth is rotating on its axis and that is why, you are moving as the earth moves around the Sun. The Sun, the Earth and the rest of the planets in our solar system are moving in a general rotation around our galaxy within the universe.
In the equation you studied and used so far for circular motion, v has represented the tangential or linear velocity of the body. Objects are moving in a circular path with an angular velocity, ω.

The angular velocity is the angle turned through per unit time. It is measured in radians per second. One radian is the angle when the arc length is the same as the radius of the circle. There are 2π radians (rad) in a circle, equivalent to 360^o. Therefore, one revolution equals 2π radians.

Tangential velocity (v) = angular velocity (ω) x radius (r)

$$v = \omega r \text{ or } \omega = \frac{v}{r}$$

Centripetal acceleration and the force in terms of angular velocity:

$$a = \frac{v^2}{r} = \omega^2 r \quad \text{and} \quad F = \frac{mv^2}{r} = m\omega^2 r$$

Until now, all motion has always involved linear quantities. However, another system can be used when dealing with motion in a circle. Instead of basic measurements being linear displacement and time, angular displacement and time are used. The angular displacement of an object is the angle in radians through which it has rotated and is given the symbol, $\boldsymbol{\theta}$.

The formula for angular displacement θ is, shown below.

$$\theta = \frac{s}{r}$$

where **s** = arc of the circular path of the object and **r** = the radius of the circular motion.

Note that the **s** is part of the circumference of the circle and the circumference is given by **C= 2πr.**

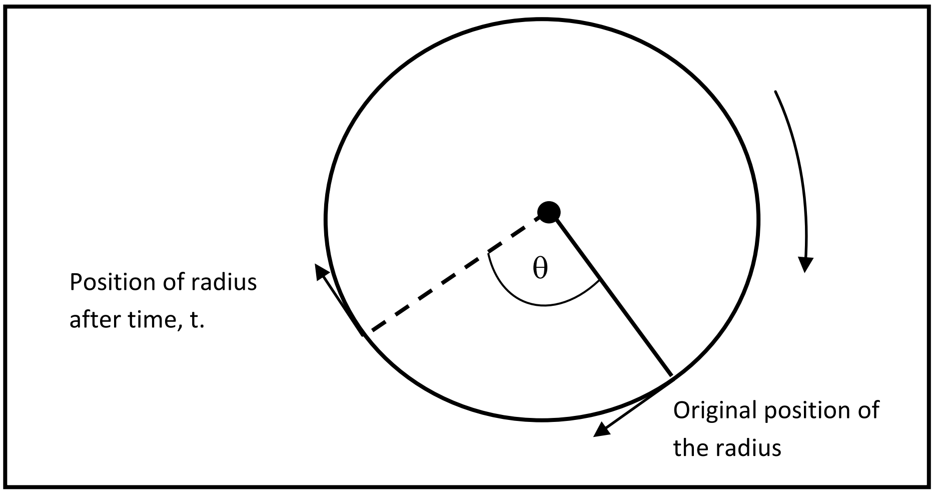


Figure 39 Angular Displacement of an object

In a full circle, **s = 2πr**

Since, $\theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi$ **radians**

In other words, 360^o represents 2π radians. Radians can be represented by unit 'rad' or by placing a small **c** where the degree symbol for an angle would be placed.

For example $\theta = \pi^c$ or $\theta = \pi$ rad.

In what you have studied earlier of motion in a straight line, velocity is defined as the rate of change of linear displacement. Similarly, an angular velocity is the rate of change of angular displacement, given the symbol **ω** (Greek letter which is called omega).

Average angular velocity has the following.

$$\omega_{av} = \frac{\theta}{t}$$

The table below shows linear and angular quantities

Physical quantities	Linear quantities	Angular quantities
displacement	s	θ
time	t	t
velocity	v	ω

Table 7 Linear and angular quantities with their symbols

Where θ is measured in radians, t, is measures in seconds and ω is measured in rad s^{-1} or rad/s.

The time for an object to complete its full circle is called its **period.** If an object undergoes a full circle, then its angular displacement is 2π radians. So if $\theta = 2\pi$ rad, then t = T.

$$\boldsymbol{\omega_{av} = \frac{\theta}{t} = \frac{2\pi}{T}}$$

It means that $\boldsymbol{\omega_{av} = 2\pi/T}$ allows the angular velocity to be determined knowing only the object's period.

Angular displacement is a scalar quantity requiring no direction. Angular velocity is a vector quantity, but its vector nature will not be considered here. The angular velocity of every point of a body undergoing uniform circular motion is the same.

Consider the second hand of a clock. For example, all points along the hand complete full circle at the same time. It means that each point has an identical period and an identical angular velocity. However, the tangential velocity of each point is different. If

$$\boldsymbol{v = \frac{2\pi r}{T}} \text{ and } \boldsymbol{\omega = \frac{2\pi}{T}} \text{ then, } \boldsymbol{v = r\omega}$$

In other words, the larger the radius of the motion, the larger is the tangential velocity.

Note: Calculators can be used with a radian mode. You can set your calculators on radian mode using the manual for instruction.

Example 1

A train is travelling on a track which is part of a circle of radius 600m, at a constant speed of 50m/s. What is its angular velocity? What centripetal force acts on the train if its mass is 2000kg?

$$\omega = \frac{v}{r} = \frac{50\text{m/s}}{600\text{m}} = 50\text{m/s} = 0.083 \text{ rad/s}$$

$$F = m\omega^2 r = 2000 \times (0.083)^2 \times 600 = 8266.8\text{N}$$

Example 2

A fly wheel of radius 2m is making 120 RPM (revolution per minute). Calculate the linear velocity of a point on the rim.

First, convert revolution per minute into rad/s

1 revolution = 2π radians

Therefore 120 RPM = 120/60 rev/s = 2 rev/s =4 rad/s

$v = \omega r = 4\pi$ rad/s x 2 = 8π m/s

Now check what you have just learnt by trying out the learning activity on the next page!

Learning Activity 10

20 minutes

Answer the following questions on the spaces provided.

1. Calculate the force that acts on a mass of 2kg which is rotating at 5 rad/s in a circle of radius 50cm.

2. The breaking strength of a string 2.5m long is 100N. What is the maximum revolution per minute (RPM) at which the string can retain a 2kg mass attached to its end?

3. A crane lifts a load at a speed of 5.4m/s. The drum onto which the wire is wounded has a diameter of 36cm. Calculate the angular velocity of the drum.

4. A merry-go–round completes one-sixth of a revolution in two seconds. If it has a radius of 8 metres, find the;

   a) angular displacement.

   b) angular velocity.

c) tangential velocity for a point 8m from the centre of the motion.

---

5. A record player has speed of 33 revolutions per minute (rpm), 45rpm and 78rpm.

   a) Find the period of revolution for the record played at each speed.

   b) A record with a 30cm diameter is now placed on the player. What is the tangential velocity at a point on the circumference of the record at each speed?

Thank you for completing learning activity 10. Now check your work. Answers are at the end of the module.

NOW REVISE WELL USING THE MAIN POINTS ON THE NEXT PAGE.

Summary

You will now revise this module before doing **ASSESSMENT 2**.

Here are the main points to help you revise. Refer to the module topics if you need more information.

- The average speed of an object is the distance travelled divided by the time taken. The average velocity is the displacement divided by the time taken.
- Speed can be measured in metres per second or kilometres per hour and is a scalar quantity.
- Instantaneous velocity is the rate of change of displacement.
- The acceleration of a body is its change in velocity divided by the time taken for the change.
- Acceleration is a vector quantity.
- Velocity is the gradient (slope) of the displacement- time graph.
- The area under the velocity -time graph indicates the distance and the displacement of the motion. Displacement is a vector and distance is a scalar quantity.
- The gradient of the velocity- time graph indicates the acceleration.
- Every object tends to accelerate towards the ground with a constant acceleration due to gravity of about $10ms^{-1}$.
- Stopping distance = reaction distance + braking distance.
- Important formulae:

Motion in uniform velocity	Motion in uniform acceleration
v = constant $v = \frac{s}{t}$ $s = vt$	**a = constant** $v = u + at$ $s = ut + \frac{1}{2} a t^2$ $v^2 = u^2 + 2as$ $s = \frac{(u+v)\,t}{2}$

- For a projectile the vertical and the horizontal motion are independent.
- The range of the projectile depends on the time of the flight.
- An object travelling along a circular path at a constant speed has an acceleration called the centripetal acceleration directed towards the centre of the circular path.

$$a = \frac{v^2}{r}$$

Because centripetal acceleration is being experienced by the object, we say it is the result of a centripetal force given by,

$$F = \frac{mv^2}{r}$$

- When a car travels around a bend on a road, the maximum safe speed for the car is given by;

$$v = \sqrt{\mu g r}$$

Where **μ** is the coefficient of friction between the tyres and the road surface, **g** is the acceleration due to gravity and **r** is the radius of the curvature on the bend.
- The centripetal force is the net force (or unbalanced force directed towards the centre of the circular path.
- A ball swung at the end of a string in a vertical circle has a non-uniform velocity around the circle because of the influence of gravity (the weight of the ball). The speed of the ball is highest at the bottom of the circle and lowest at the top of the circle. The centripetal acceleration is at a minimum at the top and at maximum at the bottom. The angular velocity (ω) is the angle turned through per unit time. It is measured in radian per second. Angular velocity = tangential velocity/radius.
- One revolution = **2π radians**
- Formulae; $\mathbf{a = \omega^2 r}$ and $\mathbf{F = m\omega^2 r}$

We hope you have enjoyed studying this module. We encourage you to revise well and complete Assessment 2.

NOW YOU MUST COMPLETE ASSESSMENT 2 AND RETURN IT TO THE PROVINCIAL CENTRE CO-ORDINATOR.

Answers to Learning Activities 1-10

Learning Activity 1

1. a) 7 metres
   b) 1m towards the board
2. distance = 10m, displacement = 4km east
3. 50 seconds
4. 30m - 20m =10m
5. a) Average speed = $5.83ms^{-1}$
   b) Average velocity = $4.16ms^{-1}$, North 53.1^{o}East
6. a) The distance travelled=2km
   b) The displacement = 2km East
   c) The speed = 1.66m/s
   d) The velocity =1.66m/s East
7. 2.5m/s
8. a) 10km
   b) 7.07km
   c) 5km/h or 1.38m/s
   d) 3.5km/h or 0.97m/s North of East
9. a) 502.4m
   b) zero
   c) 8.37m/s
   d) 5.33m/s
10. a) 400km
    b) 282.8km North of East
    c) $133.3kmh^{-1}$ =133.3km/h

d) $94.26kmh^{-1}$ N45°East

Learning Activity 2

1. 12.36m/s W14°N

2. 301m/s, N4.8°E

3. 15km/h in the opposite direction

4. a) 56.13° to the bank
   b) 3 minutes
   c) 0.3km
   d) 3 minutes

5. 100.02m/s at N1.15°E

6. a) 100m S36.87°E
   b) 140m
   c) $14ms^{-1}$
   d) $10ms^{-1}$ S36.87°E

7. Easterly component is $150kmh^{-1}$ and northerly component is $260km^{-1}$

8. $11kmh^{-1}$ at 22° east of south

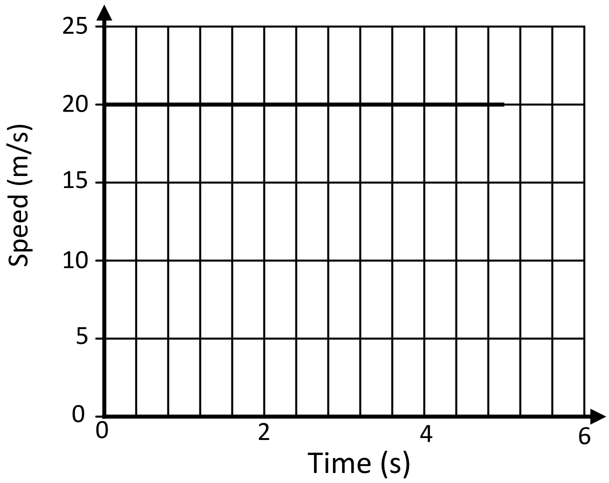
Learning Activity 3

1. a) 50km
   b) 6 hours
   c) 8.3km/h
   d) 2 stops
   e) 2 hours
   f) 12.5km/h
   g) The steepness of the graph will tell how fast she travelled (starting from point A)

2. a) 100m
   b) 20m/s

c) slows down to a stop

d)



3. a) $-5ms^{-2}$

   b) $5ms^{-2}$

4. $4m/s^2$

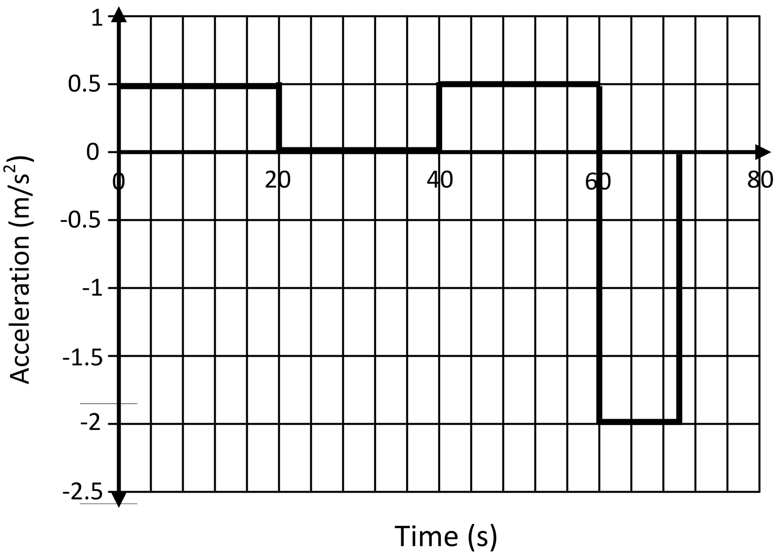
5. a) $(0.5ms^{-2}, 0)$

   b) 20m/s

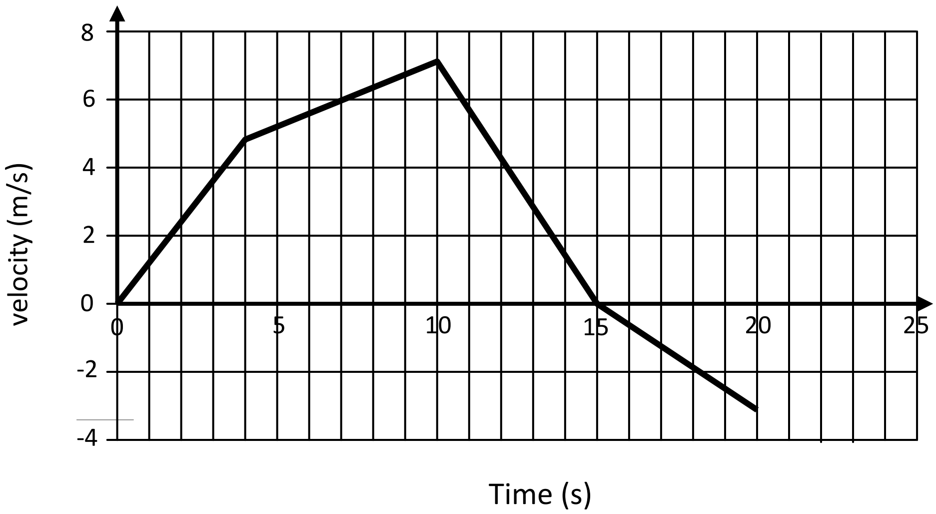
   c) 20 x 10 + (½ 20 x 10) =300m

   d) $(-2ms^{-1})$

   e)



6.

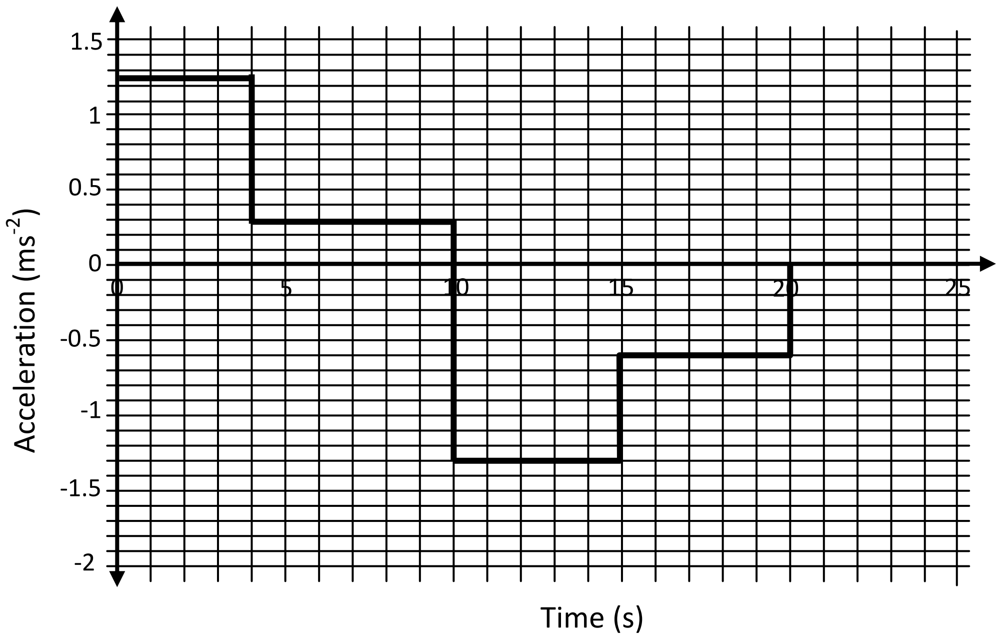


a) 71m

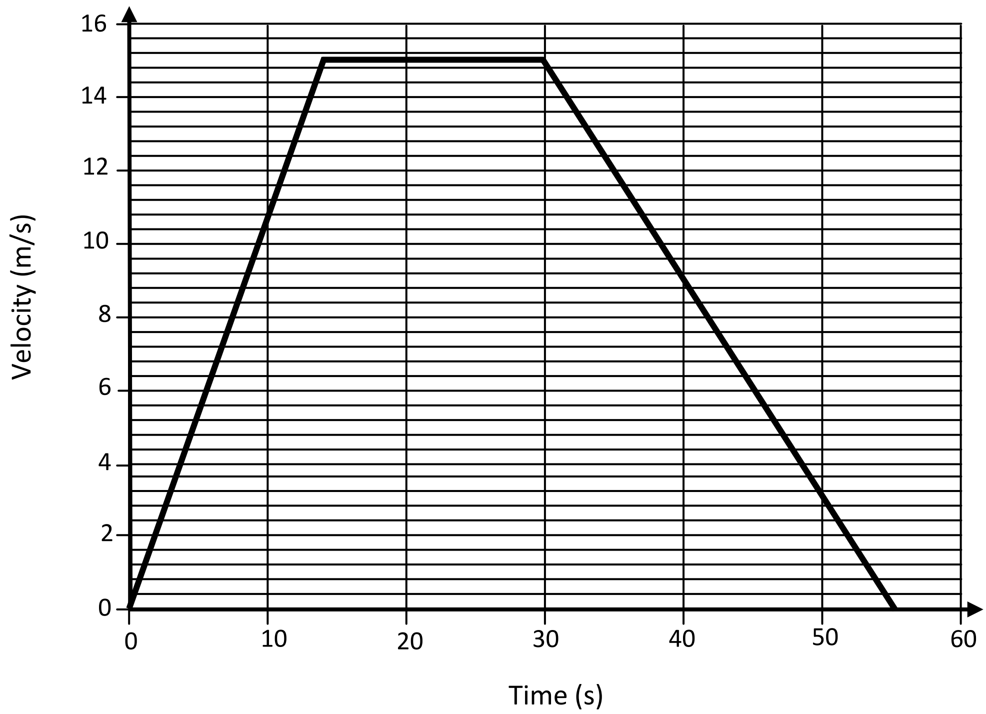
b) 56m

c) 2.8m/s

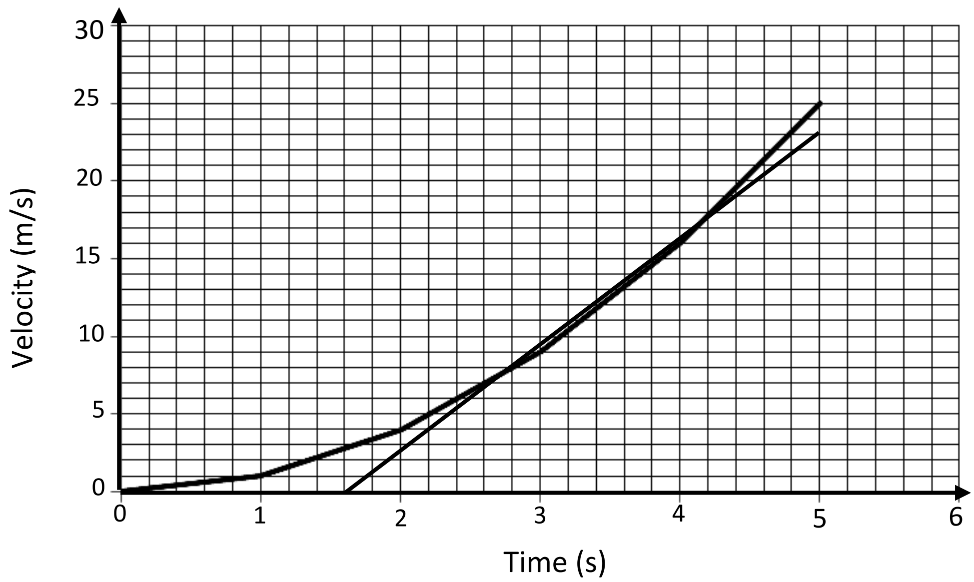
d)



7. A velocity time graph for the motion



8. Work out your answer from the graph.



a) No

b) 5m/s

c) 1m/s

d) 2m/s

e) 6m/s

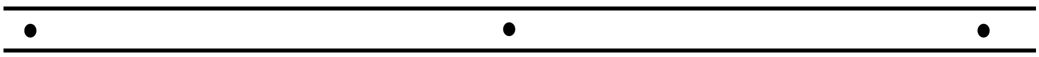
f) 6m/s

g) 6m/s

h) The same

Learning Activity 4

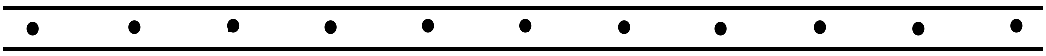
1.

a) 

2 x 0.02 = 0.04s

b) 

5 x 0.02 = 0.1s

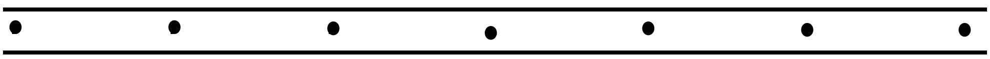
c) 

10 x 0.02 = 0.2s

2

a) 

$$v = \frac{20\text{cm}}{0.02\text{s}} = 1000\text{cm/s}$$

b) 

$$v = \frac{20\text{cm}}{(0.02) \text{ x } 6\text{s}} = 166.67\text{cm/s}$$

c) 

$$v = \frac{20\text{cm}}{(0.02) \text{ x } 13\text{s}} = 76.92\text{cm/s}$$

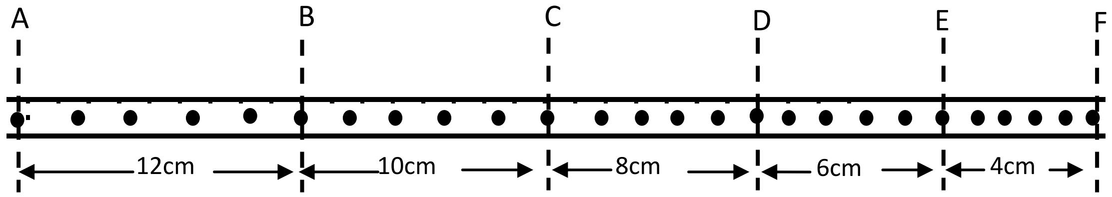
To find the time taken to make the dots, multiply with the spaces between each dot and multiply again with the length of the tape. This should be the velocity of the tape.

3. a) Y

b) Z

c) X

4.



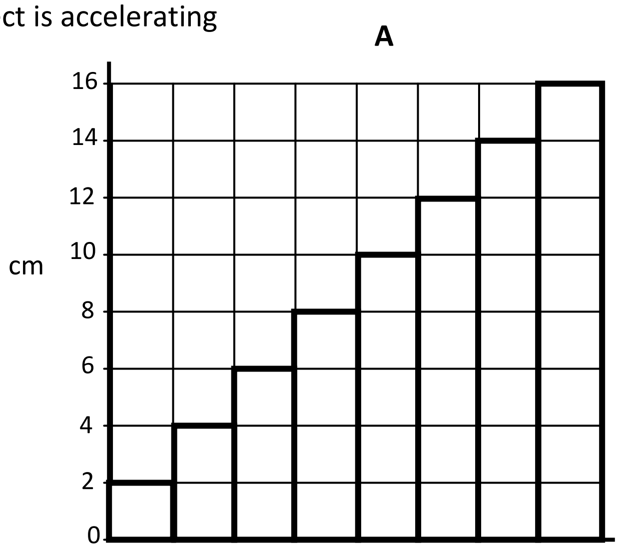
a) F

b) 0.1s

c) AB = 1.2cm/s, BC = 1.0cm/s, CD = 0.8cm/s, DE = 0.6cm/s, EF = 0.4cm/s

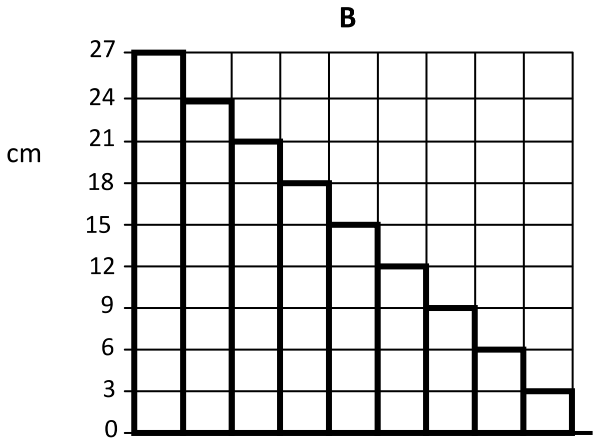
d) 2cm/s^2

5. a) The object is accelerating



b) velocity = 2cm/0.1 = 20 cm/s , 4cm/0.1 = 40cm/s.
Therefore, acceleration a = v-u/t
40 -20/0.1 = 200cm/s^2

c) The object is slowing down (decelerating)



d) velocity = 27cm/0.1 = 270cm/s , 24cm/0.1 = 240cm/s.

Therefore acceleration a = v-u/t

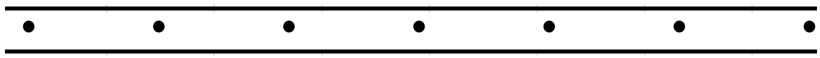
240-270/0.1 = -300cm/s^2

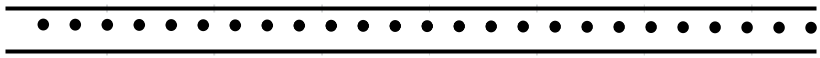
e) The object is travelling at constant speed. (no acceleration)

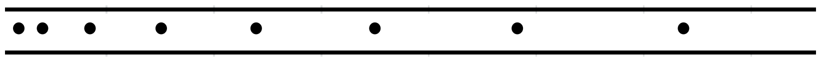
f) $v = v - {}^{u}\!/_{t} = 8cm/0.1cm/s$

$= 80 - {}^{80}\!/_{0.1} = 0cm/s^2$

6.

a) 

b) 

c) 

d) Length of tape divided by the number of dots per second.

7. a) 5mm/s

b) 37mm/s

c) 20.7mm/s

Learning Activity 5

1. 2.5ms^{-2}

2. a) 10ms^{-1}

b) s= ut + ½ at^2. =0 + ½ x 2 (5^2) = 25m

c) t = 100m/10 = 10s

3. v = u + at a = v-u /t = 20 – 4 /8 = 2m/s^{-2} . s=ut + ½ at^2 = 4 x 8 +1/2 x 2 x (8^2) = 96m

4. a) v = u + at , t = 0- 12 /-3 = 4s

b) v^2 =u^2 + 2as , s = v^2/2x3 = -144/-6 = 24m

5. a) Its final velocity. v = u + at, v = 0 + 4 x 10 = 40ms^{-1}

b) The distance travelled. s= ut + ½ at^2, s = 0 + ½ x 4 x (10^2) = 200m

6. a) Her acceleration. v^2 = u^2 +2as, a= (10^2)/2x 50 = 1ms^{-2}

b) The time taken. t = v-u/a, 10 – 0 /1 =10s

7. a) $v = u + at$, $a = 0 – 60/2 = -30ms^{-2}$
   b) $s = ut + ½ at^2 = 60 x 2 – ½ x30 (2^2) = 60m$

8. a) 15m/s
   b) 900m

9. 10 s, 100m

10. a) Its acceleration is $-2ms^{-2}$
    b) Its deceleration is $2ms^{-2}$
    c) The distance it covers in coming to rest is 36m.

Learning Activity 6

1. Reaction distance = ut, s= 20 x 0.2 =4m

   Braking distance = $v^2 – u^2/2a$, $s = 0-(20^2)/2x\ 20 = 400/40 = 10m$

   Stopping distance = 4 + 10 = 14m

2. a) $v= u + at$, $a= v-u/t = 30 – 0 /5 =-6m/s^2$
      $= -6m/s^2$
   b) What distance did the car travel before it stopped?
      75m

3. Yes reaction time is important for the driver to stop the car at a safer distance. The quicker the driver reacts by stepping on the brakes can lower the reaction distance.

4. a) $v=u + at$, $a= 0 - 20/4 = -5m/s^{-2}$
   b) $s= ut + ½ at^2$, $20 x 4 – ½ x5 x (4^2) = 80 – 40 = 40m$

5. Road surface, weather condition, inefficient brakes and smooth tyres.

Learning Activity 7

1. $s = ut + 1/2\ gt^2$, $s= 0+1/2 x 10 x(3^2)$, $=45m$
2. $s = ut + 1/2\ gt^2$, $s=0 +1/2 x 10 x (4^2) = 5 x16 = 80m$

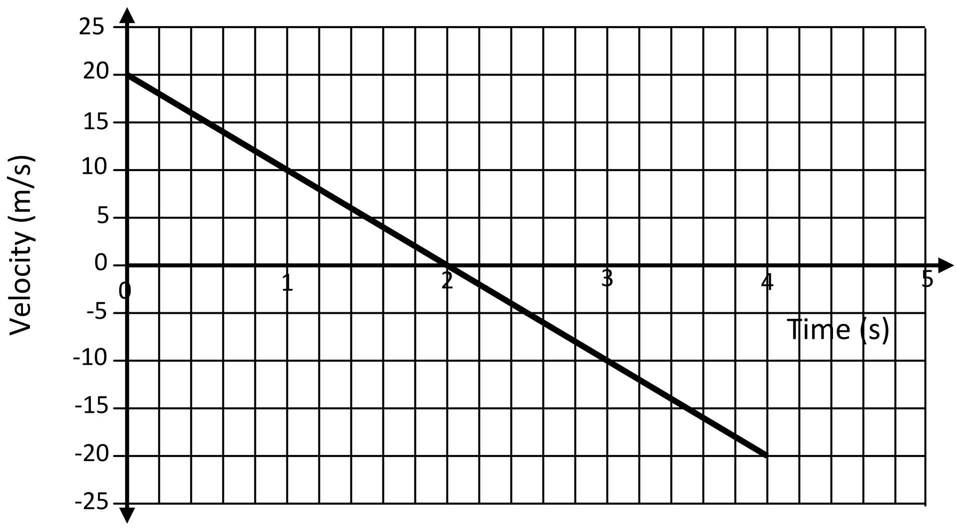
3. a) $v^2 = u^2 +2 x g x s$, $=20m/s$
   b) $v = u + at$, $20=0+ 10 x t$, $t =2s$

4. a) $v^2 = u^2 + 2as$, $0 -(20^2)/ 2x\ 10 = 20m$

   b) $v = u + at$, $v=20\ -10t$, $0-20/-10 = 2s$

   c) 2 seconds going up and 2 seconds to come back down

   d) Sketch a velocity time graph for this motion.



5. a) i) $s= ut + 1/2\ gt^2$, $40 \times 3 – ½ \times 10\ (3^2) = 75m$

   ii) $40 \times 4 – ½ \times 10\ (4^2) = 80m$

   iii) $40 \times 5 -1/2 \times 10\ (5^2) =75$

   iv) $40 \times 8 – ½ \times 10\ (8^2) =0$

   b) $40 \times 2 + ½ \times 10(2^2) =100m$

6. $s = ut + 1/2t^2$, $3.2 = 0 + g \times 4$, $g = 3.2\ x2/4 = 1.6ms^{-2}$

7. $s= ut + ½\ gt^2$, $300 \times 2 =10 \times t^2$, $t^2 = 60$, $t= 7.75s$

8. a) $s= ut + ½\ gt^2$, $20x2 =10\ t^2$, $t^2 = 4$, $t= 2\ s$

   b) $v= u + at$, $v=0 +10 \times 2 = 20m/s$

9. a) $v= u + at$, $v=10 \times 2 =20m/s$

   b) $s=ut + 1/2gt^2$, $0 +1/2 \times 10 \times 4 = 20m$

   c) Much heavier objects will fall faster than lighter object. Height will remain the same at 20m.

10. Because of air resistance the velocity cannot be constant until after some time the parachutist may have reach terminal velocity which now constant velocity.

Learning Activity 8

1. a)

$$s = ut + \frac{1}{2}at^2$$
$$= 0 \times 5 + \frac{1}{2} \times 10 \times (5)^2$$
$$= 125m$$

b)

$$\text{Horizontal} = s = ut$$
$$= 20 \text{ x } 5$$
$$= 100m$$

2. a)

$$s = ut + \frac{1}{2}gt^2$$
$$500 = 0 + \frac{1}{2} \times 10 \times t^2$$
$$t^2 = \frac{1000}{10}$$
$$t^2 = 100$$
$$t = \sqrt{100}$$
$$= 10s$$

b)

$$s = ut$$
$$= 200 \times 10$$
$$= 2000m \text{ or } 2km$$

3. a)

$$s = ut + \frac{1}{2}gt^2$$
$$= 50 \text{ x } 2$$
$$= 10 \times t^2$$
$$t^2 = 10$$
$$t = \sqrt{10}$$
$$t = 3.16s$$

b)

$$v^2 = u^2 + 2gh$$
$$v^2 = (20)^2 + 2 \times 10 \times 50$$
$$v^2 = 400 + 1000$$
$$v = 37.41ms^{-1} \text{ at } 57^0 41$$

c) $s = ut$
$= 3.16 \times 20$
$= 63.2m$

4. a) Horizontal $= v \times \cos 60 = 8 \times 0.5 = 4m/s$
Vertical $= v \times \sin 60 = 8 \times 0.86625 = 6.93m/s$

b) $v^2 = u^2 + 2gh$
$0 = (6.93)^2 + 2 \times 10h$
$20h = \frac{48.02}{20}$
$h = 2.4m$

c) $v = u + at$
$t = \frac{v - u}{g}$
$\frac{0 - 6.93}{-10}$
$= 0.693 \times 2$
$= 1.386s$

d) $s = ut + 0$
$4 \times 1.386 = 5.54m$

e) zero

5. a) No air resistance then it will reach maximum height in longer period of time .

b) The range of the projectile will be long

6. a) $v^2 = u^2 + 2gh$
$h = \frac{(250)^2}{20}$
$= 3{,}125m$ or $3.125km$

b) $t = \frac{v - u}{g}$

$= \frac{0 - 250}{-10}$

$= 25s \times 2$

$= 50s$

$s = ut$

$= 433 \times 50$

$= 21.65km$

7. a) $Horizontal = V\cos 45^0$

$= 30 \times \cos 45^0$

$= 21.21ms^{-1}$

$Vertical = V\sin 45^0$

$= 30\sin 45^0$

$= 21.21ms^{-1}$

b) $s = ut$

$= 21.21 \times 4.24s$

$= 89.9m$

c) No – increases its velocity to above 30ms.

8. $s = ut + \frac{1}{2}gt^2$

$= 1.1 \times 2 = t^2$

$t^2 = \frac{2.2}{10}$

$= \sqrt{0.22} = 0.46s$

$v = \frac{s}{t} = \frac{5}{0.46}$

$= 10.65ms^{-1}$

9. a) $s = ut + \frac{1}{2}gt^2$

$t^2 = \frac{5}{10} = 0.71s$

b) $v = \frac{18}{0.7} = 25.7m/s$

10. a) $s = ut + \frac{1}{2}gt^2$

$100 \times 2 = 98t^2$

$t^2 = \frac{200}{9.8} = 4.52s$

b) $v = \frac{s}{t}$

$= \frac{50}{4.52}$

$= 11.1m/s$

c) 43.6m/s 76^{o} down from the horizontal

11. $Horizontal = V\cos\theta$

$= 30 \times \cos 45$

$= 21.21m/s$

$Vertical = V\sin 45$

$= 30 \times \sin 45$

$= 21.21m/s$

Learning Activity 9

1. a) $a = \frac{v^2}{r} = \frac{(20)^2}{150} = 2.67 m/s^2$

b) $t^2 = \frac{4\pi^2 r}{a} = \frac{4 \times 3.14^2 \times 150}{2.67}$

$t^2 = \sqrt{2215.64}$

$t = 47.07s$

2. a) Above safe speed

$\text{Safespeed} = \sqrt{ugr}$

$= \sqrt{0.71 \times 9.8 \times 40}$

$= 16.6 m/s$

b) It is going to turn over

3. $a = \frac{v^2}{r}$

$v^2 = a \times r$

$v = \sqrt{9.8 \times 5}$

$v = 7 m/s$

4. $a = \frac{v^2}{r}$

$v^2 = 10 \times 10$

$v = \sqrt{100}$

$v = 10 m/s$

5. $F = \frac{mv^2}{r}$

$= \frac{400 \times (250)^2}{500}$

$= 50000 + 4000$

$= 54000N$

6. a) $v^2 = gr$

$v^2 = 10 \times 200$

$v = \sqrt{2000}$

$v = 44.7 m/s$

b) They felt weightless because of inertia

c) They will hit the ceiling of the plane

Learning Activity 10

1. $F = m\omega^2 r$

$= 2 \times (5)^2 \times 0.5m$

$= 25N$

2. $\frac{150}{\pi} = 47.75RPM$

3. $\omega = \frac{v}{R}$

$= \frac{5.4}{0.18}$

$= 30 rad/s$

4. a) $s = \frac{2\pi r}{6}$ ($\frac{1}{6}$ revolution)

$r = 8m$

$t = 2s$

Therefore $\theta = \frac{s}{r}$ which is

$\theta = \frac{2\pi r}{6r}$

$= \frac{\pi}{3} rad (1.05 rad)$

b) $\omega = \frac{\theta}{t}$

$\omega = \frac{\pi}{3}/2$

$= 0.52\ \text{rad.s}^{-1}$

c) $\omega = \frac{2\pi}{T}$

$= 2\pi/\frac{1}{6} = 12\text{s}$

$v = \frac{2\pi r}{T}$

$= 2\pi \times \frac{8}{12}$

$\frac{4\pi}{3} = (4.2\text{ms}^{-1}$ at the tangent to the circle)

5. a) Period for 33rpm = 1.82s.
Velocity v = 0.518ms^{-1}

b) Period for 45rpm = 1.33s.
Velocity v = 0.707ms^{-1}

c) Period from 78rpm = 0.769s.
Velocity v = 1.23ms^{-1}

If you have queries regarding the answers, then please visit your nearest FODE provincial centre and ask a distance tutor to assist you.

References

Dungan Tom. 1986. *GCSE Physics*. Fourth Edition. Hachette UK Company. England Nick. 1989. *Physics Matters* . Hodder and Stroughton. Great Britain. Grade 12 Unit Booklet. *Kinematics*. Unit One. Publisher, 1990

John Harding, Roy Burrow, Sabino Del Balso & Kim Forward. 1996. *Physics Concept And Application*. Macmillan Education Australia.

Johnson Keith. 1980. "*CSE Physics For You*. Hutchinson Ltd

Microsoft Encarta. 1993-2006. Eencyclopedia Microsoft Corporation. Moyle Allan & Miller Molde. 1986. *Gr11 Senior Physics*. Macmillan Company Australia.

Pople Steven. 2010. *Complete Physics for Cambridge IGCSE*. Second Edition .Oxford University Press.

W .Crossley & D.Christian. 1980. *Essential Physics*. Sapphire Books. Australia.

Wilkinson John. 1993. *Essentials of Physics*. Macmillan Education Australia

Quantities symbols and units

Fundamental quantities, derived quantities, their symbols including appreciations.

Quantity	Symbol	SI unit	Abbreviations
length	s	metre	m
mass	m	kilogram	kg
Time	t	second	S
Electric current	I	ampere	A
Thermodynamic temperature	T	kelvin	K

Quantity	Symbol	Units	
		In full	Abbreviation and accepted alternative
Area	A	Square metre	m^2
Volume	V	Cubic metre	m^3
Speed ,velocity	v	Metre per second	ms^{-1} or m/s
Acceleration	a	Metre per second square	Ms^{-2} , m/s^2
Electric current	I	Ampere	A
Electric charge	q	Coulomb	C
Force	F	Newton	N
pressure	p	Pascal	Pa or N/m^2 or Nm^{-2}
Electric potential	V	Volt	V
Energy/work	E /W	Joule	J ,Nm
power	P	Watt	W, J/s , Js^{-1}
Density	ρ	kilogram per metre cubed	Kgm^{-3}, kg/m^3
momentum	p	Kilogram metre per second/ newton second	Ns ,$Kgms^{-1}$, kgm/s
Plane angle	θ	Radians	rads
Gravitational field strength	g	Newton per kilogram	Nkg^{-1}, N/kg
Electric field strength	E	Newton per coloumb	Vm^{-1} ,V/m
Magnetic induction	B	Tesla/newton per ampere meter	T ,$NA^{-1}m^{-1}$

FODE PROVINCIAL CENTRES CONTACTS

PC NO.	FODE PROVINCIAL CENTRE	ADDRESS	PHONE/FAX	CUG PHONES	CONTACT PERSON		CUG PHONE
1	DARU	P. O. Box 68, Daru	6459033	72228146	The Coordinator	Senior Clerk	72229047
2	KEREMA	P. O. Box 86, Kerema	6481303	72228124	The Coordinator	Senior Clerk	72229049
3	CENTRAL	C/- FODE HQ	3419228	72228110	The Coordinator	Senior Clerk	72229050
4	ALOTAU	P. O. Box 822, Alotau	6411343 / 6419195	72228130	The Coordinator	Senior Clerk	72229051
5	POPONDETTA	P. O. Box 71, Popondetta	6297160 / 6297678	72228138	The Coordinator	Senior Clerk	72229052
6	MENDI	P. O. Box 237, Mendi	5491264 / 72895095	72228142	The Coordinator	Senior Clerk	72229053
7	GOROKA	P. O. Box 990, Goroka	5322085 / 5322321	72228116	The Coordinator	Senior Clerk	72229054
8	KUNDIAWA	P. O. Box 95, Kundiawa	5351612	72228144	The Coordinator	Senior Clerk	72229056
9	MT HAGEN	P. O. Box 418, Mt. Hagen	5421194 / 5423332	72228148	The Coordinator	Senior Clerk	72229057
10	VANIMO	P. O. Box 38, Vanimo	4571175 / 4571438	72228140	The Coordinator	Senior Clerk	72229060
11	WEWAK	P. O. Box 583, Wewak	4562231/ 4561114	72228122	The Coordinator	Senior Clerk	72229062
12	MADANG	P. O. Box 2071, Madang	4222418	72228126	The Coordinator	Senior Clerk	72229063
13	LAE	P. O. Box 4969, Lae	4725508 / 4721162	72228132	The Coordinator	Senior Clerk	72229064
14	KIMBE	P. O. Box 328, Kimbe	9835110	72228150	The Coordinator	Senior Clerk	72229065
15	RABAUL	P. O. Box 83, Kokopo	9400314	72228118	The Coordinator	Senior Clerk	72229067
16	KAVIENG	P. O. Box 284, Kavieng	9842183	72228136	The Coordinator	Senior Clerk	72229069
17	BUKA	P. O. Box 154, Buka	9739838	72228108	The Coordinator	Senior Clerk	72229073
18	MANUS	P. O. Box 41, Lorengau	9709251	72228128	The Coordinator	Senior Clerk	72229080
19	NCD	C/- FODE HQ	3230299 Ext 26	72228134	The Coordinator	Senior Clerk	72229081
20	WABAG	P. O. Box 259, Wabag	5471114	72228120	The Coordinator	Senior Clerk	72229082
21	HELA	P. O. Box 63, Tari	73197115	72228141	The Coordinator	Senior Clerk	72229083
22	JIWAKA	c/- FODE Hagen		72228143	The Coordinator	Senior Clerk	72229085

FODE SUBJECTS AND COURSE PROGRAMMES

GRADE LEVELS	SUBJECTS/COURSES
Grades 7 and 8	1. English
	2. Mathematics
	3. Personal Development
	4. Social Science
	5. Science
	6. Making a Living
Grades 9 and 10	1. English
	2. Mathematics
	3. Personal Development
	4. Science
	5. Social Science
	6. Business Studies
	7. Design and Technology- Computing
Grades 11 and 12	1. English – Applied English/Language& Literature
	2. Mathematics - Mathematics A / Mathematics B
	3. Science – Biology/Chemistry/Physics
	4. Social Science – History/Geography/Economics
	5. Personal Development
	6. Business Studies
	7. Information & Communication Technology

REMEMBER:

- For Grades 7 and 8, you are required to do all six (6) subjects.
- For Grades 9 and 10, you must complete five (5) subjects and one (1) optional to be certified. Business Studies and Design & Technology – Computing are optional.
- For Grades 11 and 12, you are required to complete seven (7) out of thirteen (13) subjects to be certified.

Your Provincial Coordinator or Supervisor will give you more information regarding each subject and course.

GRADES 11 & 12 COURSE PROGRAMMES

No	Science	Humanities	Business
1	Applied English	Language & Literature	Language & Literature/Applied English
2	Mathematics A/B	Mathematics A/B	Mathematics A/B
3	Personal Development	Personal Development	Personal Development
4	Biology	Biology/Physics/Chemistry	Biology/Physics/Chemistry
5	Chemistry/ Physics	Geography	Economics/Geography/History
6	Geography/History/Economics	History / Economics	Business Studies
7	ICT	ICT	ICT

Notes: You must seek advice from your Provincial Coordinator regarding the recommended courses in each stream. Options should be discussed carefully before choosing the stream when enrolling into Grade 11. FODE will certify for the successful completion of seven subjects in Grade 12.

CERTIFICATE IN MATRICULATION STUDIES		
No	**Compulsory Courses**	**Optional Courses**
1	English 1	**Science Stream:** Biology, Chemistry, Physics
2	English 2	**Social Science Stream:** Geography, Intro to Economics and Asia and the Modern World
3	Mathematics 1	
4	Mathematics 2	
5	History of Science & Technology	

REMEMBER:
You must successfully complete 8 courses: 5 compulsory and 3 optional.