Unit 1: Problem Solving and Investigations

Lecturer Support Material
Acknowledgements

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St Benedict’s Teachers College
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Many of the activities found within this material have been developed from the work of Mr Wally Green in ‘Problems and Investigations’ (1998) Department of Education PNG.

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Primary and Secondary Teacher Education Project
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Papua New Guinea-Australia Development Cooperation Program
## Unit outline

<table>
<thead>
<tr>
<th>Unit</th>
<th>#</th>
<th>Modules</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>1.1</td>
<td>Non-routine Problems (core)</td>
</tr>
<tr>
<td></td>
<td>1.2</td>
<td>Investigations (core)</td>
</tr>
<tr>
<td>Problem Solving and</td>
<td>1.3</td>
<td>Real World Problems (core)</td>
</tr>
<tr>
<td>Investigations</td>
<td>1.4</td>
<td>Application of Chance Processes and Probability (core)</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>Strategy Games (recommended)</td>
</tr>
<tr>
<td></td>
<td>1.6</td>
<td>Problem Posing (recommended)</td>
</tr>
</tbody>
</table>

## Icons

- 📖  Read  or  research
- 🖋️  Write  or  summarise
- 🗣️  Activity  or  discussion
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Lecturer Support Material

This Lecturer Support Material has been developed to assist lecturers in the teaching and assessing of Unit 1: Problem Solving and Investigations.

The material consists of:

- An introduction to the unit, which includes information on the overall rational for the unit as well as recommended teaching approaches and suggestions on how the unit can be integrated into practicum activities. Background readings to support the teaching of the material can also be found in this section
- Module outlines, setting out a suggested sequence of learning activities and identifying topics which could be taught within each module. Ideas for assessment activities are also provided
- A Unit Glossary

Suggestions have been made about what content should be covered within the unit as well as recommending an approach to teaching the material. It is envisaged that by working through the suggested sequence of leaning activities lecturers will be modelling good practices for teaching mathematics that students can then apply in their own teaching.

Student Support Material has also been developed for the unit ‘Problem Solving and Investigations’ to accompany this Lecturer Support Material. The Lecturer Support Material should be read in conjunction with the Student Support Material as the Lecturer Support Material makes reference to activities and material contained only within the Student Support Material.

It is not expected that the students will work only from the ideas and suggestions contained within the Student Support Material. Additional ideas and activities are set out in the Lecturer Support Material to compliment the student material. The lecturer will need to make decisions about how to present the material and make decisions about what activities will be covered during lecture periods and what students will be required to do during their study time.

When using this material it is recommended that lecturers:

- read through the whole unit prior to planning the course overview
- select the modules and topics to be cover in the time available
- plan the activities that will be presented to the students
- select the material from the Student Support Material that will be used to support the teaching of the unit
- develop the assessment tasks for the unit.

It is important to remember that this material is support material only. While lecturers are encouraged to try out the suggested activities within the material, it is hoped that people will also include their own ideas. This material, along with the Student Support Material should be seen as a living, working document which can be reviewed and changed to suit the curriculum needs of the Primary Teachers Colleges and new ideas and trends in the teaching of mathematics.
Unit 1: Problem Solving and Investigations

The Mathematics course seeks to develop beginning teachers who are:

- confident in their ability to teach mathematics in the Primary School, are familiar with the Primary School Mathematics Syllabus and have a strong understanding of the mathematical concepts covered within it
- aware of the factors which impact on the teaching and learning of mathematics
- resourceful, creative, life long learners who facilitate learning
- inclusive of all people, regardless of gender, social, cultural and language background.

Rationale

Problem solving involves the application of mathematical skills and reasoning to problems encountered in everyday life. Real world problems are not presented in a neat and orderly manner like a page of ‘sums’ so it is important for students to understand the mathematical operations and procedures so that in a problem-solving situation they know which operations and procedures to use to solve the problem. It is essential that they develop their mathematical reasoning so they will have confidence in their ability to use different approaches to solve unfamiliar problems in everyday situations. Problem solving activities provide the bridge between the mathematics classroom and the real world, allowing students to develop an understanding of the value and importance of mathematics in their lives. The ability to solve problems is crucial for the development of productive citizenship.

When children start school they love problems and puzzles and this can be nurtured through developing a problem solving approach to the teaching of mathematics. Problem solving provides an opportunity for presenting challenging, creative and enjoyable mathematical experiences.

Aims

This unit aims to produce beginning teachers who are:

- confident in their ability to solve a range of problems using a variety of strategies
- reflective and critical
- able to articulate their mathematical thinking
- confident and competent to teach problem solving in the primary school (Grades 3-8).

Objectives

As a result of studying this unit students will:

- understand what is meant by a problem in mathematics
• appreciate the importance of developing their own problem solving skills and also those of children in the primary school
• have developed strategies to help understand a problem, distinguishing between what is given and what is to be found, determining an approach and then apply it
• have solved problems in a group situation and realised the importance of cooperative learning and student centred approaches to problem solving
• understand the value of making an hypothesis and learning from the results so that the hypothesis can be improved and therefore a solution found
• be able to include problem solving in their teaching of mathematical concepts in the primary school curriculum.

Unit Outline

‘Problem Solving and Investigations’ is a 3-credit point unit.

To successfully complete this unit it is suggested that students complete the following core modules:

- Module 1.1 Non Routine problems
- Module 1.2 Investigations
- Module 1.3 Real world problems
- Module 1.4 Application to Chance Processes and Probability

Taking into consideration the time available for this unit it is recommended that lecturers select addition modules from those listed below:

- Module 1.5 Strategy games
- Module 1.6 Problem Posing

Each of these modules should take between 6 to 9 hours of lectures to complete. It is also expected that students will spend an equivalent number of hours of non-contact time studying the ideas and concepts raised in this unit. A detailed description of each module is included in this Lecturer Support Material.

Sequencing of Modules within the Unit

When considering the teaching of this unit it is important that the unit be viewed in its entirety rather than as a number of discrete modules. Lecturers are advised to read through the outlines for all modules before developing a unit overview. It is recommended that time be made available with students at the start of the unit to develop an overall view of the concepts and understandings to be developed throughout the unit. The relationships between modules need to be maintained throughout the unit as ideas are built upon as new concepts are introduced.
At the conclusion of the unit it will be important to spend some time reviewing the work covered in the various modules and considering the overall implications for teaching problem solving and investigations in the primary schools.

The concepts and suggested activities for ‘Module 1.6 Problem Posing’ may be incorporated across all modules. For example, when studying ‘Module 1.1 Non Routine Problems’ students could complete an activity which required them to develop a series of non-routine problems suitable for use in the primary school.

A suggested sequence for the delivery of this unit (3 credit points over 12 weeks) is outlined below.

<table>
<thead>
<tr>
<th>Week</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Introduction to the unit, including discussion of the work to be covered in each module and the assessment tasks. Introductory activities to co-operative learning</td>
</tr>
<tr>
<td>2-3</td>
<td>Module 1.1 Non Routine problems</td>
</tr>
<tr>
<td>3-5</td>
<td>Module 1.2 Investigations</td>
</tr>
<tr>
<td>6-7</td>
<td>Module 1.3 Real world problems</td>
</tr>
<tr>
<td>8-9</td>
<td>Module 1.4 Application to Chance Processes and Probability</td>
</tr>
<tr>
<td>10-11</td>
<td>Module 1.5 Strategy Games</td>
</tr>
<tr>
<td>12</td>
<td>Review of the unit, implications for the teaching of problem solving and investigations in the primary school, unit evaluation</td>
</tr>
</tbody>
</table>

**Teaching Approaches**

The approach recommended to teach this unit is a student centred, activity based approach. Lecturers are encouraged to build upon and respect students’ different experiences, and to provide a range of purposeful and challenging activities. Students’ thinking should be developed with the emphasis being on ‘how’ to think not ‘what’ to think. A supportive learning environment should be established, encouraging students to take risks and to learn from one another. The problems presented in this material cover a range of mathematical concepts, relate to real life context and provide students with the opportunity to express their mathematical thinking and develop appropriate mathematical language.

Suggested strategies to use in the delivery of this unit include:

- discussions, small group and whole class, open and structured, between student and teacher and among the students themselves
- seminar presentations
• research and investigation activities
• co-operative group learning
• demonstrations
• projects
• games.

Although the unit consists of a number of different modules lecturers are encouraged to adopt a holistic approach to their teaching. Connections and relationships between the concepts developed in the various modules need to be established and the understandings developed in early modules built upon throughout the unit.

When teaching selected activities from this material, lecturers will be modelling appropriate strategies that can easily be adapted to the primary school context. Class activities, followed by opportunities for group and individual work, the recording of mathematical ideas, the displaying of student work, the use of class discussions, with an emphasis on the process rather than the product, and establishing a classroom environment conducive to learning is the approach recommended in the teaching of this unit. These approaches need to be made explicit to students and consideration given to their effectiveness in the teaching of mathematics.

Inclusive curriculum

In the delivery of this unit it is expected that every person will be provided with the opportunity to participate in, and contribute to, activities without fear or favour. Activities should be presented to cater for a range of abilities and should be gender inclusive.

When developing a gender inclusive program, lecturers will need to consider:

• **Language.** The language we use shapes and represents the way we think, therefore the language we use needs to be inclusive. It is important to use language that includes woman and to avoid generic terms such as ‘man’.

• **Access and participation.** It is necessary to ensure that both male and female students receive an equal share of the lecturer’s time and attention. Both male and female students need to be treated equally and given the same opportunities to participate in discussions, ask questions and contribute to the classroom conversations. Both female and male students should have equal access to classroom resources.

• **Teaching strategies.** Examples used in the teaching of the unit need to include both female students and male students. Also students learn in different ways so a range of different teaching strategies and assessment tasks need to be developed to accommodate these differences.

When students are considering the teaching of problem solving in the Primary School context, attention should be given to catering for all children, including those with special needs. Students will need to be encouraged to focus on what children with a range of special needs can do and consider how activities can be adapted to cater for these children.

Activities planned by students to teach in the primary classroom will also need to be inclusive, presenting positive and non-stereotypical representations of people.
Language issues

Language factors contribute significantly to children’s mathematical learning and mathematics teachers have an important role to play in assisting students to acquire the specialised language of mathematics. Teachers need to establish the connections between the everyday concepts, the everyday language, and the formal language, skills, and symbols of mathematics. Teachers also need to be aware of the language and cultural diversity of children, and how this will impact on the teaching and learning of mathematics.

When teaching this unit, lecturers will need to raise students’ awareness of these issues through providing opportunities for students to explore different cultural perspectives and express their mathematical ideas in a variety of ways. Opportunities should be provided for students to use everyday language, vernacular, Tok Pisin, English, symbols, graphs, charts, written and oral texts when sharing their mathematical understandings.

When students are planning for the teaching of problem solving in the Primary School context, they will be required to consider how they can support children in developing appropriate language to discuss the mathematical ideas. Particular consideration will be given to developing strategies to supporting children who speak English as a second language, and who are in the process of bridging from the vernacular to English.

Multigrade teaching

In implementing this unit, lecturers will need to consider how they can cater for the range of student ability levels. By providing opportunities for group work, presenting activities at a range of levels and allowing students to select from these, strategies suitable for use in a multi-grade setting will be modelled. When presenting this unit and deciding how to implement problem solving into the primary school context, consideration will be given to modelling strategies suitable for use within a multigrade classroom.

Assessment Activities

Assessment is the process of identifying, gathering and interpreting information about student learning. The main purpose of assessment is to improve student learning and the quality of the learning programs. Assessment should be undertaken at the beginning of the unit (diagnostic), during the unit (formative) and at the end of the unit (summative).

A variety of assessment strategies should be used and students should be given opportunities, in a variety of contexts, to demonstrate in an authentic manner, what they know, understand and can do. The assessment strategies used need to be sensitive to the diversity that exists amongst students and take into consideration gender, culture, and language differences.

The content that is being covered, the learning objectives being assessed and the style of teaching and learning being used, will influence the method of assessment used. When developing assessment tasks lecturers will need to ensure that:

- the requirements of the task are set out clearly
- the assessment tasks chosen are relevant to the objectives and allow students to demonstrate appropriate outcomes
- marks or grades reflect the relative importance of each part of the task
• the language used is familiar to students and ideas clearly expressed
• items are not too difficult or too easy
• it does not contain bias
• a marking scheme is developed and applied consistently

The number of assessment tasks for the unit will be determined by college policy. Suggested assessment tasks have been included for each module and lecturers will need to decide on which assessment tasks they will develop, taking into consideration the learning objectives for the entire unit.

Suggested assessment strategies for this unit include:
• oral presentation e.g. seminars, tutorials,
• project work
• reflective journals
• report writing – with a focus on inquiry, analysis and reflection
• exam

A range of sample assessment tasks can be found in Sample Assessment tasks section

**Practicum Suggestions**

When studying this unit students should be provided with opportunities to:
• observe teachers teaching problem solving and investigations activities
• practice teaching problem solving and investigations activities
• critically reflect on these experiences.

The following is a list of suggestions as to how this unit may be incorporated into Practicum activities such as school experience, demonstration lessons and block teaching. These ideas would need to be negotiated with the Professional Development strand.

**Demonstration lessons**

Students observe teachers presenting problem-solving and investigation activities to children. During these observations students can keep a record of:
• what the teacher is doing
• what the children are doing
• the type of questions asked
• how the children and the classroom are organised (group or individual work, learning centres)
• what the children are learning
• the language being used

Following these observations conduct a class discussion critically reflecting on student findings.
School experience

Involve students in micro teaching, working with a small group of children over a number of weeks. Students can work with children on:

- developing co-operative learning skills
- completing problem solving activities (non-routine problems, real world problems, chance activities, problem posing)
- carrying out an investigation

At the end of this period spend time critically reflecting on the experiences and making recommendations for future teaching.

Block teaching

Students can:

- plan a series of problem solving and/or investigation activities
- teach a series of problem solving and/or investigation activities
- evaluate their teaching and critically reflect on the use of problem solving in the classroom.

Background Readings

The following readings have been selected to provide lecturers with an understanding of the issues to be covered in this unit. The readings can be found in Background Readings Section

Marr, Beth and Helme, Sue. (1991). ‘Problem Solving’ in *Breaking the Maths Barrier*, Department of employment and training: Canberra, Australia

Unit Evaluation and Reflection

On the completion of the unit an evaluation should be compiled. This should include input from both staff and students reflecting on the teaching and learning that took place during the unit. The information collated during the evaluation process should inform the review and ongoing development of the unit.

Below is an example of focus questions a lecturer may use to review the unit. A student evaluation form is also included as well as information on how the data gathered can be analysed.

Focus questions for lecturer reflection

To determine the effectiveness of the practices and methodologies employed and the content covered in a unit of work, lecturers need to reflect on their teaching. When reflecting on our teaching the areas we can consider are:

- the content of the unit
- the methodologies used in delivering the unit
- the assessment activities
- the co-ordination of the unit.

To help us reflect on our teaching we can ask ourselves a number of questions about each of these areas.

Content of the unit

- Did the content support the objectives of the unit?
- Were the activities sequenced logically?
- Was the content relevant? Did the content help the students to become competent beginning primary school teachers?
- Do you think the students are now more confident to teach this subject in the primary school?
- What recommendations can you make?

Methodology

- How did you deliver the content to the students? Were these strategies effective?
- Were the students aware of the strategies you were modelling and how they could use these strategies in their own teaching?

Assessment of the Unit

- How clear were the assessment tasks?
- How many tasks were given to students? Was this sufficient/too few or too many?
- Did you give students enough time to complete each assessment task?
- Do the students’ assessment results display what you expected of the course?
- What are your recommendations?
Unit co-ordination

- How well did you co-ordinate this unit?
- Did you produce any materials for students? Were these appropriate?
- Did you communicate effectively with other lecturers who were involved in teaching the same unit?

After considering each of these questions we can then make recommendations about the future of this unit.

Focus questions for student reflection

Below is a list of focus questions which could be used to stimulate student discussion when evaluating the unit.

- What have you learnt from this mathematics unit this semester?
- What have been the highlights/strengths of this unit?
- What problems have you encountered with this unit?
- Has this unit helped prepare you to be a beginning primary school teacher? If so, how? If not, why do you think this?
- What comments can you make about the level of work covered in this unit?
- What recommendations can you make to improve this unit?
Student unit evaluations

Unit: ___________________________   Class: ________________

*Instructions: Put an ‘X’ in the appropriate box.*

<table>
<thead>
<tr>
<th>Indicators</th>
<th>Strongly Agree (5)</th>
<th>Agree (4)</th>
<th>Not sure (3)</th>
<th>Disagree (2)</th>
<th>Strongly Disagree (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. The objectives of the unit were clearly outlined.</td>
<td></td>
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<tr>
<td>2. The unit content was clearly related to the objectives.</td>
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<tr>
<td>3. The student support material helped my understanding of the unit.</td>
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<td>4. The library was able to provide me with additional references.</td>
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<td>5. The assignments were related to the unit objectives.</td>
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<td>6. The instructions to do the assignments were clear.</td>
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<td>7. The assignments were scheduled to allow enough time for preparation.</td>
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<tr>
<td>8. I obtained useful feedback on my assignments.</td>
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<tr>
<td>9. Assignments were returned in time to help me with this unit.</td>
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<tr>
<td>10. Teaching staff were available for consultation.</td>
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<tr>
<td>11. There were sufficient opportunities to discuss the unit content in class.</td>
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<tr>
<td>12. Demonstrations and practical activities were useful to my learning.</td>
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<tr>
<td>13. I have improved in my ability to talk and write about this unit.</td>
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<tr>
<td>14. I have improved my knowledge and skills in this unit area.</td>
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<tr>
<td>15. The overall quality of teaching was good</td>
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<tr>
<td>16. The physical facilities (rooms, labs, equipment) were adequate for the unit</td>
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<tr>
<td>17. This unit was challenging and at an appropriate level.</td>
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<tr>
<td>18. I have developed my co-operative learning skills during this unit.</td>
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</tr>
<tr>
<td>19. All students (male &amp; female) were provided with an equal opportunity to participate in all activities.</td>
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<tr>
<td>20. I would recommend this unit to other students.</td>
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</tbody>
</table>
Student Unit Evaluation – Notes

The Student Unit Evaluation seeks to determine how students perceive the quality of a Unit through various indicators, objectives, texts, facilities, assignments, and teaching. It is important to note that there is a difference between Unit quality and students’ perceptions of Unit quality. What is being determined here are only students’ perceptions. Feedback from students is only one pointer which when linked to other forms of review such as lecturer peer review and self-assessment of a Unit, can provide the basis for improving student learning in a Unit.

From a completed Unit evaluation it is possible to compare the different indicators by calculating a Mean score for each of them. Each Indicator Mean score is calculated by multiplying the number of students responding for each preference, by the preference value. The preference values are 5 for Strongly Agree; 4 for Agree; 3 for Not Sure; 2 for Disagree; and 1 for Strongly Disagree.

Example of the calculation of Indicator Mean scores

Number of students: 22

The objectives of the unit were clearly outlined

<table>
<thead>
<tr>
<th>Indicators</th>
<th>Strongly Agree (5)</th>
<th>Agree (4)</th>
<th>Not sure (3)</th>
<th>Disagree (2)</th>
<th>Strongly Disagree (1)</th>
<th>Total</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student responses</td>
<td>4</td>
<td>9</td>
<td>3</td>
<td>5</td>
<td>1</td>
<td>76</td>
<td>3.45</td>
</tr>
</tbody>
</table>

The unit content was clearly related to the objectives

<table>
<thead>
<tr>
<th>Indicators</th>
<th>Strongly Agree (5)</th>
<th>Agree (4)</th>
<th>Not sure (3)</th>
<th>Disagree (2)</th>
<th>Strongly Disagree (1)</th>
<th>Total</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student responses</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>8</td>
<td>5</td>
<td>56</td>
<td>2.54</td>
</tr>
</tbody>
</table>

Making sense of the results

You can see from these examples what students’ general perceptions are. While they thought that the objectives of the Unit were clear, they did not think the content of the Unit was well related to its objectives. A lecturer may have to decide whether stronger links between objectives and content are necessary, whether the objectives should be redefined, or whether students should be made more aware of the links that do exist between objectives and content. To make an informed decision about the Unit, a lecturer would probably need to compare this information with the rest of the evaluation, take note of students’ comments at the end of the evaluation, do a self-assessment of the Unit and go through the Unit with other lecturers seeking their opinions (Peer Review).

As a guide, Units probably need fine-tuning where a particular Indicator Mean score is much lower than those of other Indicator Mean scores in that Unit; or any Indicator Mean score is below 3.0. It is possible to calculate an overall Unit Mean by adding all the Indicator Mean scores and dividing by the number of Indicators (20). The overall Unit Mean score can be compared with other Unit Mean scores in the Strand or across Strands to give a picture of which Units students perceive as being of higher or lower quality.
Resources

In addition to the material provided in this Lecturer’s Guide the following items are recommended to support the teaching of this unit:

- Butchers paper and marking pens for group activities
- Playing cards
- Variety of blocks, shells, sticks, etc for problem solving activities
- Dice
- Counters
- Grid paper

References


Marr, Beth, and Helme, Sue (1991). Breaking the Maths Barrier, Department of Employment and Training: Canberra, Australia

Sample Assessment Tasks

Written report

- Write a report which sets out the solution to an investigation and outlines the processes you worked through.
- Write your solution to a non-routine problem and describe the process you worked through.

Oral presentation

- Present a seminar sharing a solution to a problem (individual or group).
- Present a seminar sharing an investigation carried out (individual or group).
- Give an oral presentation on the effectiveness of problem solving as a strategy for developing mathematical understanding.

Journal writing activities

- Write a journal discussing the use of non-routine problems in the primary school.
- Write a journal which reflects on the value of working co-operatively to solve problems.
- Write a journal outlining how problem posing supports the development of mathematical understandings.
- Write a journal discussing how you could implement problem posing in the classroom.

Solve non-routine problems (co-operatively or individually)

- Solve a Non-routine problem. This can be assessed using the following headings and marking scale

  **Understanding the problem**
  0  completely misinterprets the problem
  ↓
  4  complete understanding of the problem

  **Devise a plan**
  0  no attempt or a totally inappropriate strategy chosen
  ↓
  6  the strategy chosen could lead to a correct solution if used correctly
Carry out the plan
- 0 no answer or answer obtained using an inappropriate strategy
- 6 correct solution

Reflect on the result
- 0 no consideration of the reasonableness of the solution
- 4 reasonableness or otherwise of the solution correctly discussed

Activities and investigations
- Complete an investigation (individually or co-operatively).
- Choose a real life problem and work collaboratively to solve it. Write a report and present your findings through a seminar presentation.
- Carry out a group investigation into the probability of a particular event occurring.

Classroom related tasks
- Pose a series of problems suitable for use in the primary school. Identify the grade, the mathematical concepts being developed through solving these problems, the types of strategies you would envisage children using and discuss how you would teach this in the school.
- Develop a range of problems you could use to teach a particular concept in the primary school.
- Develop a series of investigations which you could use with primary school children.
- Establish a portfolio of strategy games suitable for use in the primary school mathematics classroom.

Exam
- Conduct and exam which contains short answer questions and problems to be solved.
Background Readings

The following readings have been included to provide lecturers with some background information to assist in the teaching of this unit.


Problem Solving

(Adapted from ‘Problem Solving’ in Breaking the Maths Barrier, by Marr, Beth and Helme, Sue. (1991) Department of Employment and Training, Canberra, Australia)

What is problem solving?
Problem solving is the application of mathematical skills and reasoning to problems encountered in everyday life. Such problems range from the simple to the complex, for example:

• working out how much medicine to give a child and measuring it correctly;
• using or interpreting statistics at work to predict demand for goods so that an order can be placed with a wholesaler.

Such real-life problems never appear as a neatly ordered and graded page of straightforward ‘sums’. To solve real-life problems we need to be able to draw upon our mathematical skills and use them to reach a solution which is reasonable and appropriate for the particular situation.

However, many basic maths students lack confidence with anything more complex than a worksheet full of sums which reinforce a single skill or concept, and are afraid to take risks and experiment with different approaches to solving an unfamiliar problem.

Many students have difficulty in deciding for themselves which operation or procedure is needed to solve a problem. For example a problem such as How many 43t packets of Twisties can I buy with K5.00? is difficult if students are unable to decide for themselves how to go about it.

Fear of making a mistake, low self-confidence and little knowledge of the problem solving process all contribute to students’ difficulties in applying their mathematical skills and knowledge to non-standard or real-life problems.

The dangers of teaching mathematical procedures at the expense of mathematical reasoning are now well documented. For example, a national U.S. survey (Burns, 1990) found that about 80% of 17 year olds could calculate the answer to $3.04 \times 5.3 = ?$ (answer 16.112). However, less than 40% of students could select the correct alternative to the same question when asked to select the most appropriate approximation from the following alternatives:

a) 1.6
b) 16
c) 160
d) 1600
e) I don't know

This question examined students' abilities to reason that if you multiplied a bit more than 3 by a bit more that 5 the answer would be about 16. That is the only answer from the choices that makes sense, yet only one in three students successfully solved the problem.
Problem solving activities are one way of bridging the gap between the numeracy class and the real world, so that students develop skills and strategies for solving a wide range of different types of problem.

What is a problem solving activity?

In the context of adult numeracy, a problem is a question or activity which is challenging to the student. Some students may be challenged by calculating change from shopping, whereas others would need a much more complex problem if they are to be challenged in any significant way. A question becomes a problem when the answer is not immediately obvious to the student; it requires more than a guess or a simple calculation to reach a solution. There are a number of other criteria which help define a problem solving activity.

- The method of solution is not immediately obvious and students need to decide for themselves how to solve it. This may be a simple decision about whether to divide or multiply or a systematic application of a number of problem solving strategies.
- There are a number of different approaches, or strategies, which lead to a solution.
- There may be more than one correct or reasonable solution. For example, a question such as how many ways can you make K1 using 5t, 10t, 20t and 50t coins? Or asking students to brainstorm all the words that describe a shape. (This encourages creativity and divergent thinking).
- There is more than one step to a solution. For example, 'my age this year is a multiple of 7. Next year it will be a multiple of 5. I am not yet 50. How old am I?' In order to solve the problem the student has to work out the multiples of 7, then the multiples of 5 and finally match them to reach the solution of 14 or 49.

How do students benefit from problem solving?

Problem solving activities in a numeracy class benefit students in two ways. First students learn about problem solving. Problem solving activities introduce them to the problem solving process itself; the range of strategies or approaches that can be used to solve an unfamiliar problem. For example, to find all the numbers you can make with the digits 1, 2 and 3, requires students to be thorough and systematic in their approach. This is an important problem solving skill.

Second, problem solving activities can be used to introduce new concepts or reinforce existing ones. Take for example, the following problem: If A = 1, B=2, C=3 etc. how many points is your name worth? This problem involves matching letters with numbers and gives students practice in addition. To solve a challenging extension of this problem: can you find a word worth 50 or 100 points? Students need to try out different words until they are successful. Thus students are learning the age-old strategy of ‘guess and check and of course, persistence. Meanwhile students also get lots of practice in addition!

New concepts can be introduced to students using problem solving activities. For further discussion of the opportunities problem solving activities provide for exploring underlying mathematical concepts see Jeannette Thireing's (1989) article in Good Practice in Adult Literacy.
How to begin with problem solving

The best foundation for success with problem solving is a positive and supportive learning environment where students feel comfortable about taking risks.

Teachers should acknowledge students' existing problem solving skills, perhaps by reminding them that everyday life is a constant process of problem solving. Take for instance the following everyday situations:

- Have I got enough money to catch the bus and buy lunch?
- How much meat should I buy for dinner tonight with two extra guests?
- What time should I leave home to get to work on time if I have to pick up a newspaper on the way and drop my child off at school?

All these involve mathematical skills such as addition, subtraction and estimation, and problem solving strategies such as planning and carrying out the plan in a systematic way. Activities done at home such as fixing the lawnmower or making a cake need to be done in steps or stages (and in the right order!) A step-by-step approach is also a vital skill for solving more complex mathematical problems.

To encourage students with problem solving the teacher needs to model the problem solving process, by exploring and discussing different approaches, including ones that may not lead to a solution. This is important because many students are afraid of making a mistake, and believing that maths problems have instant solutions, and therefore give up very quickly. If, however, they see the teacher trying different approaches and finally reaching a solution they will be more likely to 'have a go' themselves.

Many students are still firm believers in the myth that there is only one 'proper' way to solve a problem. For example, even an apparently simple and straightforward problem such as 'Can you make 30 toes using exactly 3 coins?' can be done using the coins themselves, diagrams of coins or written numbers. Each approach is valid. This problem also has more than one correct solution which helps break down the myth that maths problems only ever have one answer.

Finally, and most importantly, problem solving is best done in pairs or small groups. Working together on problems encourages students to verbalise what they are thinking and doing; to discuss, explain and perhaps defend their method of solution. This verbal interaction of course enhances learning.

The fundamentals of problem solving

The process of problem solving can be divided into three phases:

- understanding the problem
- solving the problem
- checking the answer

Understanding the problem

Students need to start by reading (or listening to) the problem very carefully to make sure they understand what they are being asked to do. This may involve:
• several readings (or repetitions by a speaker)
• thinking about the problem for themselves for a few minutes
• discussing it with others in a small group and reaching agreement on what it is before they start
• writing the problem down using their own words, symbols or diagrams
• restating the problem verbally using their own words.

Once students have a clear idea of what they know and what they need to find out they are ready to begin solving the problem.

**Solving the problem**

Solving a problem involves exploring different approaches and finding one which successfully solves the problem. Strategies to focus on with numeracy students are listed below. These are summarised in the handout ‘A Problem Solving Guide’ which follows.

1. **Use hands-on materials**
   Manipulative aids encourage risk taking and therefore are a very valuable and important tool in problem solving. Pieces of paper or objects such as counters or matchsticks which can be moved around encourage students to try out different combinations.

2. **Draw a diagram**
   A diagram or sketch can help students visualise and organise the information in the problem and also provide a record of their solution process. For example, to solve money problems students may find it helpful to draw the coins themselves and label them.

3. **Guess and check**
   This strategy involves making a guess and seeing where it leads. For example, to solve the problem: *Can you make seven toea using exactly three coins?* students need to start by exploring different combinations of coins until they find one that works.

4. **Organise the information**
   When exploring a problem, students need to work systematically. Lists, charts and tables are a powerful way of organizing seemingly confusing information. They encourage students to be systematic and to keep track of important information. For example, the problem *what day will it be the day after tomorrow if four days ago was Friday?* is best solved if students start by making a list of the days of the week.

5. **Look for patterns**
   Awareness of patterns and relationships is fundamental to learning and understanding mathematics, as well as a valuable problem solving strategy. Take, for instance, the nine times table:
The patterns and relationships include:

- The sum of the digits in the answer is 9
- There is a descending and ascending pattern of numbers in the answers
- The initial digit in the answer is one less than the number of 9's (up to 10 times 9)

Awareness of these patterns simplifies the task of learning and remembering. For example in order to solve number patterns and magic squares problems, students need to look for such patterns and relationships.

6. Break the problem into parts

To solve many problems, more than one step is required. This involves breaking the problem into parts and solving each part in turn.

For example, solving magic squares problems requires deciding first what the numbers add up to and then filling in the rest of the squares according to the pattern.

7. Make it simpler

For students, making it simpler usually means replacing the hard numbers in the problem with easier ones to make sure they are on the right track. For example, to solve a problem such as:

A semester's school fees are K635.50 if you pay at the beginning of the semester or K37 per fortnight if you pay it each fortnight. Which is the cheapest way of paying the school fees?

This problem involves deciding which operation to use to work out a solution. Rounding off the numbers first can help students get a rough answer which will reassure them that they have used an appropriate method. For example, rounding the K635.50 to K640 and either dividing or multiplying by 20 (the approximate number of fortnights in a semester) enables students to see that dividing by 20 gives a more likely answer (K32.00) than multiplying by 20 (K12800.00). Once students are confident with their method they can go back and complete the problem using the exact figures and probably a calculator. Working the other way, rounding the K37 to K40.00 and multiplying by 20 (rather than dividing) gives an approximate answer of K800.00. Estimation is a fundamental numeracy skill and an invaluable aid to problem solving. Estimation prior to solving the problem gives an indication of the size of the answer and helps the student decide whether their final answer is reasonable. This is essential when using a calculator when wrong keys are often pressed by mistake.
Checking the answer

Checking the answer is an integral part of problem solving; a problem is not really solved unless the solution has been checked or verified. Returning to the examples of the lawnmower and the cake, the final test is whether the lawnmower starts and the cake is moist or rises properly. Students who can check their own work are also becoming independent and autonomous learners.

Students should start by re-reading the problem to make sure they answered the actual question asked. For example, a solution to 'How many ways can you find to make one kina using other coins?' should include a statement about how many different ways were actually found, and not simply a record of one or two possibilities. For open-ended questions like this one, students also need to think about whether they have included all possibilities, and how to tell whether or not they have found them all.

Calculations also need to be checked. 'Does my answer make sense?' is a question all students should ask themselves, and have the skills to answer. Students should be encouraged to solve the problems using another method in order to check their answer.

The following problem illustrates the importance of producing a sensible answer.

You are responsible for organising PMV’s to transport a group of students to a big sporting event. A PMV seats 14 and you have 119 students to transport. How many PMV’s will you need to organise?

Producing a reasonable answer to this question needs an appreciation of the situation. If a student uses a calculator to divide 119 by 14 the answer they will get is 8.5 which would not make sense. Students with an appreciation of the situation would realise that they need to organise 9 buses.

The information in this section is presented in summary form in the following handout, ‘A Problem Solving Guide’.

References


Problem Solving Guide

1. **Understand the problem**
   Make sure you understand the problem and what you are being asked to do. The following strategies may help.
   - Read (or listen to) the problem carefully, several times if necessary
   - Think about it for yourself for a few minutes
   - Discuss the problem with others and reach agreement on what to do
   - Write the problem down (or say it aloud) using your own words

2. **Solve the problem**
   Explore ways of solving the problem until you find an approach which works. Strategies to use include:
   - Use hands-on materials
   - Draw a diagram
   - Guess and check
   - Organise the information (e.g. lists, charts or tables)
   - Look for patterns
   - Break the problem into parts
   - Make it simpler (e.g. by estimating)

3. **Check the answer**
   - A problem is not fully solved until the solution is checked
   - Re-read the problem
   - Make sure you have answered the question asked
   - Check calculations
   - Check solutions using another method
   - Check that your answer makes sense
Implementing a Problem Solving Program

(Adapted from 'Implementing a Problem Solving Program' in Problem Solving Tips for Teachers, by O'Daffer, Phares G. (1996), National Council of Teachers of Mathematics, Reston, Virginia.)

Teaching problem solving can be a comfortable, interesting, and enjoyable experience for teachers if they have good problems, have a commitment to using them, and have effective techniques for doing the following:

- Using co-operative learning groups
- Developing a positive classroom atmosphere
- Teaching problem-solving strategies and skills
- Guiding students in a problem-solving situation
- Working with students with special needs
- Evaluating students' problem-solving abilities

The articles in this section give some ideas that will help teachers’ plan and carry out a classroom problem-solving program.

"Organising for Problem Solving," gives ideas for using co-operative learning groups and choosing problems amenable to group problem solving.

"Teaching the Basics through Problem Solving," shows how problem solving can be used in the crowded curriculum to help students develop basic skills.

"Helping Students Approach Problems," and "Hands-on Thinking Activities for Young Children," give ideas for helping students become independent problem solvers.

"Problem Extensions for Gifted Problem Solvers," presents ways to develop and emphasizes the importance of having slow or handicapped learners use countable objects to help choose the right operation for solving a simple problem.

"Asking Questions to Evaluate Problem Solving" focuses on the role of a unique type of questioning designed to help evaluate both students' attitudes and their problem-solving abilities.

It is hoped that the tips in this section will help you broaden your classroom problem-solving perspective and your approach to teaching problem solving. Remember that although right answers are important, students' growth in using an effective problem-solving process should be given highest priority.
1: Organising the classroom for problem solving

A growing body of research points to the benefits of having students learn in small cooperative groups. When students work in cooperative groups, the active participation of each student is maximized. More students have the chance to speak than in whole class discussions, resulting in more opportunities for students to clarify their thinking. Also, many students feel more comfortable in small-group settings and are therefore more willing to explain their ideas, speculate, question, and respond to the ideas of others. In small cooperative groups, students’ opportunities to learn with understanding are supported and enhanced.

Choosing problems suitable for cooperative groups

- Choose problems for which a collaborative effort will benefit students, both in sharing ideas, and in accomplishing the task.
- Select problems that allow for different approaches.
- Be sure students understand both the problem and how they are to present their results.
- When appropriate, have students post their findings for the class.
- Allow for discussion time to summarize, during which groups present their findings and respond to other groups’ results.

An early grade example - "making change"

Young children need opportunities for continued practice both with basic facts and with money. "Making change" is a problem-solving activity that offers both kinds of practice and also lends itself to cooperative group work.

Each group of students works to find all the ways they could be given K0.50. For example, they could receive two 20 toea and a ten toea, or one 20 toea and six 5 toea, and so on. They make a group record of their findings to present to the class. During a follow-up discussion, they also need to tell why they think they’ve found all the ways.

An example for older children - "number bracelets"

In this activity, students look for patterns while practicing basic addition facts. Several benefits emerge as students tackle this problem in groups. They share the task, thus making the work manageable, and they are able to check their conjectures and evidence with each other.

Students start by writing any two numbers from 0 to 9 and applying the following procedure: Add the two numbers and record just the digit that appears in the ones place in the sum. For example, if you start with "8 and 9," then the number that comes next is "7." Then add the last two numbers, the "9" and the "7," and record "6." Continue in this way, "8 9 7 6 3 9 2 ..." until the pattern begins to repeat.
Each group of students is asked to answer the following questions:

- How many different possible pairs of numbers can you use to start?
- What is the shortest bracelet you can find?
- What is the longest bracelet?
- Investigate the odd-even patterns in all your bracelets.
- Make up one more question and try to answer it.

Tips

Guidelines for cooperative group members

Three rules are useful when students work in cooperative groups. The rules need to be explained to the students and discussed, as they are only as useful as they are understood and practiced.

- You are responsible for your own work and behaviour.
- You must be willing to help any group member who asks.
- You may ask the teacher for help only when everyone in your group has the same question.

The third rule often puts the greatest demand on teachers when first implementing cooperative groups. Children typically ask for individual help. Asking children to check with their group, rather than giving them help at that time, is not a usual teacher response. However, it is an invaluable response for encouraging students to become more independent and to rely on each other. Assure students that you will come and discuss whatever problem the entire group faces.

How to group students

It is important for students to be willing to work and learn with all their classmates. Grouping students randomly accomplishes this objective.

- Students can move their desks into clusters of four each. The teacher labels each cluster with the number of a playing card—ace, two, three and so on. The corresponding cards are shuffled and distributed. Children who hold aces go to the ace cluster, children with twos to the cluster labelled two, and so forth.
- Numbered slips of paper can be drawn from a 'hat' to determine random groupings of either three or four.

How large should groups be?

A co-operative group requires no magic number of children to work. At some times, students work best in pairs, although groups of three to six students are successful in other situations. What is important is that groups are small enough for all students to participate.

A caution

Seating students in small groups does not magically produce instantly successful co-operative group work. Practice, encouragement, and discussion are required, but it is well worth the effort.
2: Teaching the basics through problem solving

Too often, teaching problem solving and teaching basic skills are seen as separate emphases, each requiring its own time allotment in a crowded curriculum. However, double mileage is possible when problem solving is used as the vehicle for helping students develop an understanding of the basics.

Guidelines for lessons that integrate problem solving into other areas

- Present a problem that is interesting and requires that students think and reason
- Permit students the opportunity to develop and extend their understanding of basic mathematics concepts and skills.
- Give frequent opportunities for students to express their thinking in words, both to the teacher and to other students. This verbalization helps them formulate their own thoughts, clarify their ideas, get reactions from others, and hear others' points of view.
- Use concrete materials whenever appropriate to help make mathematics concepts real and meaningful.

An early grade example

Young children need many experiences with numbers less than ten to help them see relationships and learn the basic addition facts. Using interlocking cubes of only two colours, children build all the different colour arrangements of trains that are five cubes long. Teachers will see and learn a good deal about their students thinking abilities from watching how they approach the problem of finding different arrangements. (Though finding all the possible trains may be beyond the ability of young children, it is a challenging problem for older children, and even adults.)

Children record each train pictorially by colouring on squared paper and symbolically by writing an arithmetic sentence to match. Thus, a concrete experience is preserved in a visual record and is connected to appropriate symbolism. See figure 1.

Having children report their findings, either by sharing in a whole-class discussion or by having small groups of children compare their individual results, can help stimulate them to investigate the problem further. Though the goal in this lesson is to explore the number five, the problem is appropriate for other numbers as well.

An example for older students

For many students, teaching about fractions focuses too early on teaching the rules and procedures for working with fractional symbols. In contrast, problem solving lessons put the emphasis on developing students' understanding of
fraction concepts and relating these to appropriate symbolism.

For example, the following is a problem for students to work on in small groups:

Three people are to share two small pepperoni pizzas equally. Show how much one person gets and describe it in writing.

Figure 2 shows solutions from four groups of students. Though they differ, each was verified to be correct when the students explained their thinking. From group and class discussion of solutions and ideas, children re-examine their ideas and deepen their understanding.

![Figure 2: Student sketches and descriptions of the pepperoni pizza problem](image)

Each person gets 5 pieces and $\frac{1}{5}$.

One person gets $\frac{2}{6}$.

$\frac{2}{3}$ of one pizza

that allow this experience. A three-step format is suggested for these activities:

- Students make a prediction about the outcome.
- They use objects to collect the information.
- They count, grouping the objects into tens and ones and, if required, hundreds.

Example: "How many buttons do we have on our clothes today altogether?" Discuss individual estimates. Children then count their own buttons and set out that number of small counters. In small groups, they organise their counters into piles of tens and ones. (Small cups are useful for collecting tens.) Collect the tens and ones so that extra ones can be combined to make more tens, and 10 tens can be made into a group of a hundred. The total is recorded to reveal the actual count and compared with estimates.

☑ Other estimation activities

- The number of cubes to cover a book or desktop.
- The number of linked paper clips to frame a piece of paper.
- The number of brothers and sisters for all class members.
- The number of beans to fill a small jar.
- Children can help think of other things to estimate and count.

☑ Place Value through Estimation Problems.

Second and third graders benefit from many experiences in which they use tens and ones to organise large numbers of objects. Estimation activities offer the opportunity for problem-solving lessons
3: Helping students approach a problem

What do your students do if they don't know what to do, or where to start? Do they wait for someone, namely, the teacher, to tell them what to do? What do you do to help them? Rereading the problem and having them describe the situation in their own words are two strategies that may be helpful but are often not sufficient.

Other helpful ideas include identifying the known, the unknown, extra information, the relationship between the known and unknown, and what constitutes a reasonable result regarding number size and labels. These ideas are general strategies. Research indicates that questions or hints tailored specifically to the problem are more helpful to the students than asking general questions. Examples of explicit questions are given here, in contrast to general questions such as "What is known?" These questions and hints should serve as a model for the type of questioning and thinking that students should eventually do independently when they approach problems.

Example: What are two numbers whose product is 18 and whose difference is 7?

Questions: How many numbers are we looking for? What do we know about the numbers? ... You said the product must be 18. What is meant by product? ... Give an example of numbers with a product of 18. ... Julie, what is your response to John's answer? ... Could our answer be 3, 5, and 7? ... Why not? Could our answer be 6 and 3? ... Tom, do you agree with that statement? ... Could our answer be 10 and 3? ... Why not?

Note that the preceding teacher questions hold students accountable by asking them explicit questions to check for understanding. Also, throughout the discussion, class members rather than the teacher should be involved in evaluating each response to determine if it fits the problem and their thinking. Have some explicit questions planned for each problem. These questions are especially necessary if you or your students are new at problem solving.

The goal of problem solving is to help students develop strategies so that they will become independent problem solvers. Gradually students should become independent and devise questions on their own.

Another step toward helping students become independent thinkers would be introducing a problem and having the students generate questions to help them understand the problem. These questions could be listed on the chalkboard and evaluated by the class. The focus should be on understanding the problem, not devising a plan or solving the problem. To emphasize this point, the students could solve the problems later in smaller groups, as part of a homework assignment, or not solve them at all.

Tips

Drawing diagrams and acting it out

Two strategies that can be helpful for understanding a problem, as well as devising a plan, are drawing diagrams and acting it out. These strategies create student ownership of the problem by...
having students’ talk through the situation without using numbers.

Example: Jessica has twenty four-sided decorative tiles that she can use to design a square pattern. The design will be framed with trim. How much trim does she need? Each tile measures ten centimetres on an edge.

**Questions:** Imagine that you are Jessica and you are going to plan this project today. How would you organise your work? ... Describe what you plan to do first with the materials. What is Jessica making? ... How are the tiles to be arranged? ... What patterns are possible? ... You drew a picture of a four-by-five arrangement of tiles... Will she use all the tiles? ... How large will the design be? ... You drew a picture of a square and labelled it "4 x 4." What does the '4' represent? If the problem asks for how much trim, why are you concerned with the tiles? ... What is the shape of an individual tile? ... Does the trim go around each tile? Would it help to find the perimeter of each tile? The area?

**Part-whole**

Part-whole analyses are helpful in devising plans and understanding the problem in proportions and percents, as well as in addition and subtraction. Part-whole analysis is a strategy for solving percent problems, but it is crucial that students can understand and analyse the situation in these terms.

Problem: Todd had budgeted K45.00 for a new coat. Did he have enough to buy the one on sale for K59.99 with 20% off?

**Questions:** Describe the series of decisions that Todd will need to make.... In percent problems we've used part-whole strategies. Describe this problem in terms of part and whole.... Why are you considering K60.00 the whole? ... What percent is K60.00? You said 20 percent was the part. Will the coat cost K20.00? ... If 20 percent is the discount, will we subtract 20? ... What would be the whole if we could subtract 20? ... Since our whole is K60.00, will we subtract more or less than K20.00? ... Estimate about how much will be subtracted.

**Using Easier Numbers**

National assessment results have documented students' lack of understanding of the concepts of common and decimal fractions and good student performance on one-step word problems involving whole numbers. In each of the next three problems simply choose the correct operations. Do not solve!

Susan was cooking for a large group. After feeding 17/23 of the group she finished cleaning out a container that held 3 litres. How much should she have ordered to feed the entire group?

Scott looked at the work that was left. He had finished 3/11 of it by Wednesday and had assumed it would take 13 and 5/7 days. How many days has he finished on the project?

Sarah had 3.2 kilograms of hamburger that she placed into packages of 0.2 kilograms each. How many containers did she fill?

In a class of preservice primary school teachers all students missed at least one question and some missed all three. Through class discussion the students were introduced to the strategy of substituting whole numbers less than 20 for the fractions or using more ordinary fractions, such as 1/2. In re-testing, all but a few students were able to answer similar questions correctly. The students' lack of understanding of fractions resulted in not understanding a situation that they could
normally solve. These students are not untypical. How would your students do? Have they been introduced to the strategy of using simpler numbers to help understanding?
4: Hands-on thinking activities for young children

Children in kindergarten and primary grades can engage in significant problem solving through a wide variety of activities involving physical materials. The use of materials in problem-solving contexts permits children to experiment with different situations and relations, to create shapes, and to make measurements, all without symbolic or written materials. A variety of examples are suggested here just to whet the appetite.

Logic with attribute pieces

Attribute blocks with different shapes, colours, and sizes are popular and useful. A nice set of blocks with four attributes can be made from poster board. Draw two sets of large shapes and two sets of small shapes on each of three colours of poster board. After cutting these shapes out, draw a happy face on both sides on one set and a sad face on the other set. If four shapes and three colours are used, a total of forty-eight pieces will be in the set. Here are just a few activities to get you started.

How is yours different?

Let each child select a piece. The teacher holds up a piece and asks different children to explain how their piece is alike or different. "Does anyone have a piece that is like (or different from) mine in just one way?" (For example, does someone have a different shape with the same attribute of face, colour, and size?)

One doesn't belong

Select four pieces, and arrange them as shown in figure 2. Play the "Sesame Street" game of "One of these things is not like the other." Ask children to decide which piece is different and why. Frequently, more than one solution will be correct, as in the current example.

Secret sorting rules

Decide on a sorting rule such as "red or not red", but do not tell the class. Begin to sort the pieces into two piles. The children try to decide what the rule is. When identifying this rule becomes easy, choose compound rules, such as large and smiley or "red or triangle."

Patterns

Select one attribute and begin a simple repeat pattern. Let children try to find a piece that could go next in the pattern. Encourage children to explain their choices. Praise good ideas even when they are not the ones you had in mind. The pattern shown in figure 3 is an ABB pattern using shape.

Geoboard problems

How many ways?

Many good problems on the geoboard stem from questions such as "How many
ways .. ? A few are suggested here, but you will enjoy making up many more.

How many ____________ can you find?

- shapes with five sides
- shapes with four corners but no nails inside
- shapes that touch only six nails
- shapes that you can put this piece inside (small cardboard cut outs that cover three or four squares of the board)

Very young children may feel more comfortable with searching for three or four shapes instead of trying to find many.

**Mirror images**

Symmetry is a good concept for offering a challenge and promoting thinking. Stretch a band making a straight line down the centre of the board or from corner to corner. Have children make a simple shape on one side of the line so that it touches or joins the line (see fig. 4). Now challenge them to make the mirror image of their shape on the other side of the line. A mirror can be used to check the result.

**Halves**

Make a large rectangle on the geoboard and challenge your students to find four or five ways to divide it into two equal parts with rubber bands (see fig. 5). The actual number of possibilities will depend on the size of the rectangle.

**Measurement**

**Length**

Give groups of two or three children two paths between two points as shown in figure 6. These could be drawn on poster board, made with tape on the floor, or drawn with chalk on the playground. Different pairs of paths can be given to different groups. The problem-solving and measurement task is to decide which path is longer. You can give more or less help in arriving at a solution, depending on the age of the children. The possibilities include laying units such as paper clips or drinking straws along each path and comparing. A string or rope could be placed along each path and compared directly. A single unit of length could be moved end over end to measure each path. Or perhaps standard units such as meters or centimetres could be used. Each group may invent its own methods and then discuss and compare how different groups made decisions.

**Area**

A similar, but generally a bit more difficult, task involves deciding which of two regions is larger. Again, these regions could be drawn on poster board, or larger regions could be drawn on the floor or playground with tape. More assistance will be needed to help children understand that size or area means how much surface is inside the region and not how tall or wide it is. The problem involves finding ways to measure that surface.
5: Determining the correct operation with the slow or handicapped learner

An extension of patterning underlies a student's ability to decide which of the four basic operations apply in solving a simple oral or written word problem. Many learning handicapped and slower students cannot "see" what is set forth by the words of a problem - that a situation really fits the pattern of addition, or division.

**Problem:**

Roy has 26 biscuits. He wants to give them to his 3 friends - the same number to each. How many biscuits will each friend get? How many extras will be left?

Miscued by the phrase "how many will be left," a typical response of slow or handicapped students is to subtract. An approach to building students' success in problem-solving situations like this is outlined in the following small steps:

- Use countable objects (or another appropriate model) to analyse and discuss the word problem. Use vocabulary that emphasizes the basic concept of the operation. (In this problem, we share among, or divide the biscuits.) The students themselves should move or group the objects. Encourage them to verbalize what they are doing: "Sharing 26 among 3."

- Select the best arithmetic expression from a presented list (e.g. 26 - 3, 26 ÷ 3, 26 + 3) to represent the action in the problem. Use objects to verify that rejected expressions "don't work." When possible, help students "see" how the chosen expression simulates the action in the problem:

- Reverse the foregoing procedures. (This is a very important step!) Direct students to read the problem, write from a presented list the arithmetic expression they think "fits" the problem, and then use an appropriate model to check their expression.

- Have students change the numbers in given, similar problems to form new problems. Then ask them independently to write an expression they think "fits" the action of the problem and use an appropriate model to check the expression.

- Have students identify which problems among those given involve the specified operation (e.g. division) and use an appropriate model to check their thinking.

The basic thrust of the instructional sequence just outlined is to highlight the operation embedded in the basic structure or action of a problem. Parallel examples are then provided so that basic similarities between problems begin to emerge. Identifying just those problems that involve a given operation is a final step. Note that hands-on involvement by the child and modelling of appropriate language by the teacher are integral to the interaction between teacher and student.

This sequence has been extended successfully and applied to multistep problems.
Many handicapped and slower students have reading difficulties that interfere with success in problem solving. To help:

- Rely more on small, co-operative groups
- Create a "buddy" system. Ask a student to be Johnny or Mary's buddy for a while. This buddy can read and interpret difficult words. Allow the system to function even during tests. If the child with the learning difficulty becomes too dependent on the helper or is too easily drawn off task, the buddy assignment can be changed without issue.

**Sponge activities**

Routinely provide occasions that force students to think about the basic structure of a problem situation. Fill otherwise wasted gaps during transition periods with short sponge activities that invite on task behaviour from even the slowest, most affected learner.

- Put thumbs up if you think I should divide to solve the problem. Otherwise, put thumbs down.
- Use fingers, chest high, to form +, -, x, ÷ to show me what you think I should do first to solve this problem.

Alternatively:

- Create bulletin-board displays or furnish individual cards and invite Students to sort problems according to the arithmetic operation needed to solve them. Make an answer key available for self-checking.
- Structure assignments specifically to include sorting by problem type. For example, "just do the division word problems on this page." Before students begin work, call on individuals to tell, in their own words, how they sort out division problems as opposed to other types of problems.

**Look ahead**

Provide activities that build prerequisites to small successes in problem solving:

- Include a basic-fact review as part of a daily assignment. Present the review in a way that forces decision making, which is the heart of problem solving. For example, "Find and do five problems that have an answer of 7."
- Once every two weeks observe slower or handicapped learners as each use countable objects, base- ten blocks, or other appropriate materials to check a written answer. Interact to help students use appropriate language to describe what they are doing, ("Start with 35; share among 4", or "517 is the whole. We'll take away part, the 231..."
- Provide numeric problems with answers, but without signs. Help students develop reasoning skills by asking them to decide which sign "fits" and to tell why it does. As mentioned previously, emphasise appropriate language. From time to time have students use appropriate models to check their thinking
  
  \[
  \begin{align*}
  8 \div 2 &= 6 \\
  8 \div 2 &= 4 \\
  42 &= 38 \\
  1586 &= \\
  4387 &=
  \end{align*}
  \]
Problem corner: special help

Many students with learning difficulties "go blank" when they see problems with troublesome fractions, decimals, or larger whole numbers. Teach these students to help themselves. Help them learn to substitute small whole numbers for difficult numbers until they decide what operations to carry out.
6: Problem extensions for gifted problem solvers

Teachers often search for ways to differentiate learning experiences for their students with high abilities. Problem solving with appropriate extensions offers a very viable tool for meeting needs at various ability levels. With practice, creating extensions becomes easy and interesting. In fact, students will soon help by creating their own extensions through brainstorming in cooperative learning groups.

Extensions seem to fall into four major categories: (1) reaching for a pattern or generalization, (2) introducing new concepts or vocabulary from other areas of mathematics, (3) extending divergent thinking (creativity), and (4) initiating discussions that present opportunities for value judgments. The grade level suggestions for using the following problems should be interpreted with flexibility. Wide variations of interest, ability, and experience occur at any grade level.

Pattern and generalization

Problem: If you use pattern blocks, how many triangles are required to make a larger, similar triangle (fig 1)?

Extensions
(K - 2) Can you make an even larger triangle? If nine triangles were in the bottom row, how many triangles would you have used? (81)

(2-4) How many smaller squares are needed to make a larger square? If twelve were in the bottom how many, squares would you need? (144) How many hexagons are required to make a larger hexagon? (Can you try to explain why it cannot be done?)

(4-8) If you use only two colours (shapes) of blocks, can you make a hexagon? A regular hexagon? What combinations of blocks will work? How many of each shape will you need? Can you make a hexagon with three shapes? Four? Five?

New concepts and vocabulary

Problem: If a book that has 167 pages is open and the sum of the page numbers is 137, what pages are showing? (68, 69)

Extensions
(1-3) If page 1 is on the right side of the book and page 2 is on the left, will page 39 be on the right, or the left? (right) Will page 86 be on the right or the, left? (left) How many sheets of paper, are in the book? (84)

(4-8) If the product of the pages that are showing is 16 002, what are the page numbers? (126, 127)

What is the probability that when you open the book, one of the page numbers is a multiple of three? (55/84) Or that on
exactly one of the pages the sum of the
digits is greater than ten? (7/84) Or that the
tens digit of the page number on the right is
greater than the ones digit? (29/79; 79 right-hand pages have a tens digit.)

Creativity

Problem:
A farmer has a hen, a cat, and a bag of seed to carry across the river. His boat can
only carry him and one other item. He cannot leave the cat alone with the hen or
the hen alone with the seed. How can he get them all across the river?

Extensions (1-8) Add another animal or object to the farmer’s load. Now how many
trips will it take to get everything across safely? What kinds of things (or animals) can the farmer add? What kinds of things will not work? (Note: Any
additional animal that can cause harm to one of the original animals or seeds provides an experience with a problem with no solution. Students can prove that no solution exists!)

To make value judgments
- Select problems that involve some form of personal or societal dilemma (food supplies, animal experimentation, nuclear power, and so on).
- After solving the initial problem propose an open-ended dilemma that forces students to make value judgements.
- Discuss the consequences of various "solutions" on everyone or everything potentially affected.

To enhance creativity
- Choose problems that lend themselves to acting out, drawing a picture, working backward, and logical thinking.
- Have students hold onto their creative nonmathematical solutions until a solution has been found using the actual situation and restrictions in the problem. (Example: Taping the hen’s beak shut in the creativity problem would violate the original problem.) Then make it a point to discuss these solutions later.
- Encourage students to alter a problem so as to create a new situation. Decide if the new problem has a solution, look for further variations, or prove the impossibility of a solution.

Leading students to form generalisations
- Start with a problem that lends itself to an organised list or table from which a pattern can be noticed.
- Change the numbers in a systematic manner
- Jump to a much larger number to force a generalisation.
- Have students verbalise the pattern.
- Express the pattern using mathematical symbols
7: Asking questions to evaluate problem solving

Sometimes we ask questions that are designed primarily to stimulate students to think. Other times questions are asked to give students help in solving a problem. But have you thought about how you could ask questions to help evaluate your students' thinking and attitudes about problem solving?

Checking a student's written work for a problem gives you important information about that student's ability to solve problems, but it does not provide much information about the student's thinking or how the student feels about problem solving. Evaluative questions of the following types can help fill this information gap:

- How did you...?
- Why did you...?
- What did you try...?
- How do you know that...?
- Have you...?
- How did you happen to...?
- How did you decide...?
- Can you describe...?
- Are you sure that...?
- What do you think...?
- How do you feel about...?

Consider the following problem:

A frog is at the bottom of a well 10 metres deep and 2 metres across the top. On the first day she climbs up 5 metres but at night slips back 4 metres. If she does this each day, on what day will she get out of the well?

Here are some sample questions you might ask while a student is solving this problem or after a solution has been found.

- To help evaluate the student's ability to understand the problem, ask –
  "Can you describe this problem in your own words?
  What does 'get out of the well' mean?"
  "Does this problem remind you of any others you have solved before?"
  "What did you do to help you understand this problem?"
  "How do you know that 10 is not the correct answer?"

- To help evaluate the student's ability to deal with data, ask –
  "How did you decide which data are needed to solve this problem?"
  "How do you know that the depth of the well is needed to solve the problem?"

- To help evaluate the student's ability to choose appropriate strategies, ask –
  "Can you suggest some strategies that might help you solve this problem?"

- To help evaluate the student's ability to carry out a plan, ask –
  "Have you used strategies in solving the problem? Which ones?"
  "What did you do first to solve this problem?"
  "Can you describe your solution to the problem?"

- To help evaluate the student's ability to determine the reasonableness of an answer, ask –
  "Are you sure this is the answer to the problem? Why?"
  "Do you think there might be other answers?"
To help evaluate the student's attitudes and beliefs, ask -
"Do you like to solve problems like this? Why or why not?"
"How do you feel about your experience with this problem?"
"Do you think there might be another way to solve this problem?"

As your students answer questions like these, you may wish to record your evaluations using a checklist like the one in figure 1. Remember, good questions will allow you to evaluate aspects of your students' performance and attitudes that are difficult, if not impossible, to evaluate using other evaluation techniques. The information gleaned from your evaluative questions can then be used to help you plan subsequent instruction.

The coin dealer who wanted to buy them suspected that one of the pieces was heavier than the other seven. She was right and found the heavier coin by using a balance scale and only two weighings. How do you think she did it?

(Solution: Place three coins on each balance. If they balance, the heavier coin can be found by trying to balance the other two coins. If they don’t balance, try to balance two of the coins from the heavier set of three. If they balance, the third coin is the heavy coin. If they don’t balance, the heavier coin is determined.)

Classroom Climate

When you ask questions for evaluation purposes, remember these considerations:

- Ask the questions in a friendly, relaxed, non-threatening manner.
- Reduce students' anxiety by discussing with them your use of questioning techniques for the purpose of assessment.
- Be sure students know that you are evaluating them to find ways to help them become better problem solvers and not for the purpose of assigning a grade.
- Share with students insights gleaned from your evaluative questions that they could use to improve their problem solving skills.

Tips

Problem Corner

You may wish to ask evaluative questions as students in grades 6-8 solve this "golden" problem. It is a good problem for assessing a student's ability to describe and explain his or her solution.

A seller had eight gold coins that he claimed had the same weight.
Module 1.1 – Non-Routine Problems

Rationale

Module 1.1 Non-Routine Problems is a core module within the ‘Problem Solving and Investigations’ unit. During this module students will be introduced to problem solving and in particular to non-routine problems. A range of strategies that can be applied in solving non-routine problems will be explored and consideration given to adopting a problem solving approach in the teaching of mathematics in the primary schools. A range of problems will be presented, varying in degree of difficulty, catering for all students. Co-operative learning skills will be developed as students work together to solve problems.

Objectives

By the end of this module students will be able to:

- differentiate between routine and non-routine problems
- identify a range of strategies to solve non-routine problems and apply these
- articulate the processes used to solve non-routine problems
- identify how non-routine problems could be used in the primary classroom.

Concepts and Skills to be developed

- Co-operative learning skills
- Problem solving skills
- Comprehension skills
- Analytical skills
- Mathematical content knowledge associated with the selected problems
Topics

- Co-operative Learning
- Non-Routine Problems
- Strategies for Solving Problems
- Problem Solving in the Primary School

Suggested teaching strategies

- Co-operative learning
- Class discussions
- Oral Presentations

Suggested assessment tasks

- Journal writing activity: ‘Using non routine problems in the primary school.’
- Solve non-routine problems (co-operatively or individually).
- Develop a series of non-routine problems which could be used for a specific grade in the Primary School.

Resources

- Butchers paper
- Markers
- Linking cubes
- A range of concrete materials to support selected problems

References


A Suggested Sequence of Learning Activities

Introduction – Co-operative learning

Co-operative Learning
Introduce students to co-operative learning as a strategy for developing mathematical ideas. Work though a range of activities with students which develop their co-operative learning skills. Examples of activities such as ‘Fractured Figures’ and ‘Who’s doing the talking’, can be found in Co-operative Learning Activities section. Further examples can be found in Unit 2 Teaching and Learning Mathematics, Module 2.2 Teaching Mathematics.

Have students discuss the advantages and disadvantages they see in working co-operatively and identify strategies that can be used to overcome any disadvantages. Construct a chart to display these ideas.

Problem Solving
Working as whole class brainstorm ‘What do we understand by the term ‘problem solving?’ and list down ideas.

Homework
Refer students to the Student Support Material 1.1 Activity 1 and have them complete the journal writing activity outlined for homework.

Sharing Journals
Ask students to work in small groups of 3 or 4 and to share their journals. As a class reconsider the question ‘What do we understand by the term ‘problem solving?’ and add ideas to list.
Topic 1 – Non-routine problems

Problem Solving

Give students a number of problems to solve, include both routine and non-routine problems (individually and co-operatively). Discuss what makes a good non-routine problem.

For example:
- the solution is not immediately obvious
- the method of solution is not immediately known
- the student must feel that he/she has a reasonable chance of finding a solution
- the student should want to try to find the solution i.e. the problem should be of interest.

Non-routine Problems

Present students with a range of non-routine problems (Student Support Material 1.1 Activity 2). These problems should vary in degree of difficulty and cover concepts from the various mathematics content areas (number, measurement and space). They should also require the application of a range of different strategies to solve.

Have students complete 1.1.Activity 2 which requires them to identify which of the problems they consider to be non-routine problems and then work in pairs to solve three of the problems.

Reading

Ask students to read Part 2 ‘What is Problem Solving’ (Student Resource Book 1.1. Activity 3) and then to identify the process and strategies they used to solve the problems presented in 1.1 Activity 2.

Have students present their solutions to the class, identifying the strategies they used to solve one of the problems. Construct a Retrieval Chart listing the problems attempted, the strategies used, the mathematical ideas developed and the solutions. An example of a retrieval chart can be seen below.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Strategies Used to Solve the problem</th>
<th>Mathematical ideas developed</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
Topic 2 – Strategies

Class Discussion
Review the retrieval chart with the students and focus on the strategies used to solve the problems. Discuss the process people worked through and the range of strategies used. Identify the common strategies adopted by students to solve problems. Some focus questions you may use to generate discussion are:
- What strategies did people use to solve the problems?
- Did people use different strategies to solve the same problem?
- Are some strategies more effective than others? Why do you think this?
- Are some strategies more suitable to particular types of problems? Give examples to justify your response.

Problem Solving
Refer students to the Student Support Material 1.1 Activity 4. Ask students to solve the problems and then to read through the suggested strategies and solutions. Compare the strategies students used to those outlined in the Student Support Material.

Discussion
Discuss each of the strategies outlined in the Student Support Material – Strategies for Solving Problems and the steps involved. Work through an example using each of the strategies with the students.
Topic 3 – Non-routine problems in the primary school

Primary Mathematics Syllabus
Review the Primary Mathematics Syllabus documents (Lower and Upper Primary) with the students. Consider how problem solving is dealt with in the curriculum and how it is incorporated across all strands.

Problem Posing
Review the criteria for a ‘good’ non-routine. Identify a topic within the Primary Mathematics Syllabus and a particular Grade level. Provide a number of examples of non-routine problems that could be posed for children. For example, Grade 4 Common Fractions

In John’s family there are 8 children. Half of the children are girls and half are boys. In Dorothy’s family there are 4 children. One quarter of the children are boys and three quarters are girls. When the two families met together what fraction of the children are girls and what fraction are boys?

Have students work in pairs to pose problems for use in the classroom. Students would need to identify the grade level and the mathematical topic which the problem is related to.

Share problems that different groups have written. Consider the problems and the difficulties students had in posing problem. Ask students to identify what factors they need to consider when posing problems.

Equity Issues
Discuss with students how the problem posed:
- need to be gender inclusive
- could be adapted to cater for children with specific special needs

Extension activity – Micro-teaching
Micro-Teaching
Have students plan and teach a lesson which requires children to solve a non-routine problem. Ask students to take notes on the strategies children used, the difficulties they experienced, their level of interest and involvement in the activity, and the learning that took place.

After teaching ask students to write a journal about what they have learnt from the experience.

Discuss as a class
Conclusion

Class Discussion

Discuss the following questions with the class

- What do you see as the value in solving non-routine problems?

- What role do you see non-routine problems having in the teaching of mathematics in the primary school?

Glossary

Routine problems

A problem is a routine problem when we know immediately the method to use to reach a solution. It is type of problem which we have solved many times before and involves simple practice.

For example, there are 5 children and 25 bananas. How many bananas will each child get if we share them equally?

NOTE: This problem would be considered a routine problem for Grade 7 children. However for Grade 3 children who are just beginning to learn about division it may be a considered a non-routine problem.

Non-routine problems

A problem is a non-routine problem when:
- the solution is not immediately obvious
- the method of finding a solution is not immediately known

A good non-routine problem should also
- make students feel that they have a reasonable chance of finding a solution
- motivates students to try to find a solution

e.g. The head of fish is 1/3 as long as its body. The tail of the fish is as long as its head and its body combined. The total length of the fish is 48cm. How long is each part of the fish?
**Co-operative Learning Activities**

Co-operative learning activities help us develop our mathematical understandings and require students to work in groups. Working in-groups is more than just sitting around the same table, it involves considering others, listening to others and supporting one another.

When planning different co-operative learning activities consider how different groups can be formed. For example have students choose their own groups, form random groups or work in provincial groups. You can then discuss how working in different groups effects how people work together.

**Fractured Figures**


This activity aims to introduce students to co-operative learning, and develop an awareness of the importance of being sensitive to the needs of others, as well as an awareness of aspects of behaviour that may contribute to or obstruct co-operative learning. The follow-up questions focus on the students’ feelings during the activity, the role of language in influencing the behaviour of others and the need to work together to achieve a solution.

This activity has been used many times, particularly with teachers, and it is interesting to watch how people disengage from the Fractured Squares problem when their square is formed. They will frequently lean back with their arms folded while one or two poor remaining group members attempt to rearrange pieces that cannot possibly make a square. Participants are often very reluctant to dismantle their own completed square for the sake of the group solution.

The material required for this activity is a set of fractured shapes, which have been cut up. Cut out the shapes and put complete sets in different envelopes. One nominated person in each group should distribute the pieces. Each person receives three pieces marked with the same letter.

The aim of the activity is to have each member of the group with a completed shape in front of them. Two specific rules also apply.

- No one may speak, point or signal.
- Each member of the group **must wait** for a piece to be **given** to them. They are not allowed to take a piece or signal that they want a specific piece. They can only give away pieces.

**Circle bits**

In Circle Bits the angle sizes have been included to reinforce the angle sum of a circle. This allows students to work in either space or number mode, or link number and space. The additional two circles allow for larger groups (with new labels) or alternative problems. Is more than one solution possible?
Fractured squares
Fractured squares for 4 can reinforce the need to work co-operatively towards the group's goal. Individual solutions are possible without achieving a group solution. The group solution is unique!

Fractured squares for 5 or 6
Fractured squares for 5 or 6 may be used for larger groups or to provide alternative problems.

Rhombus parts
This rhombus shape is based on equilateral triangles and suggests questions on area and fractions. Component shapes include the regular hexagon, equilateral triangles, isosceles trapezium and parallelograms. Which shapes have the same area? What fraction is the isosceles trapezium of the parallelogram? By re-labelling the parts you may create sets for five or six.

Equilateral triangles
The number of straight edges and the similarity of the pieces makes this problem very challenging! The orientation of the labelling acts as an additional clue to assist students in solving this problem.

The Activity- Fractured Figures
To complete this task students will need to work as a group.

- The aim of the activity is to put the pieces of the fractured figures together so each student within the group has a completed shape.
- During the activity students are not allowed to speak, point or make signals.
- Each person in the group is to construct their own shape. No one else is to show another person how to do it.
- You cannot take a shape from another person; they must give it to you.
- The activity is not finished until everyone in the group has a complete shape.

Group Reflection
After students have completed the activity the following questions can be used to stimulate discussion.

- What was this activity all about? (e.g. constructing shapes, working together, problem solving)
- What things did you do in your group that helped you to succeed? (e.g. worked together, considered others, supporting one another)
- What things did you do which made it harder? (e.g. just thought of self, completed own shape without considering others, didn’t cooperate).
Who’s doing the talking?
This activity aims to make students more conscious of the role they play in group discussions. It aims to encourage more equal participation by all group members.

Materials: tokens (sticks, stones, blocks etc)

The Activity: Who’s doing the talking
- This is a group activity (4 to 6 students in each group)
- Give each group a topic to discuss (something interesting, controversial)
- Each time a person speaks they are to take a token from the middle of the table and place in front of them.

Reflection
Often when we get involved in a group we lose our awareness of how much we are contributing. Often some people do most of the talking, while others are silent listeners.

Following the discussion ask students to individually consider the following questions:
- What did I discover about my own group participation?
- Did I participate fully?
- Did I encourage everyone else in my group to participate fully?
- What personal goals can I set for the next time I participate in a group discussion?

Conduct a whole group reflection and ask students to consider the number of tokens in front of each person.
- What does this tell us about the way we worked as a group?
- Are we satisfied with what we see or are there ways we could work together better?
- Did everyone in the group have a fair chance to participate in the discussion?
- Did anyone ever get left out of the discussion?
- What group goals could we set for next time we work together?
FRACTURED SQUARES

Diagram showing fractured squares with labeled parts A, B, C, and D.
Module 1.2 – Investigations

Rationale

*Module 1.2 Investigations* is a core module within the ‘Problem Solving and Investigations’ unit. During this module students will investigate open-ended problems and have the opportunity to make important mathematical discoveries for themselves. Students will work collaboratively and investigate a range of problems across the various mathematics content areas.

Objectives

By the end of this module students will be able to:

- work co-operatively to carry out a range of investigations
- articulate the processes they have worked through to complete their investigations
- identify how mathematical investigations develop mathematical thinking
- consider how investigations can be used in the primary school.

Concepts and skills to be developed

- Co-operative learning
- Analytical skills
- Data collection
- Recording mathematical ideas
- Making and testing conjectures
- Explanation
Topics

- Investigations
- Investigations in the Primary School

Suggested teaching strategies

- Co-operative group work
- Class discussion
- Investigation

Suggested assessment tasks

- Complete an investigation (individually or co-operatively) and write a report on the findings.
- Develop a series of investigations which you could use with primary school children, specifying the topic and grade level.

Resources

Concrete materials to support investigations

References


A Suggested Sequence of Learning Activities

Introduction – What is an investigation?

Definition     Have students discuss in small groups what they understand is meant by the term ‘investigation’ and when this term is commonly used. Make a list of student responses and discuss these together. Some of the responses you could expect are:

- used by the police when they are trying to collect information about a crime
- used in science when we are trying to find out how something works
- to collect more information to explain something

Introduce students to the way the term ‘investigation’ is used in mathematics.

An investigation is an open-ended problem in which students may choose to go in different directions. Investigations give the students the opportunity to make important mathematical discoveries that they will remember much longer than if they were told them by the teacher. They encourage students to use high order intellectual skills which are far more important than the mathematics concepts involved.

An important aspect in carrying out investigations is the need to make guesses and to test them. What matters is not whether the guess is right or wrong, but the willingness to deduce information from the data gathered, to predict what will happen in other cases, and then to learn from the extra information obtained. The fear of being wrong needs to be overcome as it hampers exploration and understanding.

(Adapted In Green, Wally, 1998, Problems and Investigations, Department of Education PNG pp 14)

Topic 1 – Investigations

Modelling     Complete a number of investigations with the class (Student Support Material 1.2 Activity 1). Together with students work through the different stages of the investigation and discuss what is required at each stage. Most investigations require you to:

- explore the investigation
- gather and record data
- look for patterns
- make a conjecture (guess) i.e. think and then make a guess
- test the conjecture and then refine it and test again
- explain and prove results
- make generalisations
- extend the problem
- communicate findings
Carrying out an investigation
Present a range of possible investigations to the class. Working in small groups have students select an investigation to carry out, allow students to work at their own level. Encourage each group to choose a different investigation. Examples of investigations can be found in Investigations Sections.

Allow students time to complete their investigation.

Once students have completed their investigation provide an opportunity for the different groups to share their finding.

Processes
Ask students to identify the stages they worked through when completing their investigation and to discuss the mathematical understandings they have developed by completing the investigation. Record these ideas.

Topic 2 – Investigations in the primary school

Teaching Investigations
Refer students to the Student Support Material Topic 2: Investigations in the Primary School. Discuss with students how you can use investigations to teach mathematics at a primary school level. Young children would not be expected to complete all the steps outlined above. Gathering and recording data, looking for patterns, making and testing conjectures, refining and testing again, followed by a discussion of their findings would be reasonable expectation for primary school children. Have students complete 1.2 Activity 2 in the Student Support Material and discuss their findings.

Review the Primary School Mathematics Syllabus and ask students to identify topics which they feel could be taught through investigations
For example, Grade 8 Topic: Number and Application. Volume:
Construct and investigate the volume of cylinders

Planning an Investigation
Have students choose a topic from the Mathematics Syllabus and plan an investigation activity suitable for young children (Student Support Material 1.2 Activity 3).

Equity Issues
Discuss with students how the investigations used in the mathematics classroom
- need to be gender inclusive (language used, participation, strategies)
- could be adapted to cater for children with specific special needs (give examples of specific disabilities and how you could adapt the investigation to cater for child).
**Extension activity**

*Micro or peer teaching*

Allow students an opportunity to teach their investigation activity. Discuss the opportunities for active participation that were provided and the learning that occurred.

**Conclusion**

*Journal Writing*

Have students complete a journal reflecting on the use of investigations to teach mathematics at a Primary School level.
**Glossary**

**Investigation:** An open-ended problem in which you choose the direction of your inquiry. Investigations provide an opportunity to make important mathematical discoveries and encourage students to use high order intellectual skills. An important aspect in carrying out investigations is the need to make guesses and to test them. From the data gathered you need to collect information, to predict what will happen in other cases, and learn from any the extra information obtained. It is a mathematical inquiry to obtain facts.

**Conjecture:** A guess  
An opinion formed without sufficient evidence

**Generalisation:** To give a general example rather than a specific example

For example, consider the following sequence 2,4,6,8,10, __, __

The third number in the sequence is 6, the fifth number is 10 the eighth term is 16. These are all specific examples.

If we make a generalisation we would say that we could find any number in the sequence by multiplying it by 2, that is the nth term = 2n. For example the 10th number in the sequence will be 20
**Investigations**

(Investigations are from: Green, Wally, 1998, *Problems and Investigations*, Department of Education PNG)

1. **Three-digit numbers**
   Write down a three-digit number.  
   e.g. 541
   Reverse the digits and subtract smaller from larger  
   - 145
   Reverse the digits of the results and add  
   396
   Investigate  
   + 693
   1089

2. **Two-digit number.**
   Choose any 2-digit number. Add its digits and subtract this sum from the number.  
   Continue. What always happens? Can you explain it?

3. **Proper Factor Chain**
   The proper factors of a number EXCLUDE the number itself,  
   e.g. the proper factors of 16 are 1, 2, 4, 8,  
   The sum of the proper factors is 15.  
   Investigate the proper factor chains of various numbers formed as follows
   
   ![Factor Chain Diagram]

4. **‘Squareable’ numbers**
   A number is ‘squareable’ if a SQUARE can be subdivided into that number of smaller squares. For example, 4 and 6 are ‘squareable’ as shown in these diagrams
   
   ![Squareable Diagram 1]
   ![Squareable Diagram 2]
   
   Investigate ‘squareable’ numbers.

5. **Pick’s Rule**
   For this problem you need 1cm dot paper.  
   Form polygons by joining dots.  
   Record the area, A, the number of points on the boundary, B, and the number of points inside the figure, I.  
   Can you find a relationship between A, B, and I?
In this case, \( A = 10 \frac{1}{2} \quad B = 11 \quad I = 6 \)

6. **Quadrilaterals**  
A quadrilateral is any four-sided polygon. 
Investigate the polygons formed by joining the mid points of the sides of various quadrilaterals  
e.g.

7. **Odd numbers, the power of 2 and Primes**  
Investigate the claim that ‘Every odd number greater than one, can be expressed as the sum of a power of 2 and a prime.’ Note: 1 is not a prime number.  
For example:  
\[ 3 = 2^0 + 3 \]
\[ 49 = 2^5 + 17 \quad \text{or} \quad 2^3 + 41 \]

8. **Rectangles**  
In this rectangle there are 18 rectangles, of which 8 are square. Investigate.

9. **Consecutive whole numbers**  
Investigate the conjecture that ‘any whole number can be expressed as the sum of two or more consecutive whole numbers’.  
For example  
\[ 9 = 4 + 5 \]
\[ 15 = 7 + 8 \quad \text{and} \quad 4 + 5 + 6 \quad \text{and also} \quad 1 + 2 + 3 + 4 + 5 \]
\[ 18 = 5 + 6 + 7 \quad \text{and} \quad 3 + 4 + 5 + 6 \]
\[ 1 = 0 + 1 \]
10. **Perimeters of rectangles**
   Investigate the perimeters of rectangles that have the same area.
   Material needed: 1 cm square grid paper

   For further examples and hints for solving the investigations above see
   Green, Wally, 1998, *Problems and Investigations*, Department of Education
   PNG
Module 1.3 – Real World Problems

Rationale

*Module 1.3: Real World Problems* is a core module within the ‘Problem Solving and Investigations’ unit. During this module students will develop an understanding of the purpose of mathematics as it is applied to real life situations. The module allows mathematical content to be presented in an integrated way and provides students with an opportunity to articulate their mathematical thinking.

Objectives

*By the end of this module students will be able to:*

- solve a range of real world problems
- consider how real world problems can be used to develop mathematical understandings.
- articulate processes used to solve problems.

Concepts and Skills to be developed

- Problem solving
- Collaborative learning skills
- Research
- Report writing
- Oral presentation skills
- Mathematical content knowledge associated with the various problems
Topics

• Real world problems
• Real world problems in the Primary School

Suggested teaching strategies

• Co-operative group work
• Seminar presentation
• Retrieval Chart

Suggested assessment tasks

• Choose a real life problem and work collaboratively to solve it. Write a report and present your findings through a seminar presentation.

Resources

Primary Mathematics Syllabus documents
Butchers paper for retrieval chart

References


Department of Education PNG, (1993). *Secondary Mathematics 8A, 8B*

Green, Wally (1998). *Problems and Investigations,* Department of Education. PNG

Marr, Beth and Helme, Sue (1990). ‘Problem Solving’ in *Breaking the Maths Barrier,* Department of Employment and Training: Canberra, Australia
A Suggested Sequence of Learning Activities

Introduction – Real world problems

**Brainstorming**  Conduct a class discussion about the types of problems which effect different communities in PNG. Make a list of these. Some of the types of problems you may expect students to raise are:

- transport
- lack of a reliable water supply
- poor nutrition
- urban drift
- high crime rate

Identify questions which you would need to be asked to investigate each of the problems e.g. what would be the cost of improving the roads between A and B? How much water would be required to supply a family for a given period of time?

Discuss the process you would need to work though in order to carry out a mathematical investigation into a real world problem. The processes should include:

- making a clear statement of the problem which is to be investigated
- identify the methods to be used to gather information in order to answer the question
- a presentation of the results, incorporating tables, graphs, diagrams
- an explanation of the results obtained and the reasons for any conclusions or recommendations that may be made
- an explanation of the mathematical content and process that were used.

Topic 1 – Investigating a real world problem

**Excursion**  Visit a local service provider or business e.g. Telikom, Water Authority, a coffee plantation, a tuna factory. Find out about the issues involved in running the organisation.

Consider the questions which would need to be answered to successfully run the organisation and what mathematical understanding would be required. Make a list of the mathematical understandings needed.

**Investigating**  Working in small groups, have students choose a real world problem to investigate. This could be a project that students work on during class time and/or during their study periods.
Refer students to the *Student Support Material Topic 1 – A Written Report*. Ask students to read the written report on the mathematical investigation into the problem. Students should note down the information included in the report e.g. the headings, the description of how people tackled the problem, the results, use of graphs, tables and/or diagrams, the findings, recommendations, the conclusions drawn and the mathematics used. *(1.3 Activity 1 Student Support Material)*

**Report writing**

Have students write a report on their investigation into a real world problem. The written report should include:
- the topic heading
- a description of how people tackled the problem
- the results, using graphs, tables and/or diagrams
- the findings, explaining or giving reasons for results
- the mathematics used.

Examples of real world problems which students may like to investigate can be found in Real World Problems Section.

**Retrieval Chart**

Ask students to summarise the main ideas from their investigation into a real world problem onto a retrieval chart. The retrieval chart could record the following information:

<table>
<thead>
<tr>
<th>Real World Problem</th>
<th>Strategies Used to Solve the problem</th>
<th>Mathematical ideas used</th>
<th>Findings</th>
</tr>
</thead>
<tbody>
<tr>
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</tbody>
</table>

**Seminar Presentation**

Organise students to present a seminar, outlining the problem they investigated and the processes they worked though. The information recorded in the retrieval chart should help focus the presentation. During the seminar presentation you should expect students to:
- introduce their problem
- discuss the process the group worked through to reach a solution
- discuss their results and findings (showing any tables, graphs, diagrams etc)
- identify the mathematical understandings used when solving the problem
- allow time for questions and discussions
- ensure all group members participate in the presentation
- present information in a way that others can hear and understand what is being said.

**Class Discussion**

Following the seminar presentations conduct a class discussion. Focus on:
- the range of different strategies students used to solve the problems
- the mathematical concepts which were used during the investigation
- the findings.

Emphasis that although real world problems are rarely mathematical questions, there is generally a need to apply mathematics when the investigations takes place.

**Topic 2 – Real world problems in the primary school**

**Brainstorming**

Ask students to think about the types of real world problems primary school children would be interested in investigating. Make a list of these problems and the investigation questions children could explore.

**Syllabus Documents**

Review the Primary Mathematics Syllabus documents for Lower and Upper Primary. Look at the different topics and consider which of these topics could be explored through investigations into various real world problems. Draw a chart which shows the following information:

<table>
<thead>
<tr>
<th>Real world problem</th>
<th>Investigation question</th>
<th>Maths Topics covered</th>
</tr>
</thead>
<tbody>
<tr>
<td>I need new pencils and exercise books for school</td>
<td>How can I get enough money to buy the things I need</td>
<td>Problem Solving – strategies</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Number – estimation, Addition and subtraction of decimals</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Measurement - money</td>
</tr>
</tbody>
</table>
Extension activity

Teaching
Have students plan, teach and evaluate an activity which involves children solving a real world problem.

Following the teaching allow students to share their reflections on their experiences. Conduct a class discussion which considers what people see as the benefits and problems associated with using real world problems in the classroom.

Conclusion

Journal
Ask students to complete a journal which discusses the role of mathematics in the community.

Share journals and discuss how mathematics is applied in real life situations.
Glossary

Real World Problems: Problems which apply to real life situations. In solving real world problems the mathematics is integrated and you are able to see how mathematics is applied in a real situation.
Real World Problems

- Investigate the use of measurement in different rural communities in PNG?

- Water Supply
  - In urban and semi urban communities, how much water is used in an average house per year in washing, flushing the toilet, drinking and food preparation?

- Transport
  - how much does it cost to run a vehicle?
  - transporting coffee from the Highlands to then coast for export

- Establishing a business

- Recycling - is it worth it?

- Investigate problems related to business such as:
  - hire purchase agreements
  - discounts
  - profit and loss
  - discounts
  - sales tax

- Making a living from the land
  - how much coffee do I need to grow to make a living?
  - Mining for gold, how much mining do I need to do to make a profit?

- Raising money to pay for college fees

- Cost associated with running a PTC

- Mess costs per student per year at PTC

- Book allowance, how far does it go?

- Rubbish disposal
Module 1.4 – Applications of Chance Processes, Probability and Statistics

Rationale

Module 1.4: Applications of Chance Processes and Probability is a core module within the ‘Problem Solving and Investigations’ unit. During this module students will collect their own data and investigate concepts of chance and consider experimental and theoretical probability.

Objectives

By the end of this module students will be able to:

- identify the probability of an event occurring
- discuss the relationship between experimental and theoretical probability
- differentiate between dependent and independent events

Concepts and Skills to be developed

- Data collection
- Interpreting data
- Calculating Probability
Topics
- Probability
- Independent and Dependent Events
- Probability in the Primary School

Suggested teaching strategies
- Investigation
- Modelling
- Small group work

Suggested assessment tasks
- Conduct a group investigation into the probability of a particular event occurring.
- Plan, teach and evaluate a series of activities from the probability strand of the Primary Mathematics Curriculum.

Resources
- Dice
- Coins
- Spinners
- Coloured beads or blocks

References
A Suggested Sequence of Learning Activities

Introduction – What are my chances?

Working in pairs

Organise students into pairs. Give each pair a card which states a particular event or outcome e.g.
- throwing a coin and a kumul turning up
- buying a prize winning lotto ticket
- becoming head teacher of a school
- living to 80
- after having three girls in a row the fourth born child being a boy
- owning a car
- travelling overseas
- the sky falling down
- it will rain today

Ask students to discuss the probability of the particular events happening and the reason why they believe this.

Have students swap cards with different pairs of students and repeat the activity. Share results and discuss any differences and possible reasons for these e.g. the probability of owning a car for a male student may be a likely, whereas a female student may think it is highly unlikely. The reason for this difference may be related to gender issues and can be discussed.

Language of probability

Identify the language we use to talk about the probability of particular events occurring. Organise the words along a scale which illustrates the likelihood of an event occurring

- impossible, never, unlikely, possibly, not sure, maybe, 50-50 chance, probably, fair chance, sure, likely, highly likely, certainly, definitely, absolutely certain.

Identify events which have different degrees of probability of occurring and mark these on the scale.

Probability scale

Using the scale developed in the above activity, develop a probability scale with the students. An event that is absolutely certain to happen (e.g. one day we will die) has a probability of 1, where as an event which can never happen (e.g. a person can jump unaided 10 metres) has a probability of 0. Discuss with students that all probabilities must therefore have a value between 0 and 1.
and can be expresses either as a fraction or a decimal. For example tossing a coin and getting a kumul has a probability \( P \) (kumul) of 0.5 or \( \frac{1}{2} \).

**Topic 1 – Probability**

*Calculating probability*

Introduce the idea of theoretical probability. Give each student a lucky number. Together discuss with students the chances of the following people winning and the reasons for these believes:
- an individual student winning
- a student from a particular province winning (e.g. a Sepik student)
- a female winning
- a male winning
- a person over 20 years of age winning
- a person less than 150 cm tall winning.

Construct a table to record findings.

Draw out a lucky number and note down the winner. Return the number to the draw and repeat this process a number of times (if you have 25 students, draw a lucky number 25 times) recording the results each time. Construct a table to show the results and compare them with the theoretical probability. Discuss the results and the reason for any differences.

<table>
<thead>
<tr>
<th>Lucky Number</th>
<th>An particular individual</th>
<th>Student from the Sepik</th>
<th>Female</th>
<th>Male</th>
<th>Person over 20</th>
<th>Person less than 150cm tall</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Theoretical Probability</strong></td>
<td></td>
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</tr>
<tr>
<td><strong>Experimental Probability – what actually happened</strong></td>
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</tr>
<tr>
<td><strong>Difference</strong></td>
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</tbody>
</table>

*Investigating probability*

Provide students with an example of an investigation (*Student Support Material 1.4 Activity 1*) Work through the example with students and discuss the process and the results.

Have students work in a small group to carry out an investigation into the probability of a certain event occurring. Sample investigations can be found in
the Student Support Material 1.4 Activity 2. The process student should work through would involve:
- choosing a topic
- identifying people's perceptions of the outcome
- designing an experiment
- carrying the experiment out and collecting the data
- analysing the data
- confronting and trying to explain the reasons for any differences between perceptions and the results of the experiment.

**Topic 2 – Independent and dependent events**

**Group Activities**

Organise students into small groups. Present students with a range of different activities and ask them to consider the probability of certain events happening. The activities presented should include both independent and dependent events.

Examples of independent events (an event which have no effect on the events which follow) which you could use with students are listed below.

- A coin is tossed and a card is then drawn from a pack of 52 playing cards. What is the probability that a kumul and a King will result?
- A die is rolled five times. What is the probability that you will get five 6’s?
- A K1 coin is tossed 10 times. What is the probability that you will get 2 kumul’s followed by 5 pukpuk’s and then 3 kumul’s?

Examples of dependent events (where the first event has an effect on the event which follows) which you could use with students are listed below.

- A box contains 10 blocks, 6 blue, 3 red and 1 yellow. A block is drawn from the box and not replaced. Find the probability of drawing 1 yellow block followed by 2 red blocks and 1 blue block.
- You have a pack of 52 playing cards. You draw a card from the pack and do not replace it. What is the probability of drawing 4 aces in a row?

Revise with students the law of probability

To calculate the probability of events occurring we use the multiplication law of probability.

If \( P(E_1) = \) the probability of the first event happening
\( P(E_2) = \) the probability of the second event happening
\( P(E_3) = \) the probability of the third event happening
and \( P(E_1E_2E_3) = \) the probability that \( E_1, E_2, \) and \( E_3 \) occur then
\[ P(E_1E_2E_3) = P(E_1) \times P(E_2) \times P(E_3). \]

In general,
\[ P(E_1E_2\ldots E_n) = P(E_1) \times P(E_2) \times \ldots \times P(E_n) \]
Extension activity

Provide students with a range of activities which involve mutually exclusive and non-mutually exclusive events. Investigate the probability of these events occurring.

Examples of mutually exclusive events (that is events which can not happen at the same time) are:

- A 3 or 4 occurring in a single roll of a die
- A pack of 52 playing cards are cut once. Find the probability that the card which is cut will be the Ace of Spades, a King, or the Queen of Hearts

(Note: If \( E_1, E_2, \ldots, E_n \) are mutually exclusive events then the probability of one of the events occurring is \( P(E_1 + E_2 + \ldots + E_n) = P(E_1) + P(E_2) + \ldots + P(E_n) \))

Examples of non-mutually exclusive events (that is events that can happen at the same time are:

- Cutting a pack of playing cards and getting an Ace and a Spade
- Three people A, B and C work independently to solve a cross work puzzle. The probability that A will solve the puzzle is \( \frac{2}{3} \), the probability that B will solve the puzzle is \( \frac{3}{4} \) and the probability that C will solve the puzzle is \( \frac{4}{5} \). What is the probability that the puzzle will be solved?

(Note: If \( E_1, E_2, \ldots, E_3 \) are non-mutually exclusive events then the probability of one of the events occurring is \( P(E_1) + P(E_2) - P(E_1E_2) \))

Topic 3 – Probability in the primary school

Syllabus Review

Have students review the Primary Mathematics Syllabus documents to consider how probability is covered.

Ask students to develop a probability investigation they could use with children in the primary school. They would need to identify the grade level, the objective to be taught, the context for the teaching and the investigation to be carried out.

Conclusion

Class Discussion

Conduct a class discussion that reviews the learning which took place during this module. Focus questions could include:

- What have you found out about your perceptions of the probability of certain events occurring?
- What approach would you use to teach probability to children in the primary school?
Glossary

**Chance:** The possibility or probability of an event happening

**Theoretical probability:** The calculated probability of an event occurring based on the possible outcomes. For example if you toss a K1 coin 10 times, the theoretical probability of getting a kumul is $\frac{5}{10}$ and getting a pukpuk is $\frac{5}{10}$.

**Experimental probability:** The number of times a particular event occurs during trial. For example if I toss a K1 coin 10 times the actually results may be $\frac{6}{10}$ kumuls and $\frac{4}{10}$ pukpuks.

**Probability:** Probability is the likelihood or chance of an event occurring. For example if a coin is tosses, the probability of getting a kumul is $\frac{1}{2}$ or 0.5.

\[
\text{Pr (event occurs)} = \frac{\text{number of favourable outcomes (or results)}}{\text{total number of outcomes (or results)}}
\]

**Independent event** An independent event is one which has no effect on the events which follow. For example if a die is rolled three times, what happens on the first roll does not affect what happens on the second or third roll. The three rolls of the die are independent events.

**Dependent events** A dependent event is one which has an effect on the event which follows. The probability of the second event occurring depends on what happened in the first event. For example consider a bag contains 3 red balls and 2 blue balls. A ball is drawn at random from the bag and not replaced. The probability that it is red is $\frac{3}{5}$. If you draw a second ball from the bag the probability that this is also red is $\frac{2}{4}$. Therefore the probability of drawing two red balls is $\frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$.

**Multiplication law of probability.** To calculate the probability of events occurring we use the multiplication law of probability. If $P (E_1)$ = the probability of the first event happening $P (E_2)$ = the probability of the second event happening $P (E_3)$ = the probability of the third event happening and $P (E_1E_2E_3) = $ the probability that $E_1$, $E_2$, and $E_3$ occur then $P (E_1E_2E_3) = P (E_1) \times P (E_2) \times P (E_3)$. 
In general,
\[ P(E_1E_2\ldots E_n) = P(E_1) \times P(E_2) \times \ldots \times P(E_n) \]

**Mutually exclusive events**
If events cannot happen at the same time they are said to be mutually exclusive. Consider throwing a die once, and finding the probability of a 3 or a 4 occurring. It is not possible for a 3 and a 4 to occur together. Therefore the event of throwing a 3 and a 4 in a single roll of the die are mutually exclusive.

If \( E_1, E_2, \ldots, E_n \) are mutually exclusive events then the probability of one of the events occurring is
\[ P(E_1 + E_2 + \ldots E_n) = P(E_1) + P(E_2) + \ldots + P(E_n) \]

**Non – mutually exclusive events**
If events can happen at the same time they are said to be non-mutually exclusive events. For example if you have a pack of playing cards and cut it once, the event of drawing a Jack and drawing a Diamond are not mutually exclusive, because the Jack of Diamond can be cut.

If \( E_1, E_2, \ldots, E_3 \) are non-mutually exclusive events then the probability of one of the events occurring is
\[ P(E_1) + P(E_2) - P(E_1E_2) \]

In the example above this would mean that the probability of cutting the Jack of Diamonds would be calculated by
\[ (\text{Jack of Diamonds}) = P(E_1) + P(E_2) - P(E_1E_2) \]
\[ = \frac{4}{52} + \frac{13}{52} - \left(\frac{4}{52} \times \frac{13}{52}\right) \]
\[ = \frac{17}{52} - \frac{1}{52} \]
\[ = \frac{16}{52} \]
\[ = \frac{4}{13} \]
Module 1.5 – Strategy Games

Rationale

Module 1.5 Strategy Games is a recommended module within the ‘Problem Solving and Investigations’ unit. During this module students will become familiar with a range of strategy games and consider how these games support the development of mathematical thinking. The application of strategy games into the primary school curriculum will be considered.

Objectives

By the end of this module students will be able to:

- differentiate between games of chance and games of strategy
- identify the strategies used to play a range of different games
- suggest ways in which strategy games could be used in the teaching of mathematics in primary schools.

Concepts and Skills to be developed

- Problem solving skills
- Mathematical content knowledge associated with the various games
- Oral presentation skills
Topics

- Games of Chance and Games of Strategy
- Strategy Games
- Playing Games in the Primary School

Suggested teaching strategies

- Games
- Problem solving
- Debate

Suggested assessment tasks

- Establish a portfolio of strategy games suitable for use in the primary school mathematics classroom.

Resources

- Playing cards
- Die
- Butchers paper for retrieval chart
- Counters
- Range of games

References

A Suggested Sequence of Learning Activities

A suggested sequence of learning activities can be found below. Alternatively, this module could be taught through establishing a learning centre in the classroom. By developing a series of task cards which students can complete independently, this module could be introduced at the beginning of the semester. Students can complete a set number of cards throughout the semester during their study periods. Towards the end of the semester time can be spend review the games which have been played and the learning which has occurred.

Introduction - Games

Games
Ask students to share with one another different games they are familiar with. These should be games such as card games, board games, and games using dice, rather than sports. Identify the rules for the different games and spend some time playing them. Discuss what is required to win the games (e.g. luck or certain skills, logic, reasoning).

Topic 1 – Games of chance and games of strategy

Strategy games
Introduce students to the term ‘strategy game’ and what it means by the term.

A strategy game should be for two or more players and must have a set of rules for the players to follow. The rules should establish the goals for the players and their individual goals should be in conflict, that is each player is trying to beat the other players. In a strategy game the players should be able to choose their own path or action in an attempt to reach their individual goals. It should be apparent when one of the players has won the game.

(Adapted from Gough, J. Playing Mathematical Games, When is a game not a game? in APMC Vol. 4, no 2 1999)

Emphasis the difference between games of strategy and games of chance e.g. Snakes and Ladders is not a ‘strategy game’: even though the players take ‘turns’, and the first to the end is the ‘winner’, the players have no choice about what they can do in their turn – they just follow the dice, plus the possible consequences of ending on a ladder or snake. The outcome of a game such as Snakes and Ladders is based on pure luck, as the player has no choice on where they move. There is no interaction between players, so that what one person does on their turn can effect what the next person does on their turn.

Reading
Refer students to the Student Support Material Topic 1- Games of Chance and games of Strategy. Have students read the article and complete the activity asking them to identify ‘games’. (1.5 Activity 1, Student Support Material)
Retrieval Chart

Construct Retrieval Chart of the games discussed during the introductory session. Identify whether the games are games of ‘chance’ or ‘strategy games’.

<table>
<thead>
<tr>
<th>Game and materials required</th>
<th>Rules</th>
<th>What is required to win the game</th>
<th>Game of Chance or Strategy</th>
</tr>
</thead>
</table>
| Snakes and ladders           | - 2 or more players  
- take turns  
- throw dice and move token the number of spaces indicated by dice  
- if land on a square with the bottom of a ladder, move up it  
- if land on a square with the head of a snake on it move down it  
- first one to reach the finish wins | - Luck as it depends on the number which comes up on the dice | Game of Chance |
|                              | - 2 or more players  
- take turns  
- throw dice and move token the number of spaces indicated by dice  
- if land on a square with the bottom of a ladder, move up it  
- if land on a square with the head of a snake on it move down it  
- first one to reach the finish wins | - Luck as it depends on the number which comes up on the dice | Game of Chance |
| Star Duel                    | - 2 or more players  
- take turns  
- throw dice and move token the number of spaces indicated by dice  
- if land on a square with the bottom of a ladder, move up it  
- if land on a square with the head of a snake on it move down it  
- first one to reach the finish wins | - Luck as it depends on the number which comes up on the dice | Game of Chance |
|                              | - 2 or more players  
- take turns  
- throw dice and move token the number of spaces indicated by dice  
- if land on a square with the bottom of a ladder, move up it  
- if land on a square with the head of a snake on it move down it  
- first one to reach the finish wins | - Luck as it depends on the number which comes up on the dice | Game of Chance |

Topic 2 – Strategy games

Playing Games

Introduce students to a strategy game ‘Hearts’ outlined in the Student Support Material Topic 2 – Strategy Games. Allow students an opportunity to play the game a number of times and to develop some ‘winning’ strategies. (1.5 Activity 2)

Discuss with students the different strategies they tried when playing Hearts and the process they worked through to come up with these strategies. Some strategies and tips for playing Hearts are listed below.

- Avoid winning a trick (a round of cards) containing hearts or the queen of spades. The only time you will want to win such a trick is when you are trying to or trying to prevent someone else from successfully ‘Shooting the Moon’.
- On hands that begin by passing cards to an opponent, pass cards with high values, such as aces or face cards like Kings, Queens and Jacks.
• Play your highest cards early in the game, while your opponents are likely to have some cards in each suit and will have to play those cards instead of hearts. Tricks that do not contain any hearts or the queen of spades do not add to your score.

• Keep track of which cards have been played, particularly whether the queen of spades has been played and whether hearts have been broken (that is, whether a heart has been discarded on an earlier trick).

Point out the similarities between the process involved in playing strategy games and solving problems.
- **Read the rules.** Understand what is required. What are the moves? What constitutes a win?
- **Explore.** Have you played a similar game before? Select possible strategies of play.
- **Carry out the strategies.** Can you counter your opponent’s move as the game progresses?
- **Check your results.** If your strategy worked (i.e. you won the game), is it a general strategy? Will it work in other games against other opponents?

Set up a number of workstations around the room, each with a different strategy game. Suggested games can be found in the *Student Support Material*. Divided the students into groups and allow time for students to rotate around and play each game. After playing a game (and before moving onto the next) have each group discuss the winning strategies they were using and list these. After students have rotated around all games have each group share and compare the strategies used. *(1.5 Activity 3, Student Support Material)*

**Topic 3 – Playing games in the primary school**

*Maths games in schools*  
With the class construct a list of the mathematics ‘games’ played in schools. Consider what skills are being developed through these games and whether these are games of chance or strategy

*Debate*  
Consider the statement  
‘While strategy games may be fun and interesting for children, they have no place in the mathematics classroom’.

Have students form groups and debate this issue.

*Portfolio*  
Have students establish a portfolio of strategy games suitable for use in the primary school mathematics classroom.
Extension activity – Changing the game

Changing the game  After playing a strategy game have students explore what happens when:
- you change the expected outcome e.g. changing the first to finish from winner to loser
- change the configuration of the game board
- change the allowable moves e.g. allowing diagonal moves when previously they were banned.

Conclusion

Journal Writing  Have students write a journal reflecting on the use of strategy games in developing mathematical thinking.
Glossary

**Strategy games**
A strategy game should be for two or more players and must have a set of rules for the players to follow. The rules should establish the goals for the players and their individual goals should be in conflict, that is each player is trying to beat the other players. In a strategy game the players should be able to choose their own path or action in an attempt to reach their individual goals. It should be apparent when one of the players has won the game.

**Trick**
In cards you are said to win a ‘trick’ when you win a round of cards. For example if you are playing a card game such as Hearts, the player with the highest card of the same suit wins. If Player1 leads the 4 Diamonds, Player 2 plays the King of Diamonds, Player 3 the 7 of Diamonds and Player 4 the 2 of Diamonds, then the winner is Player 2. Player 2 is said to have won the trick.

**Suit**
In a pack of playing cards you have four different suits. They are Hearts, Diamonds, Clubs and Spades. In the card game Hearts you must follow suit. If a Club is lead for example, then the following players must follow suit and play a Club.
Module 1.6 – Problem Posing

Rationale

Module 1.6: Problem Posing is a recommended module within the ‘Problem Solving and Investigations’ unit. During this module students will pose a range of different problems and consider what is involved in developing good problems. Also students will consider how the process of problem posing can support the development of their own mathematical thinking.

Objectives

By the end of this module students will be able to:

• write a range of problems suitable for their peers
• write a range of problems suitable for use in the primary school
• solve a range of problems developed by their peers
• identify what makes a good problem and the skills that are developed when posing problems.

Concepts and skills to be developed

• Problem posing
• Problem solving
• Critical thinking and reasoning
• Validation skills
• Mathematical concepts related to the types of problems posed and solved
Topics

- Problem Posing
- Problem Posing in the Primary School

Suggested teaching strategies

- Discussions
- Problem solving
- Modelling

Suggested assessment tasks

- Pose a series of problems suitable for use in the primary school. Identify the grade, the mathematical concepts being developed through solving these problems, the types of strategies you would envisage children using and discuss how you would teach this in the school.
- Write a journal:
  - outlining how problem posing supports the development of mathematical understandings.
  - discussing how you could implement problem posing in the classroom.

Resources

- Primary Mathematics Syllabus
- Butchers paper
- A range of concrete materials

References


A Suggested Sequence of Learning Activities

Introduction - Curriculum review

Discussion
Review the Primary Mathematics Syllabus and point out that one of the objectives for the various strands and topics is for teachers to pose problems for students to solve. For example:
- Grade 7, Shape and Space Strand, Co-ordinates, has an objective requiring students to solve problems relating to co-ordinates.

Highlight the need for teachers to be able to pose challenging and interesting problems for students.

Make a list
Based on the work covered in previous modules within this unit ask students to make a list of what they think are the characteristics of a ‘good’ problem.

For example, A good problem should ……….

Topic 1 - Problem posing

Working in pairs
Present the class with a topic and ask students to work in pairs to pose a problem related to the topic. The problem should be at a level suitable for their peers.

Organise pairs to swap their problem and to read the problems posed by their peers. Do this a number of times so students see at least 4 different problems.

Ask students to consider the various problems they have read and to identify which of the problems they would be interested in solving and consider to be ‘good’ problems. Ask people to give their reasons. Use this information to reinforce the characteristics of a good problem.

Work stations
Set up a number of work stations around the classroom. At each work station have an activity which requires the students to pose different types of problems (refer to Student Support Material 1.6 Activity 1) For example:
- pose a problem that gives a particular answer e.g. 24 m²
- pose a problem about a particular topic e.g. percentages
- pose a problem which involves the use of a specific mathematics concept e.g. equivalent fractions
- pose a problem which involves the use of a specific mathematics method e.g. division
- pose a problem based on a specific problem structure e.g. John and Susan have a total of 15 bananas, Susan and Matthew have a total of
19 bananas. Matthew and John have a total of 22 bananas. How many bananas does each person have?
- pose a problem which has more than one solution e.g. draw shapes with a perimeter of 10 cm

Allow students an opportunity to move around the different work stations and pose problems. For each work station ask students to pose a problem:
- suitable for their peers
- suitable for use in the primary school

**Solving Problems**

Allow each student to select one problem posed by their peers and solve it.

**Discussion**

Conduct a class discussion which looks at the skills needed when posing problems. For example:
- need to use appropriate language
- need to be aware of student interests
- need to have the mathematical understandings related to the topic covered by the problem
- need to be creative

**Topic 2 – Problem posing in the primary school**

**Reading**

Refer students to the *Student Support Material 1.6 Activity 2*. Have students read the article ‘Posing Problems and Solving Problems’ and complete the activity which requires them to identify the advantages of encouraging children to posed problems.

**Group discussion**

Identify the issues you would need to consider when writing problems for children.

The ideas that you need to emphasis here would include:
- when considering a topic/concept to be taught think about a real life context you could use to developing this idea
- consider if you wanted the children to work in groups or individually to solve these problems
- what language would be the most appropriate to use, vernacular/Tok Ples/English
- what strategies the problems would encourage children to use
- problems posed should cater for different ability groups
- problems posed should cater for children with special needs
- problems posed should be gender inclusive
Discuss with students the idea of having children pose their own problems for their peers. How could you implement this idea? What would be the advantages of this? What difficulties might you encounter?

Conclusion

*Journal writing* Have students write a journal which discusses the learning which takes place when posing problems.
Glossary

Problem
Posing The generation of a new problem