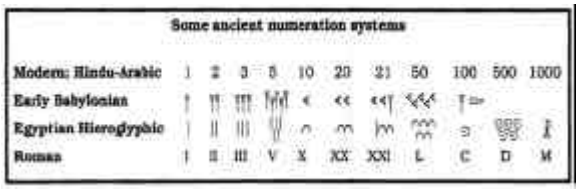
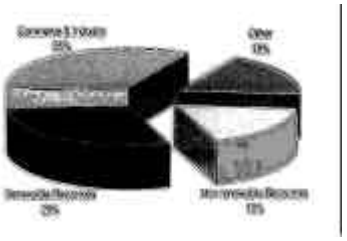
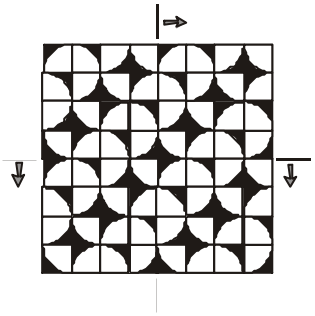
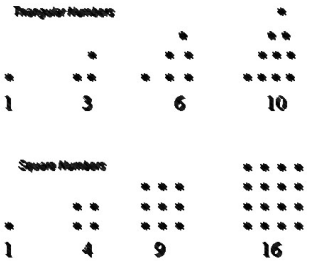
Mathematics Science Strand
Mathematics

Unit 3: Enrichment Topics in Number









Lecturer Support Material

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PASTEP

Primary and Secondary Teacher Education Project

Australian Agency for International Development (AusAID)
GRM International

Papua New Guinea-Australia Development Cooperation Program

Unit outline

Unit	#	Modules
Unit 3 Enrichment Topics in Number	3.1	Alternative Systems of Numeration and Computations (Core)
	3.2	Factors, Primes and Composites (Core)
	3.3	Number Sequences and Patterns (Core)
	3.4	Basic Statistics (Core)
	3.5	Using Calculators in the Primary School (Recommended)
	3.6	Clock Modulo Arithmetic (Recommended)

Icons

 Read or research

 Write or summarise

 Activity or discussion

Table of contents

Lecturer Support Material

This *Lecturer Support Material* has been developed to assist lecturers in the teaching and assessing of *Unit 3: Enrichment Topics in Number*.

The material consists of:

- An introduction to the unit, which includes information on the overall rational for the unit as well as recommended teaching approaches and suggestions on how the unit can be integrated into practicum activities.
- Module outlines, setting out a suggested sequence of learning activities and identifying topics which could be taught within each module. Ideas for assessment activities are also provided
- A Unit Glossary

Suggestions have been made about what content should be covered within the unit as well as recommending an approach to teaching the material. It is envisaged that by working through the suggested sequence of leaning activities lecturers will be modelling good practices for teaching mathematics that students can then apply in their own teaching.

Student Support Material has also been developed for the unit *'Enrichment Topics in Number'* to accompany this *Lecturer Support Material*. The *Lecturer Support Material* should be read in conjunction with *the Student Support Material* as the *Lecturer Support Material* makes reference to activities and material contained only within the *Student Support Material*.

It is not expected that the students will work only from the ideas and suggestions contained within the *Student Support Material*. Additional ideas and activities are set out in the *Lecturer Support Material* to compliment the student material. The lecturer will need to make decisions about how to present the material and make decisions about what activities will be covered during lecture periods and what students will be required to do during their study time.

When using this material it is recommended that lecturers:

- read through the whole unit prior to planning the course overview
- select the modules and topics to be cover in the time available
- plan the activities that will be presented to the students
- select the material from the *Student Support Material* that will be used to support the teaching of the unit
- develop the assessment tasks for the unit.

It is important to remember that this material is support material only. While lecturers are encouraged to try out the suggested activities within the material, it is hoped that people will also include their own ideas. This material, along with the *Student Support Material* should be seen as a living, working document which can be reviewed and changed to suit the curriculum needs of the Primary Teachers Colleges and new ideas and trends in the teaching of mathematics.

Unit 3: Enrichment Topics in Number

The Mathematics course seeks to develop beginning teachers who are:

- confident in their ability to teacher mathematics across the Primary School, are familiar with the Primary School Mathematics Syllabus and have a strong understanding of the mathematical concepts covered within it
- aware of the factors which impact on the teaching and learning of mathematics
- resourceful, creative, life long learners
- inclusive of all people, regardless of gender, social, cultural and language background.

Rationale

Fundamental to learning mathematics is an understanding of number. Number forms such a large part of every school mathematics curriculum, and underlies all of the computational procedures as well as setting up the patterns of thinking on which mathematics relies. As future teachers students need to further develop their own understanding of number concepts as well as develop an awareness of how to teach these concepts to children. The modules offered in this unit will revitalise students' interest in numeration and enrich their knowledge across a range of areas.

Aims

This unit aims to produce beginning teachers who are:

- able to solve a range of number problems
- able to articulate their mathematical thinking
- confident and competent to teach number concepts to primary school children (Grades 3 to 8).

Objectives

As a result of studying this unit students will:

- consider how cultural and social influences have impacted on the development of numerical understandings
- further develop their understanding of factors, primes and composites
- determine number patterns and sequences
- compile statistical information and analyse it
- debate the use of calculates in the primary school
- create modulo art designs.

Unit outline

'Enrichment Topics in Number' is a 3-credit point unit.

To successfully complete this unit it is suggested that students complete the following core modules:

Module 3.1	Alternative Systems of Numeration and Computations
Module 3.2	Factors, Primes and Composites
Module 3.3	Number Sequences and Patterns
Module 3.4	Basic Statistics

Taking into consideration the time available for this unit it is recommended that lectures selected additional modules from those listed below:

Module 3.5	Using Calculators in the Primary School
Module 3.6	Clock Modulo Arithmetic

Each of these modules should take between 6 to 9 hours of lectures to complete. It is also expected that students will spend an equivalent number of hours of non-contact time studying the ideas and concepts raised in this unit.

A detailed description of each module in included in this Lecturers Support Material

Sequencing of modules within the unit

When considering the teaching of this unit it is important that the unit be viewed in its entirety rather than as a number of discrete modules. Lecturers are advised to read through the outlines for all modules before developing a unit overview. It is recommended that time be made available with students at the start of the unit to develop an overall view of the concepts and understandings to be developed throughout the unit. The relationships between modules need to be maintained throughout the unit as ideas are built upon as new concepts are introduced. At the conclusion of the unit it will be important to spend some time reviewing the work covered in the various modules and considering the overall implications for teaching problem solving and investigations in the primary schools.

It is recommended that *Module 1: Alternative Systems of Numeration and Computations* be taught first. This module will provide students with an understanding of the historical development of number as well as an opportunity to review the Hindu Arabic System.

The remaining modules can be taught in any order.

At the conclusion of the unit it will be important to spend some time reviewing the work covered in the various modules and considering the overall implications for the teaching of number in the primary schools. A unit evaluation should also be carried out.

A suggested sequence for the delivery of this unit (3 credit points over 12 weeks) is outlined below.

Enrichment Topics in Number

Week	Activity
1	Introduction to the unit, including discussion of the work to be covered in each module, assessment tasks, study expectations.
2-3	Module 3.1 Alternative Systems of Numeration and Computations
4-5	Module 3.2 Factors, Primes and Composites
6-7	Module 3.3 Number Sequences and Patterns
8-9	Module 3.4 Basic Statistics
10- 11	Module 3.5 Using the Calculator and/or Module 3.6 Clock Modulo Arithmetic
12	Review of the unit, implications for the teaching of Number in the primary school, unit evaluation.

Teaching approaches

The approach recommended to teach this unit is a student centred, activity based approach. Lecturers are encouraged to build upon and respect students' different experiences, and to provide a range of purposeful and challenging activities. A supportive learning environment should be established, encouraging students to take risks and to learn from one another.

The skills and understanding developed during ***'Unit 1 'Problem Solving and Investigations'*** should be reinforced during this unit. Students should be provided with an opportunity to solve problems, complete investigations, work co-operatively and to discuss their mathematical thinking.

While exploring number concepts students will engage in a range of mathematical activities. These activities will further develop students' own mathematical understandings as well as their ability to teach number concepts in a primary school context.

Suggested strategies to use in the delivery of this unit include:

- discussions, small group and whole class, open and structured, between student and teacher and among the students themselves
- seminar presentations
- research and investigation activities
- co-operative group learning
- demonstrations
- peer teaching
- micro-teaching.

Although the unit consists of a number of different modules lecturers are encouraged to adopt a holistic approach to their teaching. Connections and relationships between the concepts developed in the various modules need to be established and the understandings developed in early modules built upon throughout the unit.

When teaching selected activities from this material, lecturers will be modelling appropriate strategies that can easily be adapted to the primary school context. Class activities, followed by opportunities for group and individual work, the recording of mathematical ideas, the displaying of student work, the use of class discussions, with an emphasis on the process rather than the product, and establishing a classroom environment conducive to learning is the approach recommended in the teaching of this unit. These approaches need to be made explicit to students and consideration given to their effectiveness in the teaching of mathematics.

Inclusive curriculum

In the delivery of this unit it is expected that every person will be provided with the opportunity to participate in, and contribute to, activities without fear or favour. Activities should be presented to cater for a range of abilities and should be gender inclusive.

When developing a gender inclusive program lecturers will need to consider:

- **Language**. The language we use shapes and represents the way we think, therefore the language we use needs to be inclusive. It is important to use language that includes woman and to avoid generic terms such as ‘man’.
- **Access and participation**. It is necessary to ensure that both male and female students receive an equal share of the lecturer’s time and attention. Both male and female students need to be treated equally and given the same opportunities to participate in discussions, ask questions and contribute to the classroom conversations. Both female and male students should have equal access to classroom resources.
- **Teaching strategies**. Examples used in the teaching of the unit need to include both female students and male students. Also students learn in different ways so a range of different teaching strategies and assessment tasks need to be developed to accommodate these differences.

When students are considering the teaching of number in the Primary School context, attention should be given to catering for all children, including those with special needs. Students will need to be encouraged to focus on what children with a range of special needs can do and consider how activities can be adapted to cater for these children.

Activities planned by students to teach in the primary classroom will also need to be inclusive, presenting positive and non-stereotypical representations of people.

Language issues

Language factors contribute significantly to children’s mathematical learning and mathematics teachers have an important role to play in assisting students to acquire the specialised language of mathematics. Teachers need to establish the connections between the everyday concepts, the everyday language, and the formal language, skills, and symbols of mathematics. Teachers

also need to be aware of the language and cultural diversity of children, and how this will impact on the teaching and learning of mathematics.

When teaching this unit, lecturers will need to raise students' awareness of these issues through providing opportunities for students to explore different cultural perspectives and express their mathematical ideas in a variety of ways. Opportunities should be provided for students to use everyday language, vernacular, Tok Pisin, English, symbols, graphs, charts, written and oral texts when sharing their mathematical understandings.

When students are planning for the teaching of number in the Primary School context, they will be required to consider how they can support children in developing appropriate language to discuss the mathematical ideas. Particular consideration will be given to developing strategies to supporting children who speak English as a second language, and who are in the process of bridging from the vernacular to English.

Multigrade teaching

In implementing this unit, lecturers will need to consider how they can cater for the range of student ability levels. By providing opportunities for group work, presenting activities at a range of levels and allowing students to select from these, strategies suitable for use in a multi-grade setting will be modelled. When presenting this unit and deciding how to implement number activities into the primary school context, consideration will be given to modelling strategies suitable for use within a multigrade classroom.

Assessment activities

Assessment is the process of identifying, gathering and interpreting information about student learning. The main purpose of assessment is to improve student learning and the quality of the learning programs. Assessment should be undertaken at the beginning of the unit (diagnostic), during the unit (formative) and at the end of the unit (summative).

A variety of assessment strategies should be used and students should be given opportunities, in a variety of contexts, to demonstrate in an authentic manner, what they know, understand and can do. The assessment strategies used need to be sensitive to the diversity that exists amongst students and take into consideration gender, culture, and language differences.

The content that is being covered, the learning objectives being assessed and the style of teaching and learning being used, will influence the method of assessment used. When developing assessment tasks lecturers will need to ensure that:

- the requirements of the task are set out clearly
- the assessment tasks chosen are relevant to the objectives and allow students to demonstrate appropriate outcomes
- marks or grades reflect the relative importance of each part of the task
- the language used is familiar to students and ideas clearly expressed
- items are not too difficult or too easy
- it does not contain bias
- a marking scheme is developed and applied consistently

The number of assessment tasks for the unit will be determined by college policy. Suggested assessment tasks have been included for each module and lecturers will need to decide on which assessment tasks they will develop, taking into consideration the learning objectives for the entire unit.

Suggested assessment strategies for this unit include:

- oral presentation e.g. seminars, tutorials,
- project work
- investigations
- peer and micro teaching
- report writing – with a focus on inquiry, analysis and reflection
- examination.

Practicum suggestions

When studying this unit students should be provided with opportunities to:

- observe teachers teaching number activities
- practice teaching number activities
- critically reflect on these experiences.

The following is a list of suggestions as to how this unit may be incorporated into Practicum activities such as school experience, demonstration lessons and block teaching. These ideas would need to be negotiated with the Professional Development strand.

Demonstration lessons

Students observe teachers presenting number activities to children. During these observations students can keep a record of:

- what the teacher is doing
- what the children are doing
- the type of questions asked
- how the children and the classroom are organised (group or individual work, learning centres)
- what the children are learning
- what language is being used
- what difficulties the children are experiencing
- what concrete materials are being used
- how the children are being assessed.

Following these observations conduct a class discussion critically reflecting on student findings. The discussions could incorporate topics such as:

- the prior understandings children needed to participate in the activity
- identification of the new learning that took place

- the difficulties children experienced and how these could be overcome
- the strategies adopted by the teacher to develop children's understandings and to support the development of mathematical language
- how you could assess the children's learning
- how you would follow up this lesson
- critical reflection on the effectiveness of the lesson and recommendations.

School experience

Involve students in microteaching, working with a small group of children over a number of weeks. Students can work with children and plan, teach and evaluate a sequence of number lessons using a variety of different strategies. Students can try out ideas such as:

- teaching in context
- using concrete materials
- developing children's mathematical language
- recording mathematical thinking in different ways
- supporting children bridging from the vernacular to English when learning mathematics.

Have students observe a number of different teachers from across the primary school, teaching number lessons. Students can write a journal reflecting on what they have learnt about the teaching of number from these observations.

At the end of this period spend time critically reflecting on the experience, sharing what has been learnt and making recommendations for future teaching.

Block teaching

Students can:

- plan a series of number activities
- teach these activities
- evaluate their teaching.

On the completion of block teaching and when students return to the mathematics class, follow up activities should be planned to:

- share successful experiences and identify the reasons why these experiences where successful (good planning, strong understanding of the content area to be taught, use of concrete materials, concepts taught in context etc)
- discuss problems experienced, the reasons why these occurred and possible solutions
- identify the areas where students require additional support and assistance
- make recommendations for future teaching experiences.

Unit evaluation and reflection

On the completion of the unit an evaluation should be compiled. This should include input from both staff and students reflecting on the teaching and learning that took place during the unit. The information collated during the evaluation process should inform the review and ongoing development of the unit.

Below is an example of focus questions a lecturer may use to review the unit. A student evaluation form is also included as well as information on how the data gathered can be analysised.

Focus questions for lecturer reflection

To determine the effectiveness of the practices and methodologies employed and the content covered in a unit of work, lecturers need to reflect on their teaching. When reflecting on our teaching the areas we can consider are:

- the content of the unit
- the methodologies used in delivering the unit
- the assessment activities
- the co-ordination of the unit.

To help us reflect on our teaching we can ask ourselves a number of questions about each of these areas.

Content of the unit

- Did the content support the objectives of the unit?
- Were the activities sequenced logically?
- Was the content relevant? Did the content help the students to become competent beginning primary school teachers?
- Do you think the students are now more confident to teach this subject in the primary school?
- What recommendations can you make?

Methodology

- How did you deliver the content to the students? Were these strategies effective?
- Were the students aware of the strategies you were modelling and how they could use these strategies in their own teaching?

Assessment of the Unit

- How clear were the assessment tasks?
- How many tasks were given to students? Was this sufficient/too few or too many?
- Did you give students enough time to complete each assessment task?
- Do the students' assessment results display what you expected of the course?
- What are your recommendations?

Unit Co-ordination

- How well did you co-ordinate this unit?
- Did you produce any materials for students? Were these appropriate?
- Did you communicate effectively with other lecturers who were involved in teaching the same unit?

After considering each of these questions we can then make recommendations about the future of this unit.

Focus questions for student reflection

Below is a list of focus questions which could be used to stimulate student discussion when evaluating the unit.

- What have you learnt from this mathematics unit this semester?
- What have been the highlights/strengthens of this unit?
- What problems have you encountered with this unit?
- Has this unit helped prepare you to be a beginning primary school teacher? If so, how? If not, why do you think this?
- What comments can you make about the level of work covered in this unit?
- What recommendations can you make to improve this unit?

Student unit evaluations

Unit: _____ Class: _____

Instructions: Put an 'X' in the appropriate box.

Indicators	Strongly Agree (5)	Agree (4)	Not sure (3)	Disagree (2)	Strongly Disagree (1)
1. The objectives of the unit were clearly outlined.					
2. The unit content was clearly related to the objectives.					
3. The student support material helped my understanding of the unit.					
4. The library was able to provide me with additional references.					
5. The assignments were related to the unit objectives.					
6. The instructions to do the assignments were clear.					
7. The assignments were scheduled to allow enough time for preparation.					
8. I obtained useful feedback on my assignments.					
9. Assignments were returned in time to help me with this unit.					
10. Teaching staff were available for consultation.					
11. There were sufficient opportunities to discuss the unit content in class.					
12. Demonstrations and practical activities were useful to my learning.					
13. I have improved in my ability to talk and write about this unit.					
14. I have improved my knowledge and skills in this unit area.					
15. The overall quality of teaching was good					
16. The physical facilities (rooms, labs, equipment) were adequate for the unit					
17. This unit was challenging and at an appropriate level.					
18. I have developed my co-operative learning skills during this unit.					
19. All students (male & female) were provided with an equal opportunity to participate in all activities.					
20. I would recommend this unit to other students.					

Student Unit Evaluation – Notes

The Student Unit Evaluation seeks to determine how students perceive the quality of a Unit though various indicators, objectives, texts, facilities, assignments, and teaching. It is important to note that there is a difference between **Unit quality** and **students' perceptions of Unit quality**. What is being determined here are only students' perceptions. Feedback from students is only one pointer which when linked to other forms of review such as lecturer peer review and self-assessment of a Unit, can provide the basis for improving student learning in a Unit.

From a completed Unit evaluation it is possible to compare the different indicators by calculating a **Mean** score for each of them. Each Indicator Mean score is calculated by multiplying the number of students responding for each preference, by the preference value. The preference values are 5 for Strongly Agree; 4 for Agree; 3 for Not Sure; 2 for Disagree; and 1 for Strongly Disagree.

Example of the calculation of Indicator Mean scores

Number of students: 22
The objectives of the unit were clearly outlined

Indicators	**Strongly Agree (5)**	**Agree (4)**	**Not sure (3)**	**Disagree (2)**	**Strongly Disagree (1)**	**Total**	**Mean**
Student responses	4 (4x5=20)	9 (9x4=36)	3 (3x3=9)	5 (5x2=10)	1 (1x1=1)	76	76/22 = **3.45**

The unit content was clearly related to the objectives

Indicators	**Strongly Agree (5)**	**Agree (4)**	**Not sure (3)**	**Disagree (2)**	**Strongly Disagree (1)**	**Total**	**Mean**
Student responses	2 (2x5=10)	4 (4x4=16)	3 (3x3=9)	8 (8x2=16)	5 (5x1=5)	56	56/22 = **2.54**

Making sense of the results

You can see from these examples what students' general perceptions are. While they thought that the objectives of the Unit were clear, they did not think the content of the Unit was well related to its objectives. A lecturer may have to decide whether stronger links between objectives and content are necessary, whether the objectives should be redefined, or whether students should be made more aware of the links that do exist between objectives and content. To make an informed decision about the Unit, a lecturer would probably need to compare this information with the rest of the evaluation, take note of students' comments at the end of the evaluation, do a self-assessment of the Unit and go through the Unit with other lecturers seeking their opinions (Peer Review).

As a guide, Units probably need fine-tuning where a particular Indicator Mean score is much lower than those of other Indicator Mean scores in that Unit; or any Indicator Mean score is below 3.0. It is possible to calculate an overall Unit Mean by adding all the Indicator Mean scores and dividing by the number of Indicators (20). The overall Unit Mean score can be

compared with other Unit Mean scores in the Strand or across Strands to give a picture of which Units students perceive as being of higher or lower quality.

Resources

Blocks, counters, stones, shells, sticks

Butchers paper

Markers

Coloured pencils

Graph paper

Newspapers

Calculators

Primary Mathematics Syllabus documents and support materials

References

Brinkworth, P. and Scott, P. (1994). *'The making of Mathematics – a friendly history'* Australian Association of Mathematics Teachers Inc.

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Module 3.1 – Alternative Systems of Numeration and Computations

Title page of Calandri's Arithmetic, 1491

Galley Division 16th century

Module 3.1 Alternative Systems of Numeration and Computations is core module in the unit ***'Enrichment Topics in Number'***. During this module students will develop an understanding of different systems of numeration. Students will investigate PNG counting systems as well as alternative systems from other parts of the world. Early computational algorithms will also be explored. By studying this module it is expected that students will gain a better appreciation of the Hindu-Arabic system currently taught in PNG schools.

Objectives

By the end of this module students will be able to:

- differentiate between additive, multiplicative and place value systems of numeration
- explain the characteristics of the Hindu-Arabic system
- compare different computational methods.

Concepts and skills

- Place value
- Addition
- Multiplication
- Computational skills
- Base systems
- Numbers

Topics

- Hindu Arabic System
- PNG Counting Systems
- Ancient Systems of Numeration
- Alternative Systems of Computations

Suggested teaching strategies

- Research
- Investigations
- Production of materials
- Guest speaker
- Seminar presentation
- Written report
- Interview
- Presentations
- Class display
- Timelines
- Discussions
- Debates
- Survey (what counting systems are represented by students/staff at the college)
- Data base (counting systems represented by college students)

Suggested assessment tasks

- Development of a chart on a traditional counting system and a written analysis of this.
- Written report comparing different counting systems and different ways of representing numbers.
- Production of teaching materials to support the teaching of number.
- Write a song/poem to use with children to teach number concepts.
- Seminar presentation on alternative methods of carrying out number operation.
- Seminar presentation on an alternative numeration system.
- Prepare a chart summarising alternative numeration systems. The chart must include an additive, multiplicative and place value system.
- Use different computational methods to calculate algorithms.

Resources

Butchers paper
Markers

References

Brinkworth, P. and Scott, P. (1994). '*The making of Mathematics – a friendly history'* Australian Association of Mathematics Teachers Inc.

Burnett, James (1999). *Sights, Sounds and Symbols – Classroom Activities on the History of Numbers*, Prime Education, Queensland Australia

Green, Wally (1998). *Enrichment Topics in Number*. Department of Education, PNG

A suggested sequence of activities

Introduction

Brainstorming

Establish student's understanding of the Hindu-Arabic number system currently in use by asking them to describe the features of this system. Consider the following questions:

- What are the characteristics of the system?
- What symbols are used?
- Where did this system come from?
- What is its base?
- What is the role of zero?

Topic 1- The Hindu-Arabic System

Student reading discussion

Refer students to the reading 'The Hindu-Arabic System' in the *Student Support Material.* After reading the article discuss with students the features of the Hindu-Arabic System and the significance of these features *(3.1 Activity 1 Student Support Material).*

Revising the Real Number System

Discuss with students that the Hindu Arabic System is the base for the Real Number System. Revise with students the real number system, defining integers, natural numbers, rational numbers and irrational numbers.

Integers (I): These are whole numbers, including positive and negative numbers as well as zero.

I = {….-4, -3, -2, -1, 0. 1, 2, 3, 4, …}

Natural numbers (N): These are the counting numbers, and include all the positive integers and exclude zero

N = {1, 2, 3, 4, ….}

Rational Numbers: A rational number can be expressed in the form of a/b, where a and b are integers and b ?0

Irrational Numbers: These are numbers which cannot be written in the form of a/b. For example v2, v7, π

Topic 2- PNG Counting Systems

Guest speaker Invite a guest speaker to explain a traditional PNG counting system to the students. Ask students to keep notes during the presentation. Allow the students time to practice using the counting system. Following on from the discussion have students identify the features of the traditional PNG counting system described e.g., symbols/signs used, its base, when and where it is used.

Investigation In small groups have students investigate a PNG counting system. This may be the students own traditional counting system or another one. Students will need to prepare a presentation which explains the counting system *(3.1 Activity 2 Student Support Material).*

Class activity Following the group presentations explaining various traditional PNG counting systems, develop a table which identifies the different features of each system and the context in which it is used. The following table is an example.

Counting System	Province/Region	Signs/Symbol	Base	Contexts in which it is used	Any other comments
Hindu-Arabic	World wide	0,1,2,3,4,5,6,7, 8,9	10	- Business - Education - Science - Everyday life	Uses 0 Has place value

Topic 3 – Ancient Systems of Numeration

Explanation and discussion Present students with an example of an additive, multiplicative and a place value system of numeration e.g. The Ancient Egyptian System, the Chinese- Japanese System and the Babylonian System. These can be found in the *Student Support Material Topic 3 Ancient Systems of Numeration.* Have students work in small groups to explore the three systems and identify their features.

Introduce students to the definitions of an additive, multiplicative and place value system of numeration. Categorize the three examples according to these definitions.

Revisit the PNG counting systems and the Hindu Arabic system. Categorize each system according to whether it is an additive, multiplicative or place value system of numeration.

Small group investigation

Provide students with examples of various ancient numeration systems *(3.1 Activity 3 Student Support Material*) and in small groups have students investigate these systems and categorize them.

Discussion

Conduct a class discussion on what people see as the advantages and disadvantages of additive, multiplicative and place value systems of numeration.

Topic 4 – Alternative Systems of Computations

Computations

Have students complete a series of addition, subtraction, multiplication and division problems. Identify the strategies which people used to solve these problems and share different methods with the class.

Provide students with an alternative method of completing a calculation e.g. the medieval method of subtraction or the Russian Peasant method of multiplication, and have them investigate the reasoning and logic behind these methods.

Alternative methods

Examine *Topic 4 Alternative Systems of Computations* in the *Student Support Material*, which illustrate alternative methods of computations. Working in groups, allow students to explore these methods and complete problems using these methods *(3.1 Activity 4 Student Support Material).*

Class discussion

Consider the various methods of computations and how these methods have been developed by various cultures over time. Are there PNG methods of computation?

Have students identify the methods which they find the most useful and to give their reasons for their choice.

Conclusion

Class discussion

Discuss with the class how mathematics has been socially constructed over time, with different cultural groups developing their own systems of counting and calculating. Have students select one of the following focus questions to complete a reflective journal. On the completion of the journals conduct a class discussion to consider each of the questions.

1. What have we learnt about the Hindu-Arabic numeration system and why do you think it has become so widely used?
2. Why do you think different methods of computation have developed over time? What do you see as the advantages of the current approach to teaching subtractions when compared with the 'borrow-pay-back' method?
3. What place do you feel PNG counting systems have in the education system of PNG and why do you believe this?

Module 3.2 – Factors, Primes and Composites

Module 3.2 Factors, Primes and Composites is a core module within the ***'Enrichment Topics in Number'*** unit. An understanding of prime and composite numbers is essential for all number work, therefore during this module students will have an opportunity to further extend their own understandings of these concepts. The primary school mathematics syllabus will be examined and the teaching of these important concepts will be considered.

Objectives

By the end of this module students will be able to:

- identify prime and composite numbers
- find the factors of a given number
- develop a number investigation suitable for use in the primary school
- apply rules of divisibility.

Concepts and Skills to be developed

- Primes
- Composites
- Factors
- Multiples
- Prime factors
- Lowest common multiple
- Greatest common factor
- Division

Topics

- Factors, Primes and Composite Numbers
- Rules of Divisibility
- Prime Numbers
- Exploring Pythagorean Numbers

Suggested teaching strategies

- Investigations
- Class discussions
- Charts and tables
- Problem solving

Suggested assessment tasks

- Develop an investigation into prime, composite numbers or factors suitable for use in the primary school. Identify the grade and the process you would expect children to work through
- Investigate 'Every even number greater than 2 is the sum of two primes'.

Resources

Range of concrete materials such as blocks, shells, counters, sticks
Butchers paper
Markers

References

Department of Education Papua New Guinea (1998). *Lower Primary Mathematics Syllabus Grade 3-5*

Department of Education Papua New Guinea (draft 1999). *Upper Primary Mathematics Syllabus Grade 6-8*

Green, Wally (1998). *Enrichment Topics in Number*. Department of Education, PNG

Green, Wally (May 1998). *Report on Mathematics Education in Community Teachers' Colleges in PNG, Fourth Phase, Activities Observations and Recommendations*. OHE-ADB-IDP Papua New Guinea Higher Education Project (Asian Development Bank Loan 1224)

A suggested sequence of learning activities

Introduction

Review

Review the Number Strand of the Primary School Mathematics Syllabus and identify the number topics which children are expected to cover from Grade 3 to 8.

Working in small groups identify the basic understandings and concepts that underlie the range of topics within the Number Strand which are developed across the different grade levels. Construct a table identifying the topics covered and the number concepts developed. For example

Grade	Topic	Concepts
3	Addition	Counting, place value,
3	Multiplication	Multiplication, factors

Discuss the importance of children developing a strong understanding of prime numbers, factors and composite numbers, illustrating how these concepts are essential for further developing mathematical understandings.

Topic 1- Factors, Prime and Composite Numbers

Modelling

With students complete the following investigation based on the concepts of factors, prime and composite numbers. This investigation would be suitable to use with lower primary children (Lower Primary Mathematics Syllabus Grade 5)**.**

Investigation

Using a range of concrete materials ask students explore various numbers between 1 and 12. Ask students to draw diagrams and write number sentences to demonstrate what they have discovered. For example: We had 6 blocks. We arranged them into 2 groups of three.

Introduced the students to the use of arrays to illustrate number patterns and concepts. Have students arrange numbers into:

i. equal rows in a square pattern if possible. If not
ii. equal rows in a rectangle pattern. If not
iii. form one row

Explore the numbers between 1 and 20, representing each number using arrays. Display illustrations and, working in small groups, have students note

down what they have noticed from the information they have gathered. What patterns, similarities and relationships can be seen?

Number	Pattern	Factors	Class of Number
10	***** *****	5 x 2	Rectangle
9	*** *** ***	3 x 3	Square
8	**** ****	4 x 2	Rectangle
7	*******	7 x 1	Prime
6	*** ***	3 x 2	Rectangle
5	*****	5 x 1	Prime
4	** **	2 x 2	Square
3	***	3 x 1	Prime
2	**	2 x 1	Prime
1	*	1 x 1	Class by itself

Have students share their findings and introduce the language used in mathematics to describe the concepts and numbers identified, e.g. odd and even numbers, square numbers, factors, prime numbers and composite numbers.

Have students explore the numbers from 13 –25, identifying prime and composite numbers and their factors, odd and even numbers, square numbers. Illustrate each of these numbers and label them.

Finding Factors

Refer students to the ***Student Support Material Topic 1: Factors, Prime and Composite Numbers*** and have students complete ***3.2 Activity 1,3.2 Activity 2, and 3.2 Activity 3***. These activities require students to find factors for different numbers, identify common factors and common multiples for groups of numbers, find the highest common factor and the lowest common multiple.

Investigation

Have students work in pairs to complete one of the investigations set out in the ***Student Support Material 3.2 Activity 4.*** Encourage the different pairs to complete different investigations so all investigations are covered. Allow an opportunity for students to share their findings.

Small group work

Working in groups have students jointly construct an investigation which they could use to further develop children's understandings of prime and composite numbers, or factors. Students should refer to the Primary Mathematics Syllabus and develop their investigation for a specific grade level.

Topic 2 – Rules of Divisibility

Class Discussion

Write a range of numbers on the board and discuss with students which of the numbers listed are:

- divisible by 2
- divisible by 5
- divisible by 10

Ask student to identify how they know if a number is divisible by 2, 5 or 10 without completing the actual calculations. You would expect students to provide the following reasons:

- All even numbers are divisible by 2
- A number is divisible by 5 if it ends in 0 or 5
- A number is divisible by 10 if it ends in 0

Ask students if they can identify if a number is divisible by 3, 4, 6, 7, 8, etc.

Reading

Refer students to the ***Student Support Material Topic 2 – Rules of Divisibility*** and ask people to read through the rules. Discuss the various rules and how they have been derived (see Wally Green *'Topics in Number'* p 10 – 11 for further explanation) and work through an example together.

Testing for Rules of Divisibility

Have students use the rules of divisibility to test a range of numbers for their factors ***(Student Support Material 3.2 Activity 5)***

Extension Activity

Refer students to the extension activity ***Student Support Material 3.2 Extension Activity - A Divisibility Problem.***

Topic 3 – Prime Numbers

Identifying primes

The Greek mathematician Eratosthenes invented a quick way of identifying the prime numbers in a set of counting numbers starting at 1 called 'the sieve of Eratosthenes'. Work through an example of using this method of identifying primes with students ***(Student Support Material 3.2 Activity 6)***.

For example to find all the prime numbers between 1 and 20 using the sieve of Eratosthenes, list down all the numbers and then

1. Cross out 1 because it is not a prime number
2. Go to the next number, which is 2, and circle it. Then cross out all the multiples of 2 (consider why these numbers are not primes)
3. Go to the next number which isn't crossed out. This should be 3. Circle it. Then cross out all the multiples of 3
4. Go to the next number which isn't crossed out. This should be 5. Circle it. Then cross out all the multiples of 5
5. Repeat for the next number which isn't crossed out. Keep repeating until there is no 'next number'

Individual activity

Refer students to the ***Student Support Material 3.2 Activity 7.*** Using the sieve of Eratosthenes have students find:

1. All the primes from 1 to 100
2. All the primes from 1 to 150
3. All the primes from 1 to 200

Have students note the last number that they need to circle which resulted in a number after it being crossed out. Based on their findings have students complete ***3.2 Activity 7 Student Support Material*** requiring them to investigate the following questions

(a) How can you determine how far you need to go before no new numbers will be crossed out?

(b) How far would you need to go to establish if 1003 is a prime? Use the calculator not the sieve to find this out.

Class discussion

Discuss with the class that when finding primes, once you reached the square root of a number you don't need to go any further. If there is a factor less than the square root there cannot be a factor which is larger as this would require the other factor to be less, which has been rules out.

Having established the relationship between finding the square root and identifying if a number is a prime number have students complete ***3.2 Activity 8 in the Student Support Material.*** Discuss findings.

Extension activity

Refer students to the ***Student Support Material Extension Activity Generating Primes*** and ask them to work in small groups collaboratively to complete any of the following investigations

(a) Over 2000 years ago Euclid proved that there were infinitely many primes. Over the centuries mathematicians have been fascinated by prime numbers and have looked for formulae that will:

(i) produce all existing primes

(ii) always result in a prime

By taking values for n (or p) try to determine whether the following formulae are true.

- $n^2 - n + 1$ will always result in a prime for n=1,2,3,4,……(Euler)
- $2^p - 1$ will always result in a prime for p = a prime number (Mersenner)

(b) Investigate the following conjecture about primes

'Every odd number greater than 5 can be expressed as the sum of three primes' e.g. $7 = 2 + 2 + 3$

(c) Investigate the following conjecture about primes

'Every odd number greater than 3 can be expressed as the sum of a prime and some power of 2'

e.g. to check 59

$59 - 2 = 57$ (not prime)
$59 - 2^2 = 55$ (not prime)
$59 - 2^3 = 51$ (not prime)
$59 - 2^4 = 43$ (prime)
Hence $59 = 43 = 2^4$

Topic 4 – Exploring Pythagorean Numbers

Proper factors

Greek mathematicians had a strong interest in numbers and attached 'mythical' significance to some numbers. The Pythagoreans classified numbers according to the sum of their proper factors. Have students carry out an investigate of the sum of the proper factors of a range of numbers. Discuss findings. Identify abundant, deficient, perfect and amicable (friendly) numbers ***(Student Support Material 3.2 Activity 9)***.

Extension activity

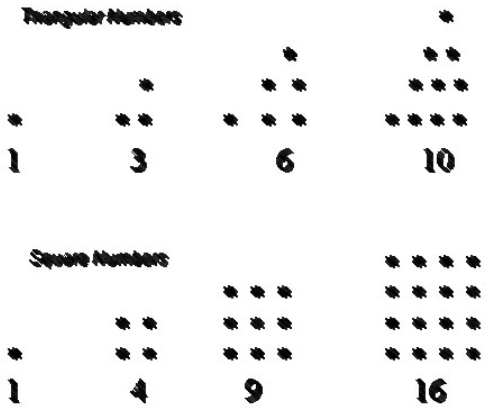
Have students complete the extension activity, requiring them to investigate odd abundant numbers ***(Student Support Material Extension Activity)***.

Conclusion

Class discussion

Revise terms and understandings developed during the module.

Module 3.3 – Number Sequences and Patterns



Module 3.3 Number Sequences and Patterns is core module in the unit ***'Enrichment Topics in Number'***. During this module students will explore different number sequences and patterns, identifying general rules to describe these, as well as creating their own patterns and sequences. Students will also investigate number sequences derived from geometric figures. It is important that students have a sound understanding of patterns and sequences as this provides the foundation for the development of further mathematical understandings.

Objectives

By the completion of this module students will be able to:

- extend given sequences
- develop a sequence of numbers based on a given rule
- find a general rules that describes the 'nth' term of a given sequence
- explore figurative numbers
- investigate difference patterns of sequences and describe their findings
- investigate the use of sequences to help generalise geometric information.

Concepts and skills

- Sequence, term and general term of sequence
- Rules to determine sequence
- Difference patterns in sequences
- Numbers described by geometric properties
- Sequences in geometry
- Computation
- Investigation
- Oral presentation

Topics

- Number sequences
- Determining differences
- Sequences in geometry

Suggested teaching strategies

- Investigations
- Seminar presentation
- Class display
- Discussions
- Questioning
- Group work

Suggested assessment tasks

- Seminar presentation reporting on an investigation carried out to identify number patterns or a geometric sequence
- Produce an item which demonstrates a pattern or sequence e.g. string art
- Use the Primary School Mathematics Syllabus documents and support materials to plan a unit of work which teaches patterns and sequences concepts (e.g. Lower Primary - Patterns and Functions, Upper Primary - Algebra)

Resources

Concrete materials such as blocks, shells, stones
Lower and Upper Primary Mathematics Syllabus documents

References

Department of Education Papua New Guinea (1998). *Lower Primary Mathematics Syllabus Grade 3-5*

Department of Education Papua New Guinea (draft 1999). *Upper Primary Mathematics Syllabus Grade 6-8*

Green, Wally (1998). *Enrichment Topics in Number*. Department of Education, PNG

A suggested sequence of learning activities

Introduction

Investigation Have students investigation a range of different number patterns to find the next three terms.

e.g. 1, -2, 4, -8, 16, _, _, _

Discuss results and the strategies people used to identify the number pattern.

Revision Revise definitions of key words such as sequence, term, nth term, used in describing number patterns and sequences. Definitions can be found in the Unit Glossary

Topic 1 – Number Sequences

Modelling As a class work through an example of determining a rule for a sequence of numbers, finding the nth term. ***(Student Support Material Topic 1 Number Sequences)***

Small group work Working in pairs have students investigate number patterns to identify the nth term. Present findings and have students discuss the strategies they used to reach a solution. ***(Student Support 3.3 Activity 1)***

Problem solving Present problems for students which require students to identify patterns and sequences. ***(Student Support Material 3.3 Activity 2)***

Topic 2 – Sequences in Geometry

Construction Have students construct figurative numbers using concrete materials e.g. triangular numbers, square numbers, or pentagonal numbers. Working in pairs, have students choose one figurative shape and investigate the pattern. Ask students to draw diagrams and tables to illustrate their findings and work out the general rule for the nth term. Present findings and discuss as a class. ***(Student Support Material 3.3 Activity 3)***

Topic 3 – Determining Differences

Guided discovery Working together, investigate with students finding the first and second difference of a number pattern ***(Student Support Material Topic 3 – Determining Differences)***

Investigation Working in small groups have students investigate difference in number patterns and discuss their findings. ***(Student Support Material 3.3 Activity 5, 3.3 Activity 6)***

Extension activity – An Investigation

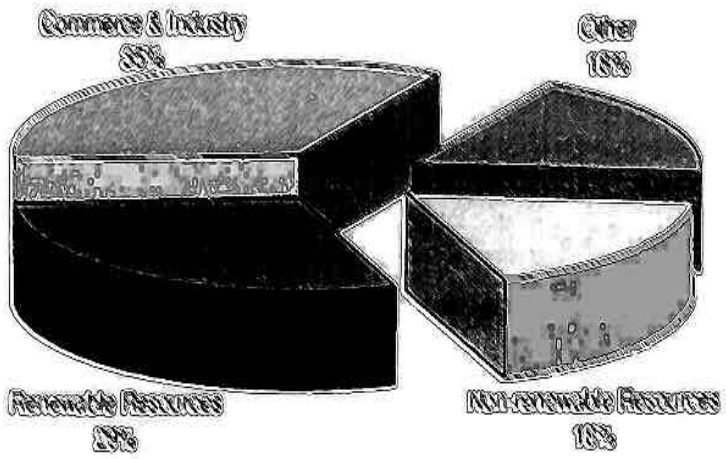
Investigation Refer students to the ***Student Support Material 3.3 Activity 7***. Working in pairs have students choose an investigation to complete. Discuss findings together as a class.

Conclusion

Class discussion Consider the place of patterns and sequences in mathematics. Where are patterns and sequences used and why is it important to develop a sound understanding of these concepts?

Implications for teaching Examine the Primary Mathematics Curriculum documents to see how concepts of patterns and sequences are covered across the different grades. Plan a sequence of activities for a particular grade level which develops children's understanding of patterns and sequences. Peer or micro teach these activities.

Module 3.4 – Basic Statistics



Module 3.4 Basic Statistics is a recommended module in the unit ***'Enrichment Topics in Number'***. Statistics provide us with a range of information and basic facts on a variety of topics which we need to make judgements and decisions about. During this module students will examine the use of statistics and their purpose. Data will be collected, interpreted and represented in a variety of ways. Students will also consider how statistical information can be misleading.

Objectives

By the completion of this module students will be able to:

- collect statistical information
- interpret data
- construct graphs to represent data
- calculate measures of central tendency
- calculate the range.

Concepts and skills

- Collecting data
- Representing data in various ways e.g. graphs, tables
- Selecting appropriate methods of analysis
- Interpreting
- Oral skills
- Writing skills

Topics

- Using statistics
- Collecting data
- Representing data
- Interpreting data
- Analysing results

Suggested teaching strategies

- Research
- Investigations
- Discussions
- Questioning
- Group work

Suggested assessment tasks

- Provide students with statistical data. Students need to write an interpretation of the results and identify the issues which would need to be considered when analysing the data.
- Present statistical data to students. Students need to represent the data in a variety of different ways e.g. pie graph, bar graph, table.

Resources

Newspapers
Graph paper
Butcher paper or chart paper
Markers
Lower and Upper Primary Syllabus documents

References

Department of Education Papua New Guinea (1998). *Lower Primary Mathematics Syllabus Grade 3-5*

Department of Education Papua New Guinea (draft 1999). *Upper Primary Mathematics Syllabus Grade 6-8*

Green, Wally (1998). *Enrichment Topics in Number*. Department of Education, PNG

Government of PNG, (1999). *Papua New Guinea Human Development Report* 1998

Zbar, Vic. Cropley, Murray and Rowland, Mike (nd). *Family of Numbers*, Curriculum Co-operation

A suggested sequence of learning activities

Introduction

Newspaper search

Have students look through newspapers and magazines and collect examples of statistical information. Make a collage that illustrates the variety of statistical information collected. As a class discuss:

- the type of statistical information gathered
- ways in which statistical information is represented
- how statistical information can be interpreted
- the purpose of the statistical information.

Reading

Ask students to read article 'Using Statistics' in the ***Student Support Materia*l.** From the reading have students identify how statistical information can be used ***(Student Support material 3.4 Activity 1)***

Case study

Present a case study to students, which provides statistical data based on a PNG context. An example 'Gender Analysis' can be found in the ***Student Support Material***. The book *'Papua New Guinea Human Development Report 1998'* provides many examples of statistical data across a range of topics.

After reading through the case study have students identify:

- the range of information gathered
- the source of the information
- how the data is represented
- how the data is interpreted
- the purpose of the information

(Student Support Material 3.4 Activity 2)

Topic 1 – Collecting Data

Brainstorm

As a class identify a variety of different ways that statistical information can be gathered.

Decide on a topic of interest to students which you can collect statistical data about. Discuss the range of statistical information that could be collated to present the relevant data. Also consider the purpose for collecting this information (to describe current situation, predict what may happen in the future, plan appropriate action, evaluate current situation and recommend changes).

For example, students could develop a college profile for the purpose of evaluating the current situation and recommending changes. The following data could be collected:

- student information (male, female, province, first language, age, etc)
- staff information (male, female, secondary/primary teaching background, academic/ancillary, qualifications, marital status, province etc)
- infrastructure (number of buildings – classroom/dorms, maintenance, etc)
- college activities (sporting activities, lecturers, community work, student and staff functions etc)
- communication (access and use of radio, TV, newspapers, phones, letters, faxes, internet etc)
- transport (vehicles available, staff/student use, etc)

Identify how data relating to each of these areas could best be collected. Working in groups have students collect the necessary data ***(Student Support Material 3.4 Activity 3).***

Topic 2 – Representing Data

Graphs and tables

Present a range of examples ***(Student Support Material Topic 2 – Representing Data)*** which illustrate how statistical data can be represented. For example, different types of tables, charts, graphs, written statements.

Using the data students have collected from ***3.4 Activity 3*** discuss appropriate ways in which the information can best be represented. Discuss the reasons for people's preferences.

Ask each group to represent their data. Encourage the use of a variety of representations ***(Student Support Material 3.4 Activity 4).***

Refer students to the ***Student Support Material – Measures of Central Tendency*** and review the material. Using the data collated by students find measures of central tendency (mean, mode, median) and the range. Consider what these measurements tell us about the data and which measures are considered the most appropriate in particular contexts. ***(Student Support Material 3.4 Activity 5)***

Topic 3 – Interpreting Data

Reading

Refer student to the ***Student Support Material Topic 3 – Interpreting Data***. Read through the interpretation of the data collated relating to the number of schools and colleges in PNG. Discuss the data collected and the conclusions made in the interpretation.

Group writing

Working in groups have students write an interpretation of the data they have collected and represented in ***3.4 Activity 3 and 3.4 Activity 4.*** The interpretation should include:

- an explanation of the results
- conclusions which can be made
- a statement of how this information can be used e.g. to evaluate the current situation and recommend changes.

(Student Support Material 3.4 Activity 6)

Topic 4 – Analysing Statistical Data

Reading

Refer students to the article 'Abusing Statistics' in the ***Student Support Material*** and have people read it.

Discussion

Re- read the interpretation of the data relating to the number of schools and colleges in PNG. As a class consider what you would need to take into account when analysing this information to ensure it provides a 'true picture'. Issues that students should raise are:

- the sample (size, gender, socio-economic background, religion etc)
- the questions (clear, misleading)
- the answers (clear, easy to interpret, misleading)

Consider how valid people feel this data is and the reasons for these opinions. ***(Student Support Material 3.4 Activity 7)***

Have students review the statistical information they have collated and reconsider in terms of the above discussion. ***(Student Support Material 3.4 Activity 8)***

Conclusion

Review

Identify the process that has been worked through in this module to develop an understanding of statistics (e.g. examine examples of statistical data, collect data, collate it and represent it, interpret and analyse).

Teaching statistics

Review the Primary School Mathematics Syllabus and consider the statistics topics that are taught at this level. Working in small groups have students plan a sequence of lessons for a particular grade which would teach the specified statistics objectives, e.g. Grade 7: Compare mean, median and mode and see how they vary.

Module 3.5 – Using Calculators in Primary Schools

Module 3.5 Using Calculators in Primary Schools is a recommended module within the ***'Enrichment Topics in Number'*** unit. Students of all ages generally enjoy using calculators and the ability to competently use a calculator is an essential and expected skill in many workplaces. Calculators allow students the opportunity to work with large numbers which are generally found when exploring real world data, and support the teaching of computation skills for children with learning disabilities. During this module students will use calculators to explore numbers, as well as consider how calculators can be used effectively in primary schools.

Opportunities for students to develop their skills in using the calculator can be integrated across all modules within this unit.

Objectives

By the end of this module students will be able to:

- use a calculator to solve a range of problems
- identify the advantages and disadvantages associated with the use of calculators in primary schools
- identify the skills children need to be able to use a calculator effectively.

Concepts and skills to be developed

- Number properties (communicative, associative, and distributive)
- Recurring and terminating decimals
- Negative numbers
- Estimation skills
- Rounding numbers
- Basic number facts
- Writing skills

- Oral skills

Topics

- Using the calculator
- Skills needed to effectively use a calculator
- Calculators in primary schools

Suggested teaching strategies

- Debate
- Class discussions
- Problem solving

Suggested assessment tasks

- Write an argument for or against the use of calculators in primary schools
- Write an essay discussing the skills people need to be able to use a calculator effectively

Resources

Calculators

References

Green, Wally (May 1998). *Report on mathematics education in community teachers' colleges in PNG*, Fourth Phase, Activities Observations and Recommendations. OHE-ADB-IDP Papua New Guinea Higher Education Project (Asian Development Bank Loan 1224).

Marr, Beth and Helme, Sue (1991). *Calculators and adult numeracy*, in Breaking the Maths Barrier.

Swan, Paul and Sparrow, Len (1999). *Calculators in the primary school*, Challenging the Myth in Australian Primary Mathematics Classroom, Vol. 3 no. 4.

A suggested sequence of learning activities

Introduction

Language issues

For the following activities each student will need access to a calculator. Allow students an opportunity to explore the calculator by presenting a range of problems to be solved which require the use of the different symbols and functions of the calculator.

Develop a class chart which identifies all the different symbols on the calculator and language we use to describe their function. Record the different words people use to describe the same function. For example,

Symbol	Language to describe function
+	Add, plus,
-	Minus, take away, subtract
.	Point, decimal point
=	Equals, makes, altogether
0	Zero, nought

Emphasis with students the importance of teaching children the language used to describe the various symbols and provide examples of where the same operation is required but requested in different ways,

e.g. 24 subtract 12 gives you?

24 take away 12 equals?

Topic 1 – Using the calculator

Number properties

Refer students to the ***Student Support Material Topic 1 Using the Calculator***. Revise number properties with students and the order in which we do calculations (BODMAS rule) Ask students to complete ***Student Support Material 3.5 Activity 1***.

Devise calculator problems which provide students with an opportunity to explore number properties (e.g. communicative, associative, distributive), place value and recurring decimals.
For example,

Enter 26795 into the calculator. Now by adding one number change the 6 to a 7.

Using only the '3' key and any of the four operation keys find as many different ways you can to make the calculator display 36.

Find the square root of a number without using the square root key.

Choose a 3-digit number; multiply it by a two-digit number many times. What happens? Start with the same 3-digit number and divide it by the two-digit number many times. What happens?

For example,

365 x 24 x 24 x 24 x 24 x 24 x24 x 24 x 24 x 24 x 24 x etc

365 ÷ 24 ÷24 ÷ 24 ÷ 24 ÷24 ÷ 24÷ 24 ÷24 ÷ 24÷ 24 ÷ etc

Topic 2 – Skills for Effective Use of the Calculator

Group activity

Using the calculator have students work in pairs to solve a variety of different problems. These problems should range in complexity, include word problems and highlight a number of different aspects associated with the use of calculators e.g. recurring decimal (65 divided by 3), zeroes dropped off (K3.60 becomes K3.6 on the calculator).

Discussion

After solving the problems have students identify the skills and knowledge they needed to be able to calculate their answers. The following ideas should be discussed:

- An understanding of the value of numbers (for example in the case of adding K1.37 and 13 toea, you need to enter 13 toea as 0.13 not simply as 13)
- Making sense of the answer (K1.37 plus 0.13 will give you K1.50 but calculators drop off right hand zeroes so the display will be K1.5. You need to be able to interpret the answer correctly)
- Estimation skills (is the answer shown on the calculator acceptable, e.g. 2.5 metres of material at K7.95 per metre should give you an answer somewhere between 2 x K8 (K16) and 3 x K8 (K24) so around K20. If the answer is significantly different an error may have been made in pressing the keys 2.5 x 79.5 instead of 2.5 x 7.95)
- Rounding skills (in the case of recurring decimals)
- Basic number facts (able to do mental calculations to support findings)
- Accuracy in entering data
- Selecting the appropriate operation (is the problem requiring you to multiply or divide?)

Reading

Refer students to the ***Student Support Material Topic 2 - Skills for the Effective Use of the Calculator***. Have students read the article 'Using Calculators Effectively'. Discuss the main points raised by the article. Have students develop a series of calculator problems suitable for their peers which

demonstrates the importance of the skills referred to in the reading ***(Student Support Material 3.5 Activity 2)***.

Have students exchange calculator problems and solve them. Discuss what knowledge and skills were needed to solve the problems.

Topic 3 – Calculators in Primary Schools

Discussion Pose the question to students 'Should calculators be used in primary schools? Why? Why not?' and have students discuss this in small groups. The PIN (Positive, Interesting, Negative) discussion strategy could be used (e.g. taking in turns each person makes a comment, the first person must make a positive comment the second person must say something interesting about the topic and the next person needs to make a negative statement. Continue in this sequence).

Reading Ask students to read the article 'Calculators in Primary Schools – Challenging the Myths' found in the ***Student Support Material*** and complete ***3.5 Activity 3***.

Debate Conduct a class debate, with sides for and against the use of calculators in primary school.

Extension activity – Research

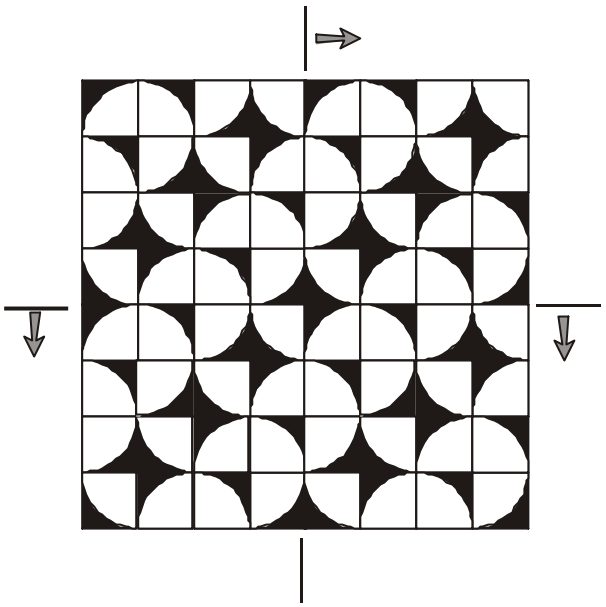
Group Activity Working in small groups have students carry out research into the use of calculators. Activities could include

- Conducting a survey to establish the use of calculators in primary schools.
- Conduct a survey to find out teacher's, children's and parent's attitudes towards the use of calculators in primary schools

Conclusion

Syllabus studies Review the Primary School Mathematics syllabus and identify opportunities where it may be beneficial for children to use calculators.

Module 3.6 – Clock Modulo Arithmetic



Module 3.6 Clock Modulo Arithmetic is a recommended module within the ***'Enrichment Topics in Number'*** unit. During this module students will develop their number sense and their ability to think logically. Students will have the opportunity to solve number problems, explore the relationship between operations and consider various ways in which numbers can be represented.

Objectives

By the end of this module students will be able to:

- explore the relationship between the four operations (addition, subtraction, multiplication and division) when solving a range of problems based around different modulo, e.g. modulo-7, -12, -24
- create patterns and designs using modulo numbers.

Concepts and skills to be developed

- Addition, subtraction, multiplication and division
- Relationship between operations, e.g. division is the inverse of multiplication
- Problem solving
- Interpreting
- Logical thinking
- Bases
- Euclidean transformation (translate, reflection or rotation)
- Rules of divisibility

Topics

- PNG Counting Systems with different bases
- Number lines
- Clocks (12hr, 24hr)
- Days of the week (modulo-7)
- Rules of divisibility

Suggested teaching strategies

- Investigations
- Class discussions
- Problem solving
- Create designs using modulo art

Suggested assessment tasks

Construct a design based on modulo art.

Resources

Butchers paper
Markers
Graph paper
Coloured pencils

References

Green, Wally (1998). *Enrichment Topics in Number*. Department of Education, PNG

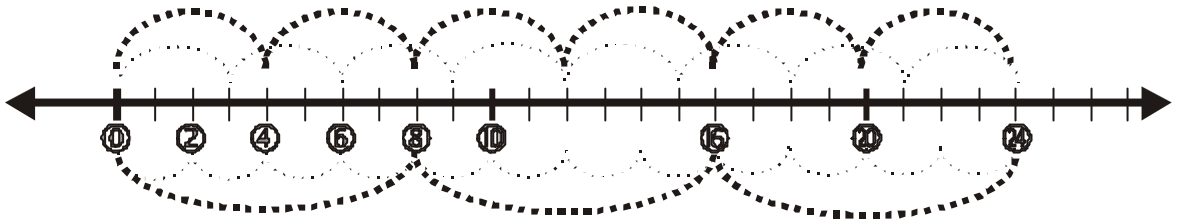
Green, Wally (May 1998). *Report on Mathematics Education in Community Teachers' Colleges in PNG*, Fourth Phase, Activities Observations and Recommendations. OHE-ADB-IDP Papua New Guinea Higher Education Project (Asian Development Bank Loan 1224)

A suggested sequence of learning activities

Introduction

Class activity

Construct number lines. Use different numbers and the four operations to create as many different 'sums' as possible, e.g. using 24, 6, 4, =, +, -, x, ÷. Represent them on a number line. Write a statement describing the number line? What can you say about the relationship between the operations?



For example,

(a) 4 + 4 + 4 + 4 + 4 + 4 = 24

6 x 4 = 24

multiplication is the same as repeated addition

(b) 8 x 3= 24 24 ÷ 3 = 8

These operations are the inverse of one another

Working in pairs

Have students create problems to present to their peers which use the number line to explore the relationship between the four operations (addition, subtraction, multiplication, division)

e.g. 72, 8, 9, =, +, -, x, ÷

28, 7, 4, =, +, -, x, ÷

Topic 1 – Clock Arithmetic

Problem Solving

Present students with problems to investigated which are based around clock arithmetic (modulo-12). For example,

5+3=8, 10+4=2 8+8=4 5-6=11

Present a clock face to the class to demonstrate how these mathematical statements make sense. For example:

It's 10 o'clock now and in 4 hours time it will be 2 o'clock.

It's 5 o'clock now, 6 hours ago it was 11 o'clock.

Solve a range of problems (+, -, x,) involving clock arithmetic modulo-7 ***(Student Support Material 3.6 Activity 1)***.

Working in pairs

Investigate the relationship between addition, subtraction and multiplication facts when represented on a number line with numbers represented around a clock. Have different groups of students investigate the different operations and construct tables to show these basic number facts. What similarities and differences can be found between the two tables? Present findings to the class (***Student Support Material Book 3.6 Activity 2)***.

Class discussion

Refer students to the ***Student Support Material 3.6 Activity 3***. Discuss the concepts of closure, identity (or neutral) element, inverse element and communitive property in respect of the relationships between the four operations.

Topic 2 – Modulo Art

Class Activity

Refer Students to the ***Student Support Material Topic 2 – Modulo Art.*** Work through the activity as a class which requires you to:

- Construct a table for addition modulo-4.
- Draw a pattern within a grid to represent each number used in the addition table.
- Copy the pattern into a 4 x 4 grid so that each corresponds in position to its number in the addition table.
- Copy this 4 x 4 grid into the first quadrant of an 8 x 8 grid.
- Use an Euclidean transformation (translate, reflection or rotation) to place your pattern into the other three quadrants.

Individual Work Have students create their own modulo art design using different operations (addition, subtraction, multiplication) and different transformations (translate, reflection or rotation – ***3.6 Activity 4 Student Support Material Book***. Student's work could be displayed.

Extension Activity – An Investigation

A Challenge Refer students to the Extension Activity in the ***Student Support Material***

Unit Glossary

Hindu Arabic System The numeration system widely used around the world and currently used in PNG for education and business purposes. The symbols of the Hindu Arabic system are 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9

Additive System of Numeration An additive system of numeration is one in which the overall value of the number represented is determined by adding the value of each symbol together. Each symbol has only one value

Multiplicative System of Numeration A multiplicative system of numeration is one in which the value of the number represented is determined by the multiplication of some pairs of symbols as well as addition e.g. in the number 26, the value of the number is determined by multiplying 2 x 10 and 6 x 1 and then adding these values together.

Place Value System of Numeration A place value system of numeration is one in which the value of the number represented is determined by the position of the numeral. For example in the number '26' the 2 represents 2 tens and has the value of 20 and the 6 represents 6 ones with the value of 6

Factors All the whole numbers that can be divided exactly into another number.

For example $6 \div 1 = 6$

$6 \div 2 = 3$

$6 \div 3 = 2$

$6 \div 6 = 1$ so 1, 2, 3, and 6 are factors of 6

Proper factors All the factors of a numbers except the number itself

For example, the proper factors of 6 are 1, 2 and 3.

Prime Number A counting number that can only be divided by one and itself. For example 2, 3, 5, 7,11.

The factors of 2 are 2 and 1; the factors of 3 are 3 and 1.

Note: 1 is not considered a prime number or a composite number.

Prime Factors A prime number that will divide exactly into a given number; for example 2, 3 and 5 are prime factors of 30. (10 is a factors of 30 but not a prime factor)

Composite Number

A number with factors other than itself and one.

For example $12 = 12 \times 1 = 3 \times 4 = 3 \times 2 \times 2$

$33 = 33 \times 1 = 3 \times 11$

but not $17 = 17 \times 1$

Both 12 and 33 are composite numbers but not 17. Numbers like 17 which have no other factors except themselves and 1 are called 'prime numbers'. Every whole number greater than one is either

1. a prime number or
2. a composite number

Multiple

A multiple of a given number is any number into which it will divide exactly.

For example multiples of 2 are 2, 4, 6, 8, 10, 12, …

Multiples of 5 are, 5, 10, 15, 20, 25, …

Lowest Common Multiple (L.C. M.)

The lowest counting number that is a multiple of given numbers.

For example, What is the lowest common multiple of 2 and 3?

The multiples of 2 are 2, 4, ***6,*** 8, 10, ***12,*** 14, 16, ***18,*** …

The multiples of 3 are 3, ***6***, 9, ***12,*** 15, ***18***, …

The common multiples are 6, 12, 18, …

The lowest common multiple is 6.

Highest Common Factor (H. C. F.)

The highest counting number that is a factor of given numbers. For example: What is the highest common factor of 18 and 24?

The factors of 18 are 1, 2, 3, ***6***, 9, 18

The factors of 24 are 1, 2, 3, 4, ***6***, 8, 12, 24

The greatest common factor is **6**.

Abundant Numbers

If the sum of a given numbers proper factors is greater than the number itself, the number is said to be abundant.

For example, the proper factors of 12 are 1, 2, 3, 4, 6. The sum of these proper factors is 1 + 2 + 3 + 4 + 6 = 16

As 16 is greater than the 12, then **12** is an abundant number.

Deficient Numbers

If the sum of a given numbers proper factors is less than the number itself, the number is said to be deficient.

For example, the proper factors of 8 are 1, 2, 4. The sum of these proper factors is 1 + 2 + 4 = 7.

As 7 is less than 8, then **8** is a deficient number.

Perfect Numbers

If the sum of a given numbers proper factors is equal to the number itself, the number is said to be perfect.

For example, the proper factors of 28 are 1, 2, 4, 7, 14. The sum of these proper factors is 1 + 2 + 4 + 7 + 14 = 28.

As the sum equals the number, **28** is a perfect number.

Amicable (friendly) Numbers

Two numbers are said to be Amicable or Friendly if the proper factor sum of both numbers are equal.

Sequence

An ordered set of number formed according to some pattern or rule.

For example, 1, 3, 5, 7 …… the rule for this sequence is 'add 2'.

Term

Each number in a sequences is referred to as a term of that particular sequence. The term refers to the position within the sequence

For example:

in the sequence 1, 3, 5, 7, 1 is the first term, 3 is the second term, 5 is the third term etc.

nth Term

The rule or formula which describes the sequence. The rule is expressed as a function of n where n is the number of the term (its position) in the sequence.

For example:

Find the nth term of the sequence 3, 5, 7, 9….

For the first term when n =1, 2n =1

For the second term when n=2, 2n = 4

For the third term, when n=3, 2n = 6

By identifying the pattern you can see that each term is one more than twice n. The nth term is therefore 2n + 1

Figurative Numbers

Different sequences of numbers associated with different geometrical figures, considered special by the Pythagoreans and other early mathematicians, are referred to as figurative numbers. For example triangular number, square numbers

Nominal Data

Data which cannot be ordered and can only be classified according to the category.

Ordinal Data

Data which can be ordered. Ordinal data can be divided into two types, discrete data and continuous data

Discrete Data

Ordinal data which can only take on particular distinct values e.g. the number of females/males in a class can only take the values 0,

	1, 2, 3, ….. but it cannot be 2½ or 4¾
Continuous Data	Ordinal data which does not have a distinct value. For example the height of a student can be 158cm 0r 178.3cm or 164.345cm depending on accuracy of measurement.
Mean	A measure of central tendency. Also called the average. For example to calculate the mean for a set of scores you add all the scores together then divide them by the total number of scores.
Median	A measure of central tendency. It is the middle number in a series of numbers.
Mode	A measure of central tendency. It is the number which occurs with the most frequency in a series of numbers.
Range	The range is the difference between the highest and lowest values of the data.
Interquartile Range	This is the difference between the upper and lower quartiles in a set of data. The data is divided in to four quarters and the lower quartile is the division between the lower half values. The upper quartile is the division between the upper- half values.
BOMAS	BOMAS represents the order in which operations are carried out when completing calculations. ‘B’ is for brackets, ‘O’ is for Operators, ‘D’ is for division, ‘M’ is for multiplication, ‘A’ is for addition and ‘S’ for subtraction.
Commutative Operations	The order in which the numbers in an operation can be reversed without changing the result e.g. 7+3 = 10 and 3+ 7 = 10. Addition is therefore a commutative operation
Associative Operation	Two operations are associative with each other if the order in which they are done can be changes without the result changing. For example 4+3-2 = 5 if we add 4 and 3 together before taking away 2. If we take 2 from 3 and then add the 4 we will also get 5. Addition and subtraction are therefore associative operations.
Distributive Operations	An operation is distributive over another operation if it can be done on individual numbers in a group e.g. individual numbers inside brackets, or on the results after the group is combined e.g. when the brackets have been calculated, and final answer is the same. For example take 4 x (2 +5). If you complete the brackets first then multiple the answer would be 4 x 7 = 28. If we multiple each number inside the bracket and then add we will get 4 x 2 + 4 x 5 = 8 + 20 =28. The same answer is obtained. Therefore multiplication is distributive over addition.

Closure	An operation is closed in the set (S) if as a result of carrying out the operation on elements within a set, the answer is also in the set (S)
Identify Element	The set (S) has an identity element 'i' if combining 'i' with any other element either first or second does not change the other element. For example in the set of integers and the operation of addition if we combine 3 + 0 or 0 + 3 the element 3 does not change. Therefore 0 is the identity element 'i' for addition.
Inverse Element	Each element in a set has its own inverse element if when combined with that element either first or second, the result is the identity 'i'. For example, in addition the identity element is 0. The inverse of 5 is therefore –5 because 5 + -5 = 0