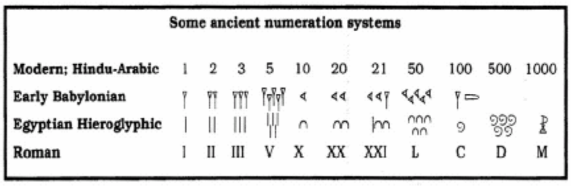
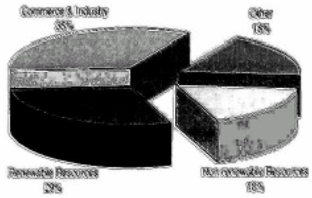
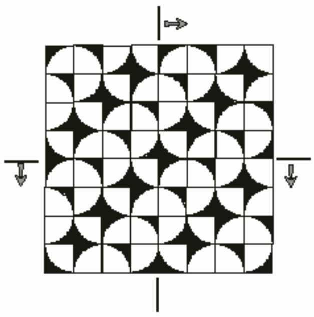
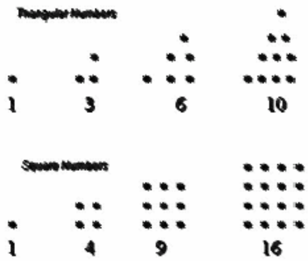
Mathematics Science Strand
Mathematics

Unit 3 Enrichment Topics in Number









Student Support Material

Acknowledgements

Materials written and compiled by Louise Quinn.

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- Mr Stephen Tapi
- r Edwin Sikil
- Mr Jazy Magia
- Br Tony Gaul
- Mr Amkat Mai
- Mr John Rumins
- Mr Synell Ko'ou
- Mr Siegeru Woda
- Mr Lengkepe Zonggareng
- Mr. Rotzoki Karkar
- Mr Peter Seth
- Mrs Mea Dobunaba
- Mr Casper Hahambu
- Mr Morgan Gwangilo
- Mr John Munum
- Mr Paul Pasingan
- Mr Robert Sine

Incorporating suggestions from the Mathematics staff of:

- Balob Teachers College
- Holy Trinity Teachers College
- Kabaleo Teachers College
- Madang Teachers College
- St Benedict's Teachers College
- PNG Education Institute
- Gaulim Teachers College
- Dauli Teachers College

Many of the activities found within this material have been developed from the work of Mr Wally Green in *Topics in Number* (1998) Department of Education PNG.

Layout and diagrams supported by Nick Lauer.

Date: 25 June 2002

PASTEP

Primary and Secondary Teacher Education Project

Australian Agency for International Development (AusAID)
GRM International

Papua New Guinea-Australia Development Cooperation Program

Unit outline

Unit	#	Modules
Unit 1 **Enrichment Topics in Number**	**1**	**Alternative Systems of Numeration and Computations** **(Core)**
	2	**Factors, Primes and Composites** **(Core)**
	3	**Number Sequences and Patterns** **(Core)**
	4	**Basic Statistics** **(Core)**
	5	**Using Calculators in the Primary School** **(Recommended)**
	6	**Clock Modulo Arithmetic** **(Recommended)**

Icons

🕮	Read	or	research
✍	Write	or	summarise
☞	Activity	or	discussion

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[Notes]

Unit 3: Enrichment Topics in Number

Rationale

Fundamental to learning mathematics is an understanding of number. Number forms such a large part of every school mathematics curriculum, and underlies all of the computational procedures as well as setting up the patterns of thinking on which mathematics relies. As future teachers, you need to further develop your own understanding of number concepts as well as develop an awareness of how to teach these concepts to children. The modules offered in this unit will revitalise your interest in numeration and enrich your knowledge across a range of areas.

Aims

This unit aims to produce beginning teachers who are:

- able to solve a range of number problems
- able to articulate their mathematical thinking
- confident and competent to teach number concepts to primary school children (Grades 3 to 8).

Objectives

As a result of studying this unit you will:

- consider how cultural and social influences have impacted on the development of numerical understandings
- further develop your understanding of factors, primes and composites
- determine number patterns and sequences
- compile statistical information and analyse it
- debate the use of calculates in the primary school
- create modulo art designs.

Unit outline

'Enrichment Topics in Number' is a 3-credit point unit.

To successfully complete this unit it is suggested that you complete the following core modules:

Module 3.1	Alternative Systems of Numeration and Computations
Module 3.2	Factors, Primes and Composites
Module 3.3	Number Sequences and Patterns
Module 3.4	Basic Statistics

The additional modules listed below may also be covered if time permits:

Module 3.5 Using Calculators in the Primary School

Module 3.6 Clock Modulo Arithmetic

Each of these modules should take between 6 to 9 hours of lectures to complete. It is also expected that you will spend an equivalent number of hours of non-contact time studying the ideas and concepts raised in this unit.

How to use this material

This material has been developed to support your studies in this unit. It aims to support the development of your own mathematical knowledge and skills, as well as prepare you to teach mathematics in primary schools.

The material for this unit has been set out according to the different modules. For each module the objectives and the concepts and skills to be developed within the module are stated. This information is followed by a series of topics. Each topic consists of readings as well as activities to complete. Extension activities have also been included. Your lecturer will guide you through the materials during the lecture program. At times your lecturer may ask you to complete activities and reading as homework. Sometimes you may work directly from the book during lectures. Your lecturer may also include additional information and topics.

A glossary can be found at the end of the unit to assist you in reading the material.

Assessment

Assessment details will be provided by your lecturer. A range of different assessment tasks will be given which require you to show your understanding of the concepts and skills covered by this unit.

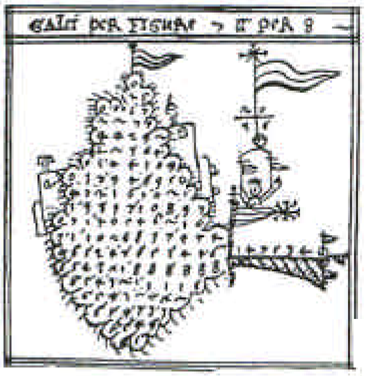
Inclusive curriculum

In the delivery of this unit it is expected that every person will be provided with an opportunity to participate in and contribute to activities without fear or favour. Activities will be presented to cater for a range of abilities and will be gender inclusive. Assessment tasks will cater for a range of different learning styles.

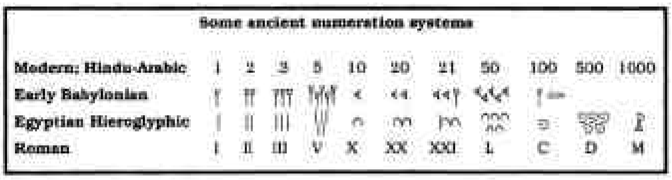
You will be encouraged to plan activities for use in the primary school mathematics classroom which are gender inclusive and present positive and non-stereotypical representations of people.

Module 3.1 – Alternative Systems of Numeration and Computations

Title page of Calandri's Arithmetic, 1491



Galley Division 16th century



Some ancient numeration systems											
Modern: Hindu-Arabic	1	2	3	5	10	20	21	50	100	500	1000
Early Babylonian	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		
Egyptian Hieroglyphic	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Roman	I	II	III	V	X	XX	XXI	L	C	D	M

Alternative Systems of Numeration and Computations is a core module in the unit ***'Enrichment Topics in Number'***. During this module you will develop an understanding of different systems of numeration. You will investigate PNG counting systems as well as alternative systems from other parts of the world. Early computational algorithms will also be explored. By studying this module it is expected that you will gain a better appreciation of the Hindu Arabic system currently in use in PNG schools.

Objectives

By the end of this module you will be able to:

- differentiate between additive, multiplicative and place value systems of numeration
- explain the Hindu Arabic system
- compare different computational methods.

Concepts and skills

During this module the following concepts and skills will be developed:

- Place value
- Addition
- Multiplication
- Computational skills
- Base systems

Topic 1 – The Hindu-Arabic System

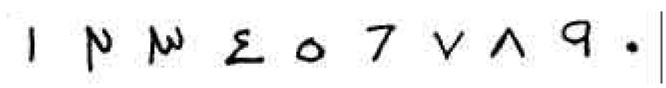
Throughout the world and over a long period of time many different counting systems have been developed by people. People use a range of objects like fingers, rows of pebbles, bones, sticks or seashells to count as well as a range of different symbols to record their calculations. The system of numbers commonly used today all over the world is called the Hindu-Arabic system.

3.1 Activity 1

Read the following article which discusses the Hindu-Arabic System. Identify the important and distinguishing features of this system and why they are significant. Prepare to discuss your ideas with the class.

The Hindu –Arabic system of numeration is believed to have been developed by the Hindus around about 300-200 BC in India. Historical records show that the Hindus used numerals from which our present symbols have evolved, although early examples do not contain zero or place value.

By 825 AD a Persian mathematician al-Khowarizmi described a Hindu system complete with place value and a zero. From AD 900 the following set of Arabic symbols were in use:



The Hindus numerals were transmitted to other parts of the world by the Arabs, through their conquest of Spain and through written records during the 8^{th} Century AD. At this time many other numeration systems were still in use. Calculations were done using the abacus and other methods.

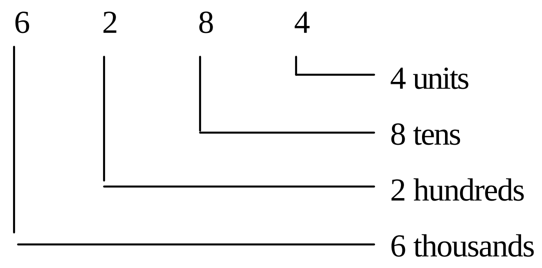
Although the Hindu-Arabic numerals were superior for recording and for calculating, they were slow to be accepted in Europe because Roman numerals were used, with the abacus for calculations. The Hindu-Arabic numerals were eventually accepted and the standard symbols to record the numerals established.

The main characteristics of the Hindu Arabic system are:

1. It is a place-value system.

   The position of each digit is important. The value of each digit is determined according to its position.

   For example



2. The system is based on 10 so is called a decimal system of numerals (from the Latin word **decem** meaning **ten**). It is referred to a base 10 system

3. Zero is used as a placeholder. That is, in the number 207, there are no tens so the zero takes the tens place and thereby retains the correct place value for the other digits.

4. The system is multiplicative. That is, the overall value of a digit is determined by multiplying the face value of the digit by the value of its place. For example in the number 437, the overall value of the 3 is determined by multiplying the face value of the digit 3 by the value of its place, 10, so the value of the 3 is 30 (3 x 10)

5. The system is additive. This means that the value of the number is calculated by finding the sum of the overall value of each digit. For example, the value of 437 is the sum of 400 + 30 + 7.

6. The system is a unique-representation system. That is, each number in the system represents only one number and there is only one way of writing any one number. For example, the number 437 has only one value and there is only one unique way of writing this number using the Hindu-Arabic symbols.

Topic 2 – PNG Counting Systems

Around the world, many people used counting systems based on parts of their bodies. Persians, Egyptians and Arabs had a system like the sign language used by deaf people which they used to express numbers from 1 – 10 000. In the nineteenth century, some Torres Strait islanders counted by touching parts of their bodies. A system used in China until the nineteenth century made it possible to count up to 100 000 using finger joints.

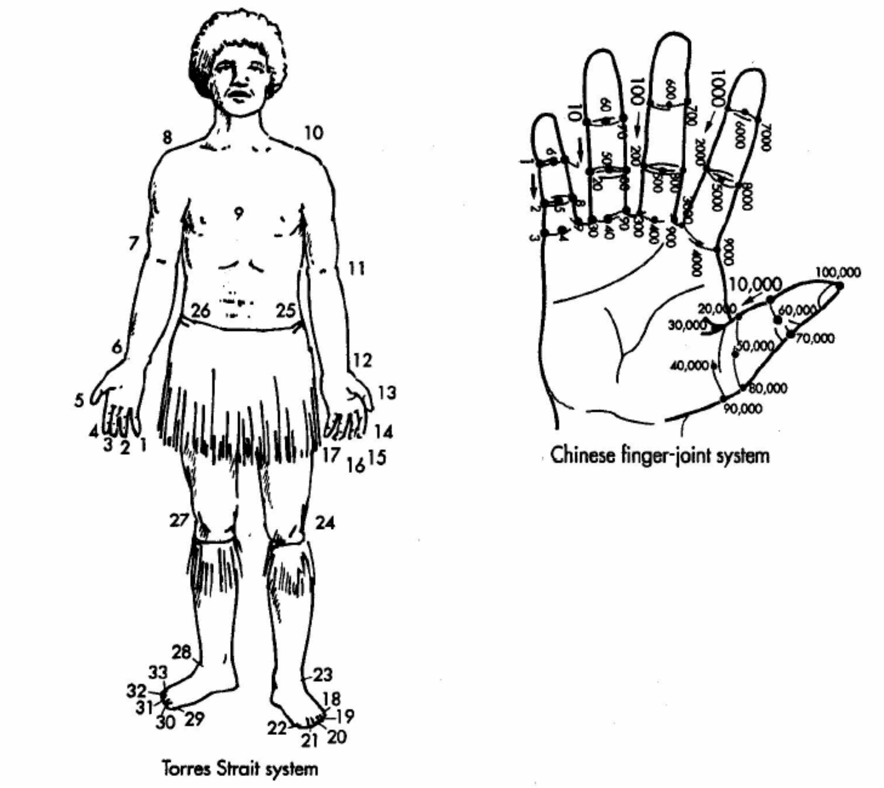


Diagram 1: Torres Strait Islander, the Chinese finger-joint systems

The Incas of Peru in South America recorded large numbers through using quipus or knotted string. One of the reasons the Incas were so efficient and were able to keep their large empire intact was because the quipus helped them to keep accurate records of taxes, harvests, budgets, births and deaths. The method worked so well that, up until the nineteenth century, herders in parts of Peru still counted their sheep and goats by using quipus.

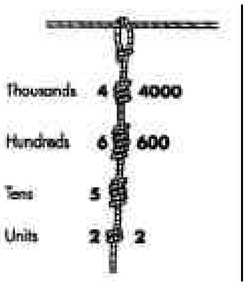


Diagram 2: Incas of Peru in South America used a quipus (a knotted string) to recorded large numbers

The Oksapmin counting system is the counting system used by all the Min people living in the Star Mountains of Papua New Guinea. This system is used in the Telefomin District of

Sandaun Province and Olsobip District in the Western Province. The same counting system is used in the Irian Jaya, in places which border these two districts.

The traditional language used to describe the numeric representation on the body parts may be same, similar or different depending on proximity of the villages and language speaking groups. However, the body parts used to represent the numbers never differs, it's the same as it was since the days of our ancestors.

Illustrated by Venantius Gadd MTC 2000

Number in English	Oksapmin Language	Body Parts Name
1	Tifanna	Thump.
2	Tifnarip	Pointer
3	Bumprip	Tall man
4	Hatdip	Ringman
5	Hathata	Small man.
6	Dopa	Wrist.
7	Besa	Forearm
8	Kir	Inside elbow joint.
9	Tuwat	Arm
10	Kata	Shoulder
11	Gwer	Girdle
12	Nata	Ear
13	Kina	Eye
14	Ruma	Nose
15	Tan Kina	Left eye
16	Tan Nata	Left ear
17	Tan Gwer	Left Girdle
18	Tan Kata	Left shoulder
19	Tan Tuwat	Left arm
20	Tan Kir	Left inside elbow joint
21	Tan Besa	Left forearm
22	Tan Dopa	Left wrist
23	Tan Tifanna	Left thump.
24	Tan Tipnarip	Left pointer.
25	Tan Bumrip	Left tallman
26	Tan Hatdip	Left ring man
27	Tan Hathata	Left small man

The counting begins on the right hand thump and finishes on the left hand little finger. **Tan** means ‘other side’. A complete cycle of counting up to 27 is called a **Fu.** Two cycles would be called **Yot Fu**. Three cycles would be called **Yatir Fu**, which means three cycles of counting up to 27 e.g. 81. Four cycles would be called **Hatdip Fu**. While five cycles would be called **Hathat Fu**.

This counting system was traditionally used to calculate bride price and for trading purposes. For example people would count the number of pigs, rolls of tobacco, pandanus nuts, bilums etc. that would be exchanged for a range of items such tapa cloth, kundu drums, bows & arrows etc. This system is still used today by the village elders for the same purpose. Younger people have adopted the Hindu Arabic system but will use their traditional counting system when completing quick calculation.

☞ 3.1 Activity 2

Investigate a PNG counting system. This may be the counting system from your place or it may be another one. Prepare a presentation which explains the counting system. Your presentation should include:

- *information about where the counting system is from*
- *an explanation of how the counting system works*
- *the signs and symbols used by the counting system*
- *examples of the context in which the counting system is used.*

Topic 3 – Ancient Systems of Numeration

Before the Hindu-Arabic counting system was widely accepted during the 16th century, many other systems of numeration were in use. The development of different numeration systems in ancient civilizations reflected the needs of the people at that time and influenced further mathematical developments within that civilization.

Examples of ancient systems of numeration can be found below.

The Ancient Egyptian system

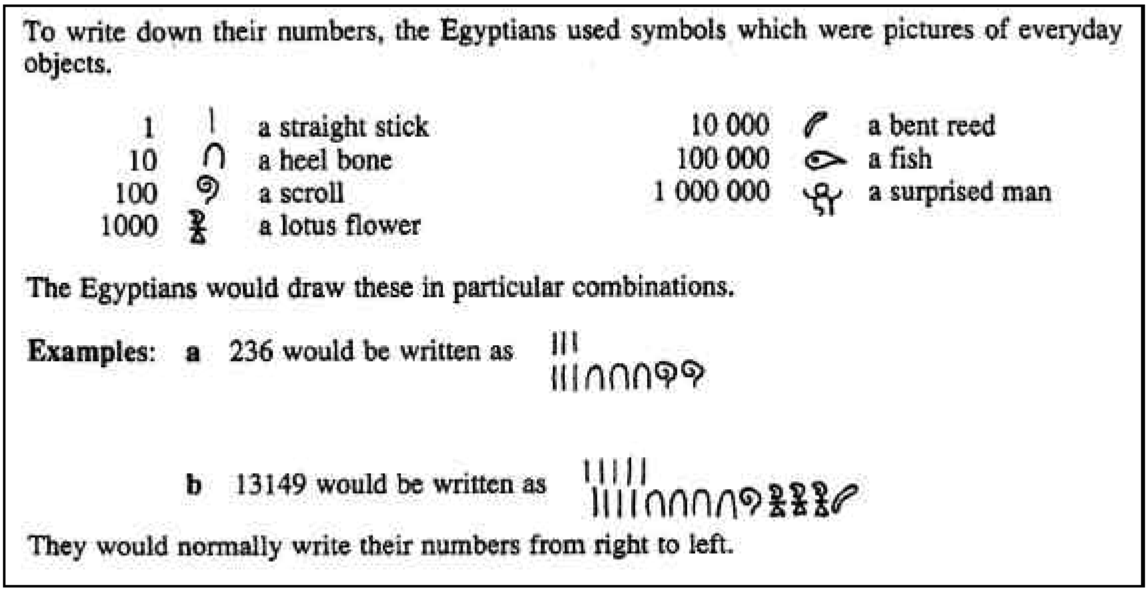


Diagram 3: Ancient Egyptian system.

The Chinese - Japanese system

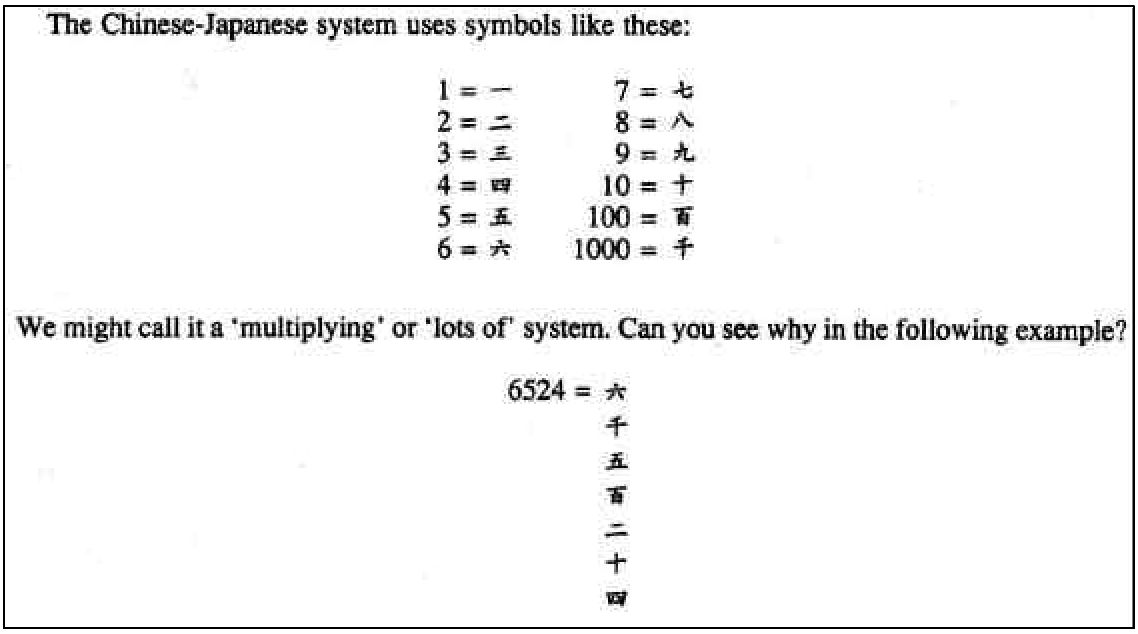


Diagram 4 The Chinese-Japanese system.

The Babylonian system

Records of the ancient Babylonian numeral system were known as early as 3500 BC. All their numbers are wedge shaped because they wrote by pressing wedge-shaped sticks into damp clay. The most striking feature of their numeration system was the use of a sexagesimal (base 60) place value system.

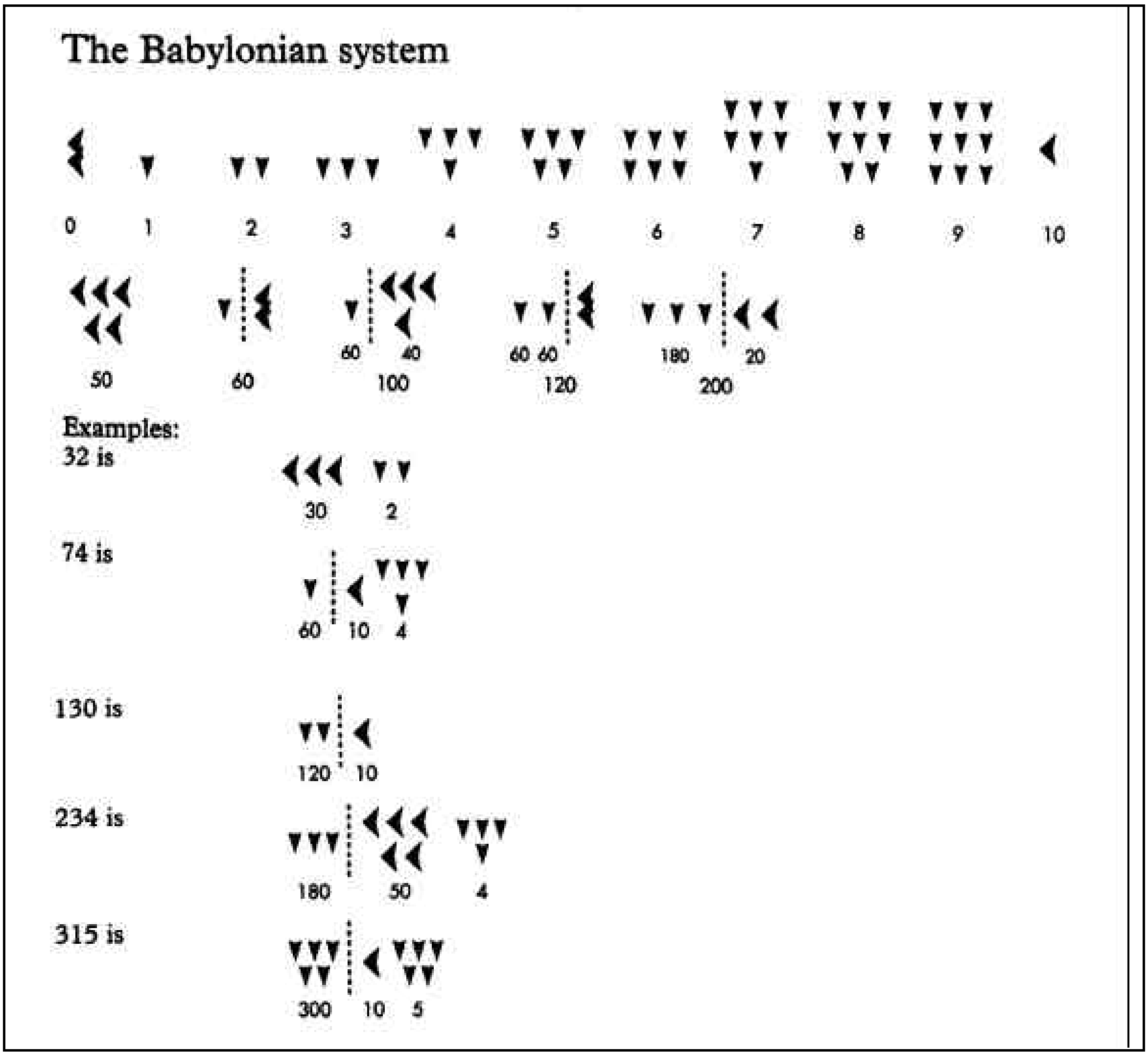


Diagram 5: The Babylonian system.

Ancient systems

Ancient system of numeration can be broadly classified as either being:

1. an additive system
2. a multiplicative system or
3. a place value system

Additive system - an additive system is one in which the value of the number represented is determined by adding the value of each symbol and each symbol has only one value.

Multiplicative system – a multiplicative system is one in which the value of the number represented involves the multiplication of some pairs of symbols used as well as addition.

Place value system – a place value system is on in which the value of a symbol varies according to where it is placed in the numeral.

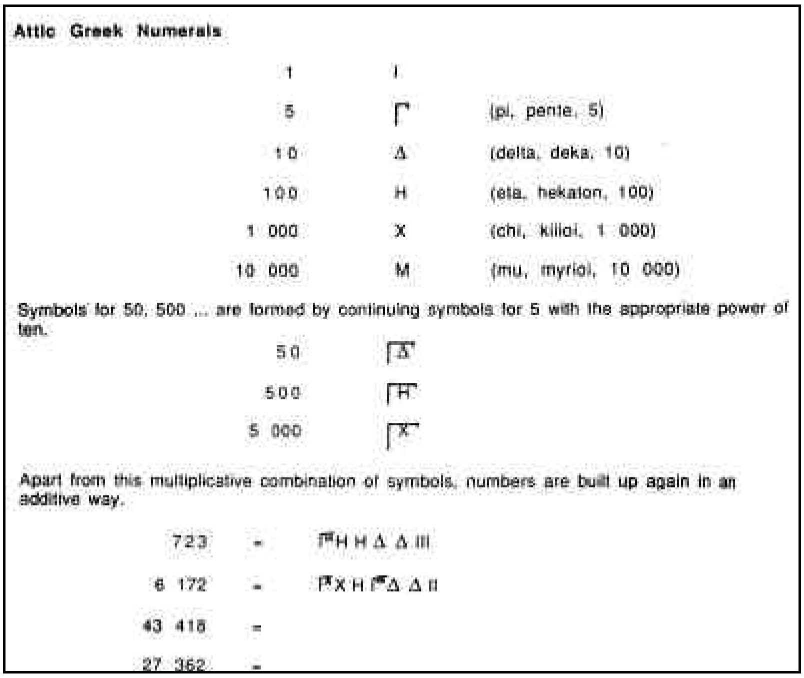
☞ 3.1 Activity 3

Examine the ancient numeration systems below. What do you notice about each of the systems e.g. is the order of the symbols important, how are large numbers written, are the symbols repeated and if so do they have the same value each time? Group the different systems according the whether they are an example of an additive system, a multiplicative system or a place value system. Use the different numeration systems to represent the following numbers:

365 *5671* *723* *43 148*

The Attic Greek numerals

The Greeks developed two systems of numeration. Around 300B.C., they developed the Attic or Heriodianic System. They had symbols for 5, 50, 500 … as well as the powers of ten.


Attic Greek Numerals

1	I	
5	Γ	(pi, pente, 5)
10	Δ	(delta, deka, 10)
100	H	(eta, hekaton, 100)
1 000	X	(chi, kilioi, 1 000)
10 000	M	(mu, myrioi, 10 000)

Symbols for 50, 500 ... are formed by continuing symbols for 5 with the appropriate power of ten.

50	𐅄
500	𐅅
5 000	𐅆

Apart from this multiplicative combination of symbols, numbers are built up again in an additive way.

723 = 𐅅HHΔΔIII

6 172 = 𐅆XH𐅄Δ II

43 418 =

27 362 =

Diagram 6: The Attic Greek numerals.

The Mayan numerals

The Maya of Southern Mexico and Central America developed a numeration system combining grouping in fives with place values based on 20. They had a symbol to represent zero that was used as a placeholder.

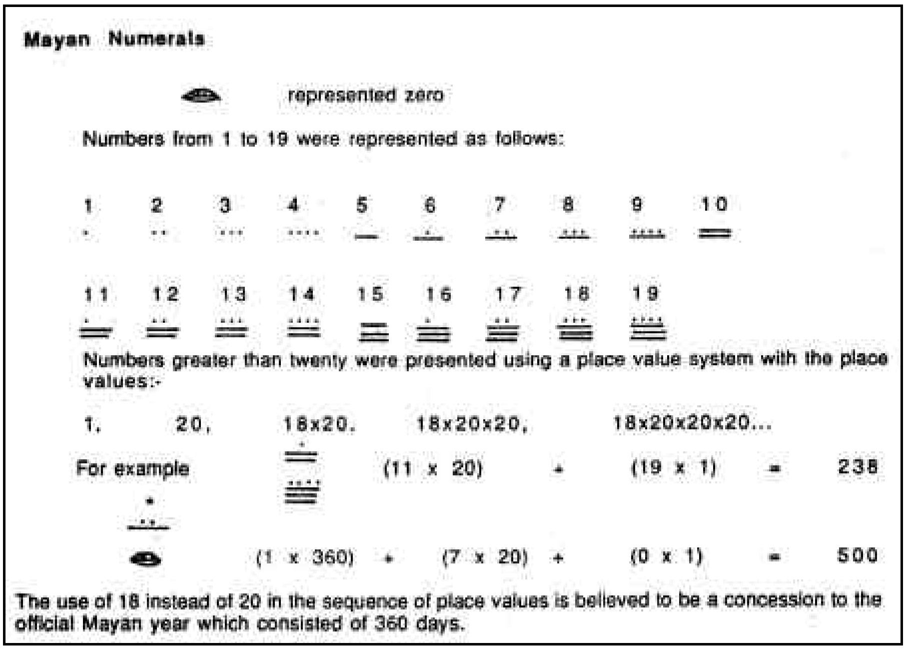


Diagram 7: The Mayan numerals

The Roman numerals

Most people are familiar with the system of Roman numerals. This system is similar to the Attic Greek numerals and has an addition and a subtractive property. The Romans are believed to have taken over the numerals from the Etruscan, who possibly came from Asia Minor, ruled in Italy north of the Tiber as early as 1000 BC. and are though to have conquered Rome in the 7th century BC.

Roman Numerals

The familiar system of Roman numerals bears many similarities to the Attic Greek numerals with in addition, a subtractive property. They are believed to have taken over their numerals from the Etruscans, who possibly came from Asia Minor, ruled in Italy north of the Tiber as early as 1000B.C., and are thought to have conquered Rome in the 7th century B.C.

1	I		
5	V	365 =	CCCLXV
10	X		
50	L	1979 =	MCM LXX IX
100	C		
500	D		
1 000	M		

A variety of methods were used to represent larger numbers:-

(i) Use of bar placed over a symbol or group of symbols indicated multiplication by 1000:-

$\overline{XXV}$ = 25 000

(ii) Use of M at the end of a group of symbols also indicated multiplication by 1000:-

XXVM = 25 000

Diagram 8: Roman numerals.

The Greek Alphabet numerals

The Greek Alphabet numerals were in use at approximately 450 BC. It employs the 24 characters of the Greek alphabet and three other symbols.

Greek Alphabet Numerals

1	α	alpha	10	ι	iota	100	ρ	rho
2	β	beta	20	κ	kappa	200	σ	sigma
3	γ	gamma	30	λ	lambda	300	τ	tan
4	δ	delta	40	μ	mu	400	υ	upsilon
5	ε	epsilon	50	ν	nu	500	φ	phi
6	ϛ	digamma	60	ξ	xi	600	χ	chi
7	ζ	zeta	70	ο	omicron	700	ψ	psi
8	η	eta	80	π	pi	800	ω	omega
9	θ	theta	90	ϙ	koppa	900	ϡ	sampi

Numerals are represented as follows:

38 = λ η

567 = φ ξ ζ

The following techniques are used to represent larger numbers:

/α = 1 000		$\overset{\alpha}{M}$ = 10 000
/β = 2 000		$\overset{\beta}{M}$ = 20 000
/γ = 3 000		$\overset{\gamma}{M}$ = 30 000

Diagram 9: The Greek alphabet numerals.

Topic 4 – Alternative Systems of Computations

In the past calculations were often done using pebbles; in fact, the word 'calculate' comes for the Latin word *calculus* which means 'small stone'. The oldest 'counting machine' known is a 20 000 year old wolf bone found in Vestonice, Czechoslovakia, in 1937. It had 55 notches in two rows divided into groups of five. The age of the bone means that counting machines were invented very early in human history.

Over time, different methods of computation have been developed by people. A number of factors influenced the development of written algorithms for number operations. For example, early number systems tended to be additive rather than place value and calculations tended to be done using fingers and abacus. Once the Hindu –Arabic numeration system was developed and later transmitted to Western Europe, around the 16th or 17th centuries, written algorithms became more widely used.

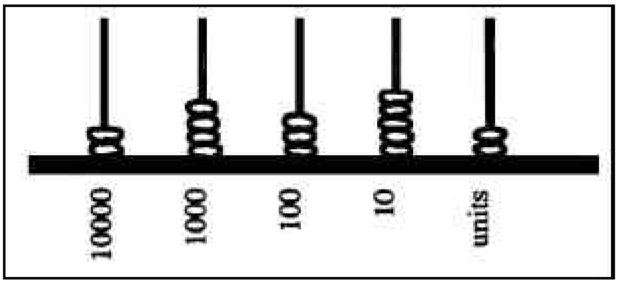


Diagram 10: An Abacus used for calculations

☞ 3.1 Activity 4

Investigate the alternative methods of computations set out below. How do they work? Complete some computations using these methods.

What do you see as the advantages and disadvantages of the various methods? Write down your ideas and be prepared to discuss this with the class.

Examples of mathematical operations

Some methods of addition

65391	9279	9 2 7 9
3279	389	3 8 9
10420	479	4 2 7 2 9
78980	27	1 0 1 4 7
909	22	
	9	
	9	
	10147	
Ancient Hindu (work from left)	Gemma Frisius (1540)	Modern version

Diagram 11: Methods of addition (Ancient Hindu, Gemma Frisius and the modern version)

Some methods of subtraction

Columbian Algorism (14th century)

7678 → 5982 → ?

2	1	16
7678	27	2796
5982	7678	7678
	5982	5982

A 'scratch' method used with the sand abacus. Work from the left.

Complementary subtraction (Ancient Hindu)

452
267
185

To take 7 from 2, find the complement of 7, (10 − 7 =) 3, and add this to the 2 to give 5. The 10 'borrowed' to find the complement is added to the 6 to make 7. Then take this 7 from 5 (tens) and so on.

'Borrow and pay back' (Arabic, possibly Hindu)

8 ¹2
6₁ 7
1 5

Work from the right. Carry figures were not always used. The 1 'borrowed' from the 5 is 'paid back' to the 2. In reality, 10 is added to both numbers.

'Simply borrowing' or decomposition (Arabic, possibly Hindu)

7
8 ¹2
6 7
1 5

Say: '7 from 12, 5; 6 from 7, 1'. Useful for the sand abacus.

Complementary addition, or 'making change' (Buteo 1559)

8 2
6 7
1 5

'7 and 5 make 12; 7 and 1 make 8', or '7 and 5, 12, and 6 and 1 make 7'. Later called the Austrian method, and still used for calculating change.

Diagram 12: Methods of subtraction (Columbian Algorism, complementary subtraction, borrow and pay back, simply borrowing and complementary addition).

Bar Method

The smaller digit is taken from the larger and a bar placed on top to indicate negative digits.

$$\begin{array}{r} 4\;3\;1 \\ -1\;4\;5 \\ \hline 3\;\bar{1}\;\bar{4} \end{array} \rightarrow \begin{array}{r} 4\;3\;1 \\ -1\;4\;5 \\ \hline 3\;\bar{1}\;\bar{4} \\ 2\;9\;\bar{4} \end{array} \rightarrow \begin{array}{r} 4\;3\;1 \\ -1\;4\;5 \\ \hline 3\;\bar{1}\;\bar{4} \\ 2\;9\;\bar{4} \\ 2\;8\;6 \end{array}$$

Diagram 13: The Bar method.

To multiply 26 by 33 using duplation

	1	33	1 . 33 =	33
→	2	**66**	2 . 33 =	**66**
	4	132	4 . 33 =	132
→	8	**264**	8 . 33 =	**264**
→	16	**528**	16 . 33 =	**528**
		858	26 . 33 =	858

Egyptian method — **Stifel (1546)**

26	**13** ↓	6	**3** ↓	**1** ↓	← Divide by 2
33	66	132	264	528	Ignore remainders

Add numbers under odd numbers

66 + 264 + 528 = 858

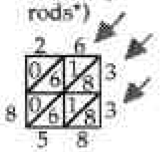
Russian peasant method

Some Methods used in Renaissance Italy

The ancestor of long multiplication: *multiplicatio per scachieri* (from the Italian: "chessboard")

multiplicandus 2 6
multiplicans 3 3
7 8
7 8
summa 8 5 8

The lattice method: ... *per gelosia* (used in Napier's rods*)



The *repegio* method:
(using decomposition of factors; the associative law for x)
26 x 33 = 26 x (3 x 11) = (26 x 3) x 11 = 78 x 11 = 858

The *scapezzo* method:
(using multiplication by the parts of the multiplier; the distributive law for x and +)
26 x 33 = (3 + 4 + 5 + 6 + 8) x 33
= 99 + 132 + 165 + 198 + 264 = 858

The Hindu 'scratch' method

Because they worked on a dust abacus or small board, the Hindus developed a setting-out which took up little space: figures which had been used in the calculation were erased (or scratched out).

To multiply 569 by 5, this is what the calculation looked like:

```
   8  4  5    Work from left to right
2  7  9       and cross out used figures
   5  6  9          5
```

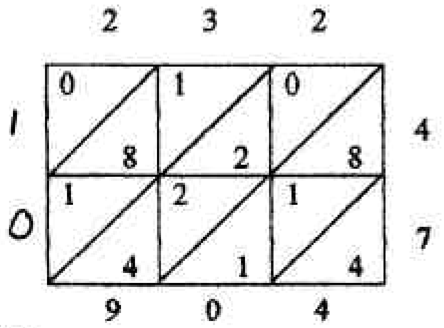
Work from left to right and erase or cross out used figures. Can you see how this works?

Diagram 14: Multiplication methods (Egyptian, Stifel, Russian Peasant, Renaissance Italy, Hindu Scratch).

The Gelosia or Grating Method

This method was popular in the 15th and 16th centuries although it was probably developed in India much earlier.

232 x 47



Product = 10 904

Does this method have merit as an algorithm suitable for primary children?

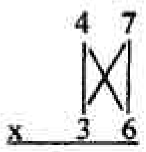
The Chessboard Method

Our current algorithm is similar to the chessboard method that appeared in Italian mathematician Luca Pacioli's book in 1494.

		9	4	3	7
				2	8
	7	5	4	9	6
1	8	8	7	4	
2	6	4	2	3	6

The Cross Multiplication Method

Pacioli also included the following cross multiplication method which is very efficient but requires good mental processes.



7 x 6 = 42, record 2, carry 4
4 x 6 = 24, 7 x 3 = 21
24 + 21 + 4 = 49, record 9, carry 4
4 x 3 = 12, 12 + 4 = 16

1 6 9 2 (The operation is performed in one line.)

Diagram 15: More multiplication methods (Gelosia/Grating, Chessboard, Cross Multiplication).

Two examples of Egyptian division by duplation

To divide 638 by 24:

1	24	Double until the next
* 2	48	doubling would be
4	96	greater than 638
* 8	192	
*16	384	

Since 638 = 384 + 254
= 384 + 192 + 62
= 384 + 192 + 48 + 14
= 24 (16 + 8 + 2) + 14

Quotient is
16 + 8 + 2 = 26, remainder 14.

To divide 21 by 8:

1	8
*2	16
*$\frac{1}{2}$	4
$\frac{1}{4}$	2
*$\frac{1}{8}$	1

Select numbers from right hand column adding to 21
16 + 4 + 1 = 21
Quotient is
$2 + \frac{1}{2} + \frac{1}{8}$
(Remember, unit fractions!)

Diagram 16: Egyptian division.

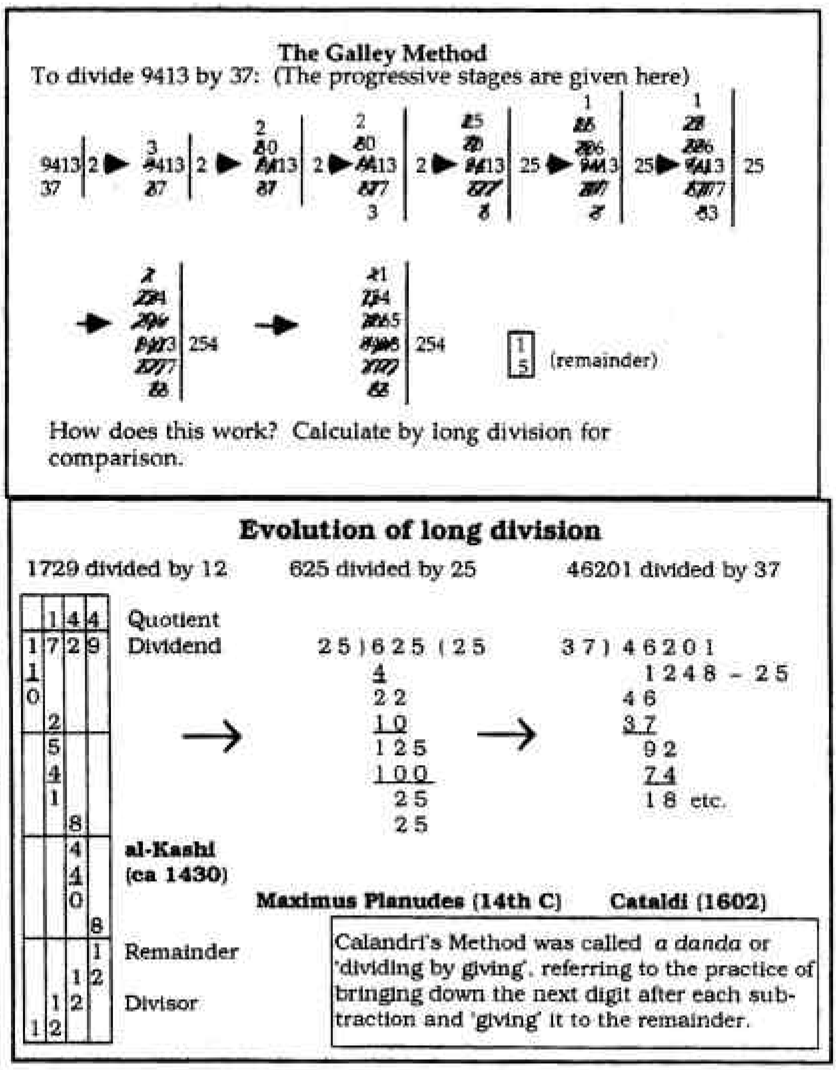


Diagram 17: Methods of division (Gallery method, evolution of long division).

References – for further reading

Department of Education Papua New Guinea (1998). *Lower Primary Mathematics Syllabus Grade 3-5*

Department of Education Papua New Guinea (draft 1999). *Upper Primary Mathematics Syllabus Grade 6-8*

Brinkworth, P. & Scott, P. (1994) '*The making of Mathematics – a friendly history*' Australian Association of Mathematics Teachers Inc.

Burnett, James (1999) *Sights, Sounds and Symbols – Classroom Activities on the History of Numbers*, Prime Education, Queensland Australia

Module 3.2 – Factors, Primes and Composites

Module 3.2 Factors, Primes and Composites is a core module within the ***'Enrichment Topics in Number'*** unit. An understanding of prime and composite numbers is essential for all number work so during this module you will have an opportunity to further extend your own understandings of these concepts. The primary school mathematics syllabus will be examined and the teaching of these important concepts will be considered.

Objectives

By the end of this module you will be able to:

- identify prime and composite numbers
- find the factors of a given number
- develop a number investigation suitable for use in the primary school
- apply rules of divisibility.

Concepts and skills to be developed

- Primes
- Composites
- Factors
- Multiples
- Prime factors
- Lowest common multiple
- Greatest common factor
- Division

Topic 1 – Factors, Prime and Composite Numbers

An understanding of factors and primes is important for the development of many number concepts. For example to be able to add or multiple fractions with different denominators requires the ability to be able to identify common factors.

Factors are all the whole numbers that can be divided exactly into another number. For example, the factors of 6 are 1, 2, 3 and 6. The proper factors are all the factors of a number except the number itself so the proper factors of 6 are 1, 2 and 3.

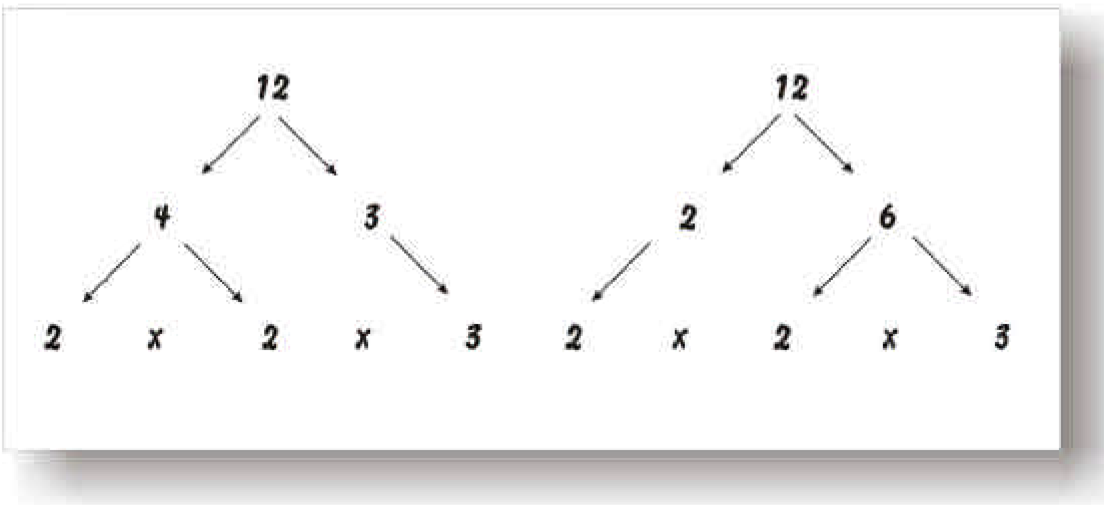
A counting number that can only be divided by one and itself is called a prime number. For example 2, 3, 5, 7,11 are prime numbers.

A prime number that will divide exactly into a given number are referred to as the numbers prime factors. For example, 2, 3 and 5 are the prime factors of 30, (10 is a factors of 30 but not a prime factor).

There are a number of different ways of finding the prime factors of a particular number.

Factor tree

A factor tree is a branching diagram which shows the prime factors of a given number. It breaks the number up into its prime factors. For example:



Note: If you get a prime (e.g. 2 or 3) continue to keep going down to the next line until all your factors are primes.

The results are recorded by using indexes to show repeated factors e.g. 2^2 x 3

Division method

If you need to find the prime factors of a larger number, you can use repeated division by primes (2, 3, 5, 7, 11 etc).

2	**2100**
2	1050
3	525
5	175
5	35
7	7
	1

Finish when the dividend is 1. The results are recorded using indexes to show repeated factors e.g. $2^2 \times 3 \times 5^2 \times 7$.

☞ 3.2 Activity 1

Find the prime factors of the following numbers using either the factor tree or division method. Record your answer using indexes to show repeated factors.

39	*52*	*32*	*48*
400	*252*	*1869*	*1225*

Common factors and common multiples

When simplifying fractions it is necessary to be able to identify common factors. The highest common factor of two numbers is the highest number that is a factor of the given numbers. For example the highest common factor for the numbers 6 and 12 is 6

the factors of 6 are 1, 2, 3, and 6

the factors of 12 are 1, 2, 3, 4, 6, and 12

the common factors are 1, 2, 3 and 6 and the highest of these factors is 6

Adding and subtracting fractions with different denominators requires the identification of a common denominator. This is done by looking for common multiples. The lowest common multiplier of two numbers is the lowest counting number that is a multiple of the given numbers. For example the lowest common multiple for the numbers 6 and 12 is 12.

The multiples of 6 are *6, 12, 18, 24, 30, 36 ……*

The multiples of 12 are *12, 24, 36, 48, 60, ……*

The lowest common multiple for the numbers 6 and 12 is 12

☞ 3.2 Activity 2 – Finding common factors and multiples

Find the common factors and then the highest common factor (HCF) for the following numbers:

(a) 45 and 48 *(b) 44 and 22*

(c) 32 and 60 *(d) 28 and 70*

(e) 32, 56 and 80 *(f) 14, 77 and 90*

Finding the Highest Common Factor and the Lowest Common Multiple by Prime Factors

If you wish to find the highest common factor (HCF) and the lowest common multiple (LCM) of a group of numbers, you can do so by finding the prime factors. For example if you want to find the HCF and LCM of 252 and 120

1. Express each number as a product of its prime factors (as you did in Activity 1, above)

   252 $= 2 \times 2 \times 3 \times 3 \times 7$ 120 $= 2 \times 2 \times 2 \times 3 \times 5$

   $= 2^2 \times 3^2 \times 7$ $= 2^3 \times 3 \times 5$

2. The HCF is the largest factor common to both and must therefore contain all the common prime factors.

   HCF $= 2 \times 2 \times 3$

   $= 12$

3. The LCM is a multiple of 252 and must therefore contain all the factors of 252. Since it is also a multiple of 120, it must also contain all the factors of 120.

   The LCM can be found by writing down all the factors of the larger number and then including the factors of the smaller number not already included.

   LCM $= (2 \times 2 \times 3 \times 3 \times 7) \times (2 \times 5)$

   $= 2520$

4. Once we understand how to find the HCF and the LCM by the method of prime factors, the problem can be solved quickly using index notation.

   252 $= 2^2 \times 3^2 \times 7$ 120 $= 2^3 \times 3 \times 5$

   HCF $= 2^2 \times 3 = 12$

   LCM $= 2^3 \times 3^2 \times 5 \times 7 = 2520$

☞3.2 Activity 3 – Finding the HCF and the LCM

Find the HCF and the LCM of the following pairs of numbers

(a) 16, 24

(b) 128, 324

(c) 36, 90

(d) 80, 112

(e) 84, 156,

(f) 45, 360

☞ 3.2 Activity 4 – An investigation

Complete one of the following investigations and be prepared to share your findings with others.

1) *Investigate the number of factors of numbers (Hint: First express each number in terms of its prime factors as* $a^x\, b^y\, c^z$*).*

2) *Investigate the truth of the following statement:*

   'All perfect squares have an odd number of factors, whereas all other positive integers have an even number of them'.

3) *Investigate the truth of the following*

   If A and B are counting numbers then:

   HCF (A, B) x LCM (A, B)= A x B

4) *A German mathematician, Goldbach, suspected that every even number greater than 2 could be written as the sum of 2 prime numbers*

   e.g. 4 = 2 + 2   6 = 3 + 3   8 = 3 + 5

   Check this statement for even numbers to 50. Does this mean you have proven Goldbach's conjecture?

Topic 2 – Rules of Divisibility

When identifying the factors of a number it is useful to be able to recognise certain number patterns and to be familiar with the rules of divisibility.

Some simple tests for divisibility are shown below:

Divisible by 2	If a number ends in 0, 2, 4, 6,or 8, i.e. even numbers
Divisible by 3	If the sum of its digits is divisible by 3, the number is divisible by 3 e.g. 238 464 is divisible by 3 because 2+ 3+ 8 + 4 + 6 + 4 = 27 and 27 is divisible by 3
Divisible by 4	If the last two digits of a number form a number which can be divided by 4, the number can be divided by 4 e.g. 378 465 236 is divisible by 4 because the last two digits, 36, is divisible by 4.
Divisible by 5	If a number ends in 5 or 0, it is divisible by 5
Divisible by 6	If an even number passes the ‘divisibility by 3’ test, that even number is also divisible by 6. That is if it is an even number and the sum of its digits is divisible by 3.
Divisible by 7	There is no simple test to determine if a number is divisible by 7.
Divisible by 8	If the last three digits of a number are divisible by 8, then the number is divisible by 8. e.g. 475 381 416 is divisible by 8 because 416 is divisible by 8.
Divisible by 9	If the sum of its digits is divisible by 9 then the number is divisible by 9 e.g. 425 327 265 is divisible by 9 because 4 + 2 + 5 + 3 + 2 + 7 + 2 + 6 + 5 = 36 and 36 is divisible by 9
Divisible by 10	If a number ends in 0, it is divisible by 10
Divisible by 11	If the sum of the odd positioned digits differs from the sum of the even positioned digits by zero or a multiple of 11, the number is divisible by 11. e.g. 837 265 825 is divisible by 11 because 8 + 7 + 6 + 8 + 5 = 34 3 + 2 + 5 + 2 = 12 and 34 – 12 = 22, which is divisible by 11
Divisible by 12	If a number satisfies the ‘divisibility by 3’ test and the ‘divisibility by 4’ test then it is divisible by 12

☞ 3.2 Activity 5

Use the rules of divisibility to test the following numbers for factors

(a) 435

(b) 516

(c) 8229

(d) 987 345 244

(e) 243 987 678

(f) 789 768 543

☞ Extension Activity– a divisibility problem (AHA)

The general was inspecting his troops as they stood two abreast ready to march into battle. He was dismayed to find that one man was left over. "That's odd" he muttered and then ordered his men to reform into threes. Now things were worse than before – there were two men left over. And when he tried fours, there were three left over. With fives there were four left over, with sixes there were five over, with sevens, six over.

Now knowing that there were not more than 800 troops on parade, exactly how many were there?

Topic 3 – Prime Numbers

A prime number is a number which has exactly two factors: itself and one. For example 5 is a prime number because it has only two factors, 5 and 1. Numbers, which are not primes, are called composite numbers. For example, 4 is a composite number because it has more than 2 factors. Its factors are 1, 2 and 4.

The number 1 is special: it is not a prime and it is not a composite.

The Greek mathematician Eratosthenes invented a quick way of identifying the prime numbers in a set of counting numbers starting at 1. It is called the sieve of Eratosthenes.

☞ 3.2 Activity 6

Work through the following method made famous by Eratosthenes to identify all the prime numbers between 1 and 20.

To find all the prime numbers between 1 and 20, list down all the numbers

1	**2**	**3**	**4**	**5**	**6**
7	**8**	**9**	**10**	**11**	**12**
13	**14**	**15**	**16**	**17**	**18**
19	**20**				

Then

1. *Cross out 1 because it is not a prime number*
2. *Go to the next number, which is 2, and circle it. Then cross out all the multiples of 2 (consider why these numbers are not primes)*
3. *Go to the next number which is not crossed out. This should be 3. Circle it. Then cross out all the multiples of 3.*
4. *Go to the next number which is not crossed out. This should be 5. Circle it. Then cross out all the multiples of 5.*
5. *Repeat for the next number which is not crossed out. Keep repeating until there is no 'next number'.*

The prime numbers between 1 and 20 are the numbers which you have circled. These should be 2, 3, 5, 7, 11, 13 and 17.

☞ 3.2 Activity 7

Using the sieve of Eratosthenes find:

- *all the primes from 1 to 100*
- *all the primes from 1 to 150*
- *all the primes from 1 to 200*

Note the last number that you needed to circle which resulted in a number after it being crossed out. Use the information you have found out to consider the following questions:

- *How can you determine how far you need to go before no new numbers will be crossed out?*
- *How far would you need to go to establish if 1003 is a prime? Use the calculator not the sieve to find this out.*

Once you reach the square root of a number, you do not need to go any further. If there is a factor less than the square root, there cannot be a factor which is larger as this would require the other factor to be less, which has been rules out.

☞ 3.2 Activity 8

Having established the relationship between finding the square root and identifying if a number is a prime number find out if the following numbers are primes

1051

891

947

Consider why is it only necessary to test for prime numbers as factors when finding out if these numbers are prime?

Be prepared to discuss your findings with the class.

☞ Extension Activity - Generating primes

Work in a small group to complete one of the following investigations.

(1) Over 2000 years ago, Euclid proved that there were infinitely many primes. Over the centuries mathematicians have been fascinated by prime numbers and have looked for formulae that will:

- *produce all existing primes*
- *always result in a prime*

By taking values for n (or p) try to determine whether the following formulae are true.

(i) $n^2 - n + 1$ *will always result in a prime for n=1,2,3,4,… (Euler)*

(ii) $2^p - 1$ *will always result in a prime for p = a prime number (Mersenner)*

(2) 'Every odd number greater than 5 can be expressed as the sum of three primes' e.g. 7 = 2 + 2 + 3

(3) 'Every odd number greater than 3 can be expressed as the sum of a prime and some power of 2'

e.g. to check 59	$59 - 2 = 57$ *(not prime)*
$59 - 2^2 = 55$ *(not prime)*	$59 - 2^3 = 51$ *(not prime)*
$59 - 2^4 = 43$ *(prime)*	*Hence* $59 = 43 = 2^4$

Topic 4 – Pythagorean Numbers

Greek mathematicians had a strong interest in numbers and attached 'mythical' significance to some numbers. The Pythagoreans classified numbers according to the sum of their proper factors.

If the sum of a given numbers proper factors is greater than the number itself, the Pythagoreans said the number was 'abundant'. For example, the proper factors of 12 are 1, 2, 3, 4 and 6. The sum of these proper factors is 1 + 2 + 3 + 4 + 6 = 16. As 16 is greater than 12, then 12 is an abundant number.

If the sum of a given numbers proper factors is less than the number itself, the number is said to be 'deficient'. For example, the proper factors of 8 are 1, 2, 4. The sum of these proper factors is 1 + 2 + 4 = 7. As 7 is less than 8, then 8 is a deficient number.

If the sum of a given numbers proper factors is equal to the number itself, the number is said to be 'perfect'. For example, the proper factors of 28 are 1, 2, 4, 7, 14. The sum of these proper factors is 1 + 2 + 4 + 7 + 14 = 28. As the sum equals the number, 28 is a perfect number.

Two numbers are said to be 'Amicable' or 'Friendly' if the proper factor sum of both numbers are equal

☞ 3.2 Activity 9

(a) Investigate the following numbers and classify then as either abundant, deficient or perfect:

20	*25*	*100*	*13*	*45*	*496*

(b) Examine a range of prime numbers to see if they are abundant, deficient or perfect. Explain your findings and the reasons for this.

(c) Show that 220 and 284 are amicable.

☞ Extension Activity –An investigation

Can you find an odd abundant number? Investigate!

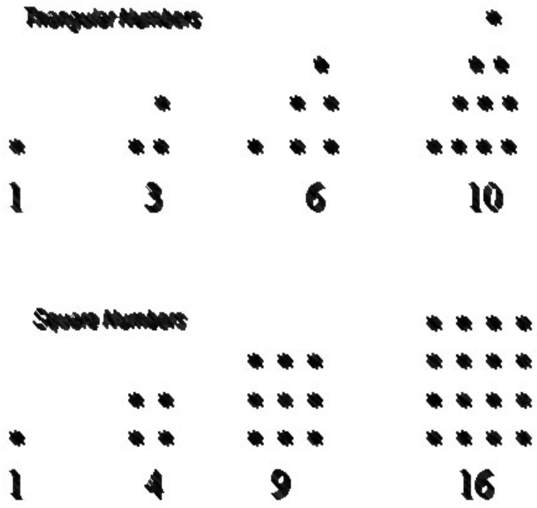
References – for further reading

Department of Education Papua New Guinea (1998). *Lower Primary Mathematics Syllabus Grade 3-5*

Department of Education Papua New Guinea (draft 1999). *Upper Primary Mathematics Syllabus Grade 6-8*

Green, Wally (1998) 'Enrichment Topics in Number' Department of Education PNG

Module 3.3 – Number Sequences and Patterns



Module 3.3 Number Sequences and Patterns is core module in the unit ***'Enrichment Topics in Number'***. During this module you will explore different number sequences and patterns, identifying general rules to describe these as well as creating your own patterns and sequences. You will investigate number sequences derived from geometric figures. It is important that, as future teachers, you have a sound understanding of patterns and sequences as this provides the foundation for the development of further mathematical understandings.

Objectives

By the completion of this module you will be able to:

- extend given sequences
- develop a sequence of numbers based on a given rule
- find a general rules that describes the 'nth' term of a given sequence
- explore figurative numbers
- investigate difference patterns of sequences and describe their findings
- investigate the use of sequences to help generalise geometric information.

Concepts and skills

- Sequence, term and general term of sequence
- Rules to determine sequence
- Difference patterns in sequences
- Numbers described by geometric properties
- Sequences in geometry
- Deduction
- Computation
- Investigation
- Oral presentation

Topic 1 – Number Sequences

Finding the rule for a particular number pattern is referred to as finding the nth term, where n is the number of the term or its position in the sequence. To find the nth term you need to closely look at the sequence of numbers and identify the pattern.

Below is an example of how to you could find the nth term for a sequence of numbers.

Take the sequence

7, 10, 13, 16, 19, ……

One of the things we notice straight away is that each number is 3 more than the previous number e.g. 10 is three more than 7, 13 is three more 10 etc. It is simple to find the next few terms by just counting by 3's. However what if we need to find the 65^{th} term without writing out the whole sequence? Can we find the relationship between the numbers in the sequence and their position in the sequence (n)?

One strategy we could use is to draw a table to show the relationship between the numbers in the sequence and their position in the sequence (n).

Position (n)	**1^{st} term**	**2^{nd} term**	**3^{rd} term**	**4^{th} term**	**5^{th} term**	**…**	**nth**
Numbers in sequence	7	10	13	16	19	…	
Pattern	2x3 + 1	3 x 3 + 1	4 x 3 + 1	5 x 3 + 1	6 x 3 +1	…	3n + 4

From the information on the table we then need then to look for patterns.

For example you may notice that:

When n = 1 2 x 3 + 1 = 7

When n = 2 3 x 3 + 1 = 10

When n = 3 4 x 3 + 1 = 13 etc.

From this we can see that each number in the sequence is multiplied by 3 and then 1 added. We can also see that we need to add 1 to n to know how many times to multiply by 3 before adding 1.

This can be expressed as (n + 1) x 3 + 1 which is 3n + 3 + 1

Which can be simplified to 3n + 4

The general rule or the nth term for this sequence is therefore 3n + 4.

To find the 65^{th} term all we need to do is substitute 65 for n to find our answer, that is the 65^{th} term will be (3 x 65) + 4, which is 112.

☞ 3.3 Activity 1

Find the next 3 terms for each of the following sequences and find the expression for the nth term.

(a) 2, 4, 6, 8, 10, …., ….., ……, *nth term ……….*

(b) 3, 6, 9, 12, …., ….., ….., *nth term ……….*

(c) - 4, -2, 0, 2, 4, ….., ….., ….., *nth term ……….*

(d) 10, 10^2, 10^3, 10^4, ….., ……, ……, *nth term ……….*

(e) 9, 5, 1, -3, -7, ….., ….., ……, *nth term ……….*

(f) 2, 4, 8, 16, 32, ….., ….., ……, *nth term ……….*

☞ 3.3 Activity 2 - Problem solving

Work with a partner and choose two of the following problems to solve. Be prepared to share your answers.

1. *The doctor ordered Michael to take tablets every 9 hours. If he took the first tablet at 11a.m. at what time will Michael take the 26^{th} tablet?*
2. *If there are 4 seats and if 2 are occupied like this _ X _ X (where X represents a person), then the next person must sit next to someone. If there are 5 seats and if 2 are occupied like this: _ X _, _ X, then the next person must sit next to someone.*

   If there are a limited number of seats in a row:

   (a) *What is the fewest number of seats that must be occupied so that the next person to be seated must sit next to someone?*

   (b) *Show that if there are 12 seats, 4 must be occupied, and if there are 13 seats, 5 must be occupied for this to happen*

   (c) *Develop a generalisation*

   (d) *If there are 150 seats in a row, how many seats must be occupied so that the next person to be seated must sit next to someone*
3. *Sandra is making square patterns with square tiles using the pattern shown. She has yellow and red tiles. When she adds 5 red tiles her*

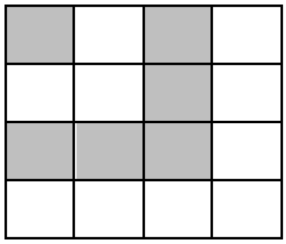
large square is 3 X 3. When she adds 7 yellow tiles, her large square is 4 X 4.

(a) *Find the size of her large square, if she has just added:*

(i) *27 tiles*

(ii) *65 tiles*

(b) *If the first tile was a red tile, work out in each case what colour tile she has added to the pattern last in the above questions.*



4. *Three hens lay four eggs in five days. How many days will it take a dozen hens to lay four dozen eggs. Generalise*

Topic 2 – Sequences in Geometry

Figurative numbers

The Pythagoreans studied number sequences derived from geometric figures. These are referred to as figurative numbers.

For example:

Triangular numbers

```
                                          *
                              *         * *
              *             * *       * * *
  *         * *           * * *     * * * *
  1                         6         10
     3
```

Square Numbers

```
                                    * * * *
                          * * *     * * * *
            * *           * * *     * * * *
  *         * *           * * *     * * * *
  1                         9         16
     4
```

☞ 3.3 Activity 3 – Figurative numbers

Work in pairs to choose a geometric figure and investigate its number sequence. Draw diagrams and tables to illustrate your findings and try to find a general rule for the number sequence. Be prepared to discuss your findings with the class.

Choose from the following geometric figures

- *square*
- *triangle*
- *pentagon*
- *hexagon*

☞ 3.3 Activity 4 – Extension activity

Square and triangular numbers

Use a diagrammatic representation to find a relationship between square and triangular numbers. State the general relationship

(Hint: take 2 consecutive triangular numbers and arrange them to form a square, repeat for other pairs of triangular numbers)

Topic 3 – Determining Differences

Difference patterns

In many sequences there is a constant difference between consecutive terms. In others sequences it may take 2, 3 or more steps to find a constant difference.

For example

1. Sequence: 3 5 7 9 11 13 ……..

   First difference 2 2 2 2 2 (constant difference)

2. Sequence: 1 3 6 10 15 21 ……..

   First difference 2 3 4 5 6
   Second difference 1 1 1 1 (constant difference)

3. Sequence: 4 11 30 67 128 219

   First difference 7 19 37 61 91
   Second difference 12 18 24 30
   Third difference 6 6 6 (constant difference)

Once you have identified the constant difference it is possible to find the next term in the sequence by working backwards. For example in (3) the third difference is constant and is 6.

Therefore the second difference is 30 + 6 = 36

Therefore the first difference is 91 + 36 = 127

Therefore the next term in the sequence is 219 + 127 = 346

☞ 3.3 Activity 5 – Finding the difference

Find the constant difference for each of the following number sequences and then find the next 3 terms.

(a) 9, 15, 38, 83, 105, ….., ……., …….. constant difference…….

(b) 18, 41, 77, 126, 188, ……, ……., …… constant difference…….

(c) 3, 26, 91, 207, 383, ……, ……., …… constant difference…….

Quadratic general term

Sequences which have a constant second difference have a quadratic general term of $an^2 + bn + c$ which can be used to find the general term of a specific sequence.

For example: Find the general term of a sequence of triangular numbers.

Sequence:		1		3		6		10		15	21
First difference	2		3		4		5		6	
Second difference				1		1		1		1

If the general term is $\mathbf{an^2 + bn + c}$

		1st Difference	2nd Difference
Then the 1st term, where n = 1 is	$a(1^2) + 1b + c$ or $\mathbf{a + b + c}$		
		3a + b	
The 2nd term, where n = 2	$a(2^2) + 2b + c$ or $\mathbf{4a + 2b + c}$		**2a**
		5a + b	
The 3rd term, where n = 3 is	$a(3^2) + 3b + c$ or $\mathbf{9a + 3b + c}$		

Therefore it follows that 2a = 1 (the second difference) so **a = ½**

and that 3a + b = 2 (the first difference) so b = ½

and a + b + c = 1 (the first term) so **c = 0**

Therefore the general term for triangular numbers is $\mathbf{\dfrac{n^2 + n}{2}}$

☞ 3.3 Activity 6 – Using the quadratic general term

Using the quadratics general term $an^2 + bn + c$ for number sequences which have a constant second difference, find the general term for the following sequences:

(a) 1, 5, 12, 22, …..

(b) 1, 2, 7, 16, 29, ……

(c) 6, 20, 42, 74, 104, ……

☞ 3.3 Activity 7 – Extension activity

Work with a partner to complete one of the following investigations. Be prepared to share your findings with others.

1. *Draw straight lines in a plane such that no two of them are parallel and no three of them pass through the same point. Investigate the number of points of intersection that are made.*

   Can you see a relationship between the number of points and the number of lines?

   Are the differences constant?

   Is there a pattern in the ratio $\frac{\text{number of points}}{\text{number of lines}}$?

   Can you make up a formula giving the number of points for n lines?

2. *A six sided polygon has 9 diagonal. Investigate the number of diagonals in a polygon.*
3. *Two intersecting chords divide a circle into a maximum number of regions. Investigate the relationship between the number of chords and the maximum number of regions*
4. *If there are 4 points on the circumference of a circle and we join them in all possible ways then the circle is divided into a maximum number of 8 regions. Investigate the relationship between the number of points and the maximum number of regions.*

References – for further reading

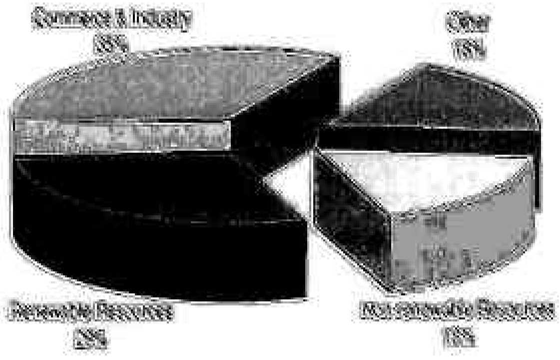
Department of Education Papua New Guinea (1998). *Lower Primary Mathematics Syllabus Grade 3-5*

Department of Education Papua New Guinea (draft 1999). *Upper Primary Mathematics Syllabus Grade 6-8*

Green, Wally (1998) 'Enrichment Topics in Number' Department of Education PNG

Module 3.4 – Basic Statistics

Module 3.4 Basic Statistics is a core module in the unit ***'Enrichment Topics in Number'***. Statistics provide us with a range of information and basic facts on a variety of topics which we need to make judgements and decisions about. During this module you will examine the use of statistics and their purpose. Data will be collected, interpreted and represented in a variety of ways. You will also consider how statistical information can be misleading.



Objectives

By the end of this module you will be able to:

- collect statistical information
- interpret data
- construct graphs to represent data
- calculate measures of central tendency
- calculate the range.

Concepts and skills

- Collecting data
- Representing data in various ways e.g. graphs, tables
- Selecting appropriate methods of analysis
- Interpreting
- Oral skills
- Writing skills

Introduction

📖✍ 3.4 Activity 1

Read the following article which discusses the use of statistics. Discuss with a peer what you understand statistical information is used for. Write down 3 different example of statistical information and how this information may be used.

> *For example, statistical information collected in 2001 on birth rates in Enga Province may be used to predict how many primary school teachers will be needed in this Province by the year 2010*

Using statistics

*(Adapted from **Family Numbers –Using Statistics** by Zbar, V., Cropley, M., & Rowland, M. Curriculum Corporation 1998)*

Statistics can provide us with the information we need to understand a topic so that we can make decisions about it. Statistics help us to:

- *Understand how things are at present*

  In order to discuss what we should do to in the future we need to collect information that will help us to understand the important things influencing what is happening today.

- *Predict*

  Examining what we know helps us to estimate what might happen in the future. For example, a consideration of trends over the previous 10 years in people's eating habits can enable health promotion agencies to work out what will happen in the future.

- *Plan*

  Considering the range of relevant available data assists us to plan appropriate courses of action. For instance, an examination of the age structure of the population is an important concern when planning for the provision of educational services.

- *Evaluate/change direction*

  Collecting information on how well programs are working enables us to decide if the plan needs to be changed. For example, an increase in the number of children attending school might lead the Government to change its policies related to school fees and subsidies.

Statistics can be used in a variety of ways depending on the nature of the material and the focus of your investigation. For example, in looking at data on birth rates we may be interested in the average or mean at any particular time. Alternatively, we may want to focus on the growth in birth rates over a specific period.

When it comes to telling other people about the data, there are many options available to us. We may choose to use graphs such as bar and line graphs or we may prefer to use tables or diagrams of other sorts. The choice is up to us. Some methods are better for displaying particular data than others.

The important thing, however, is to ensure that we use the statistics faithfully. In other words, use them to convey a true picture of what is happening, rather than trying to make things look good or bad depending on our point of view.

Case study – Gender Analysis in Papua New Guinea

Below you will find an example of a report containing statistical information collated by the Papua New Guinea Office of National Planning. This report was published in the 'Human Development Report 1998' and will be used by policy makers to make decisions for improving the lives of people at a national, provincial and district level.

3.4 Activity 2

Read the report above on 'Gender Analysis' in PNG identify:

- *the type of information gathered*
- *how the information is represented*
- *what the information tells us*
- *how this information could be used for future planning*

Write down what you have found out and be prepared to discuss it in class.

Gender analysis

*(adapted from the '**Human Development Report 1998**' Office of national Planning, 1999)*

INTRODUCTION

Analysis of human development indicators by gender reveals widespread inequality between men and women in Papua New Guinea. This section focuses on gender disparities as measured by the Gender-related Development Index (GDI) and Gender Empowerment Measure (GEM). It examines more closely the trends in women and capabilities by analysing indicators for education, life expectancy, and fertility. It also examines levels of women's participation in formal sector employment and in politics.

The GDI measures achievements in gender development by means of the three core indicators employed by the Human Development Index (HDI) - life expectancy at birth, educational attainment, and income. The GDI has a maximum value of 1.000 that represents equality and a minimum value of 0.000 that represents absolute inequality between males and females in terms of life expectancy, education, and income. The GEM focuses on opportunities by measuring inequalities between men and women in key areas of economic and political participation and decision-making. Like the GDI, the GEM values range between 1.000 and 0.000.

Three other tables of data are reviewed for this discussion. These are Women and Capabilities, Trends in Women and Capabilities, and Women and Political and Economic Participation. Gross enrolment ratios at all levels of education - elementary, primary, secondary, and tertiary are reported for 1996 and 1980 as well as total fertility rate and female life expectancy at birth. The Women and Political and Economic Participation table reports the percentage of women represented in politics in local level governments. The table also reports the percentage of females employed as a percent of total persons employed and the number of females employed to every 100 males in five main formal classifications.

This section does not discuss violence against women. That subject is reviewed in the Law and Order analysis.

SUMMARY

Achievements in gender equality have been poor throughout Papua New Guinea. The GDI shows that in every province, male achievement in life expectancy, educational attainment, and income are greater than female achievement. The GEM highlights widespread inequality between men and women in terms of economic and political participation and decision-making.

Nationally, the GDI and GEM values are 0.318 and 0. 177 respectively. All provincial GDI and GEM values are less than 0.500 with the exception of the National Capital District, which has a GDI of 0.506. The national ranking of provinces shows that the GDI rank for some provinces is less than their HDI rank, which suggests that achievements in human development do not always translate to achievements in gender equality.

While gender disparity is widespread throughout Papua New Guinea, provincial values of GDI and GEM indicate that Southern and Islands regions have been more successful than the other two regions in promoting equal opportunities for women. Women in the Momase and Highlands regions face more inequality than women in the other two regions do. Provinces in the Highlands region tend to be confined to the lower GDI and GEM rankings.

The difference between GDI and GEM national values indicates that relative improvement in human development for women over the last two decades has had minimal impact on their economic and political participation. The majority of the female labour force in Papua New Guinea is engaged in agricultural production, primarily as subsistence food producers, while only 20 percent is engaged in formal sector employment.

In politics, at the national level only two out of 109 elected members of parliament are women. The 1995 Organic Law on Provincial and Local Level Governments constitutionally mandates the appointment of women's representatives to local level councils. The proportion of local level council members who are women, which ranges from 3 percent in Southern Highlands to 9 percent in Oro. A number of provincial and local level governments have yet to make the mandated appointments.

COMPARISONS AND CONTRASTS

Gender-related Development Index

The 1998 national GDI value is 0.318. Internationally, Papua New Guinea is ranked 159 out of 175 countries. The national summary of the four variables used to measure the G131 reveals considerable gender imbalance. Female life expectancy at birth is 53.5 years whereas male life expectancy at birth is 54.6 years. Papua New Guinea is one of the few countries, maybe the only country, in the world where female life expectancy is less than male life expectancy. A major gender gap exists in earned income share. Data show that females earn only 16 percent whereas males earn 84 percent. Data on educational attainment show that the national rate for female adult literacy is only 40 percent whereas the male adult literacy rate is 50 percent. The combined gross enrolments ratios for elementary, primary, and secondary schools show that the female ratio is 46 whereas the male ratio is 49, perhaps indicating that females are beginning to attain nearly equal recognition for education.

Table 3.4.1 shows a wide range of GDI values per province from 0.506 for the National Capital District to 0.234 for Sandaun Province.

Provinces that rank in the top five after The National Capital District are East New Britain, Manus, Milne Bay, and Central, which are all located in either the Southern or Islands regions. The next four provinces are also in these two regions, indicating a regional bias in the level of development opportunities for women. East Sepik, Western Highlands, Southern Highlands Enga, and Sandaun occupy the bottom five places.

Several general observations can be made from the GDI values and ranks presented in Table 3.4.1. Firstly, the GDI values indicate that in no province do women and men have equal opportunities for achievement. With the exception of the National Capital District, all provinces have a GDI value less than 0.500, indicating that the low levels of human development throughout the country are compounded by gender disparities.

Table 3.4.1 Gender Disparities: GDI and HD1 Ranks and Values

GDI Rank	Province	HDI rank	GDI Value	HDI Value	HDI Rank Minus GDI Rank
1	National Capital District	1	0.506	0.758	0
2	East New Britain	3	0.395	0.431	+1
3	Manus	4	0.393	0.421	+1
4	Milne Bay	5	0.387	0.420	+1
5	Central	6	0.384	0.331	+1
6	Western	2	0.381	0.472	-4
7	New Ireland	7	0.376	0.396	0
8	West New Britain	8	0.368	0.394	0
9	Oro	10	0.356	0.386	+1
10	Morobe	9	0.330	0.389	-1
11	Madang	11	0.313	0.336	0
12	Gulf	12	0.312	0.331	0
13	Simbu	14	0.298	0.320	+1
14	Eastern Highlands	13	0.279	0.325	-1
15	East Sepik	15	0.282	0.336	0
16	Western Highlands	17	0.261	0.282	+1
17	Southern Highlands	18	0.244	0.274	+1
18	Enga	16	0.243	0.283	-2
19	Sandaun	19	0.234	0.262	0
N/A	Bougainville	Data not available			

Source: Gender-related Development Index (table 8), Appendix 2 1998 Human Development Report

Secondly, all provinces have a GDI value that is less than their HDI value. This means that throughout the country gender disparities limit the capabilities and choices that comprise human development.

Thirdly, comparing the GDI ranks of the provinces with their HDI ranks confirm that achievement in human development does not necessarily mean achievement in gender equality. Enga, Eastern Highlands, Morobe, and Western provinces have negative results when their GDI ranks are subtracted from their HDI ranks, which suggests that they are doing poorer in terms of gender development compared with overall human development.

Gender Empowerment Measure

The 1997 GEM value for Papua New Guinea is extremely low at only 0. 177. The national summary of the variables used to measure the GEM shows that opportunities for women to participate in economic and political life are severely limited. Data on formal sector employment for 1990 indicate that only 12 percent of administrators and managers are women, only 19 percent of professional workers are women, and only 7 percent of technical workers are women. Women hold only 1.8 percent of elected offices in national parliament, and only an average of 8.6 percent of the seats on local level councils.

Table 3.4.2 Gender Disparity: GEM Ranks and Values and GDI and HDI Ranks

GEM Rank	Province	GEM Value	GDI Rank minus GEM Rank	HDI Rank minus GEM Rank
1	National Capital District	0.275	0	0
2	Oro	0.255	+8	+8
3	New Ireland	0.216	+4	+4
4	Central	0.213	+1	+2
5	East Sepik	0.212	+10	+10
6	Morobe	0.208	+4	+3
7	Milne Bay	0.204	-3	-2
8	East New Britain	0.203	-6	-5
9	Eastern Highlands	0.200	+5	+4
10	Madang	0.199	+1	+1
11	Gulf	0.196	+1	+1
12	West New Britain	0.191	-4	-4
13	Western Highlands	0.188	+3	+4
14	Western	0.187	-8	-12
15	Simbu	0.177	-2	-1
16	Manus	0.162	-13	-12
17	Sandaun	0.144	+2	+2
18	Enga	0.143	0	-2
19	Southern Highlands	0.132	-2	-1
[illegible]	[illegible]	[illegible]		

Source: Gender Empowerment Measure (table 9), Appendix 2, 1998 Human Development Report

Table 3.4.2 shows relatively little variation in GEM values at the provincial level. The GEM values range from a high of 0.275 for the National Capital District to a low of 0. 132 for the Southern Highlands. It is apparent from the very low GEM that Papua New Guinea as a whole has a long way to go in promoting and facilitating broad economic and political opportunities for women.

Some general observations can be drawn from the provincial GEM values. As with GDI values, the GEM values for all the provinces are far removed from perfect equality. Significantly, all provinces have values less than 0.300, indicating that a large gap needs to be filled to reach the half-way mark for achieving gender parity in economic and political opportunities.

Secondly, all the provincial GEM values are less than the HDI and GDI values. This means that in every province gender inequality not only exists in terms of longevity, education, and income, but also in terms of participation in economic and political spheres of society and decision making.

Thirdly, a comparison of the provincial HDI, GDI, and GEM ranks reveals that some provinces have relatively high H-DI and GDI ranks but low GEM ranks. This shows that these provinces are making gains towards gender balance in life expectancy or educational attainment, or both but are not seeing achievements in women's economic and political participation.

Topic 1 – Collecting Data

Statistical data can be collected using a range of different methods. The method that you choose to collect statistical data will vary according to the:

- type of information you need to collect (information from people, observed trends, temperature readings etc)
- people who will be required to provided the information (e.g. literate/illiterate, urban/rural)
- purpose of the data (who is the audience, how will the data be used?)

For example a National Census was conducted in PNG in 2000 after a major publicity campaign took place advising people about what the census would involve and the dates it would be conducted. Teams of people were allocated to the different council wards and trained in how to carry out the census. A census worker visited each household in PNG. The census worker had a series of questions to ask the head of the household and the census worker recorded the information received on the census form. By employing this method of data collection, the National Census office was able to ensure:

- all people in the country were able to be considered in the census (regardless of where they were living and whether they were literate)
- all people were asked the same questions
- the data collected from all the different council wards could be collated and compared.

Types of data

There are two main types of data that is collected by statistics: nominal (categorised) and ordinal. Nominal data is data which cannot be ordered and can only be classified by the name of the category from which it is obtained. For example, if people were asked if they support Queensland Maroons or NSW Blues they could answer ‘Maroon’, ‘Blues’ or ‘neither’.

Ordinal data is data which can be ordered, for example the different speeds of PMV’s travelling along a road. Ordinal data can be divided into two types: discrete and continuous.

- Discrete data is data which can only take particular distinct values, for example the number of cans of coke that customers purchase from a trade store
- Continuous data is data which does not have a distinct value, for example the height of a group of people

Methods of collecting statistical data include:

- Questionnaires
- Surveys
- Recording observations (e.g. number of cars which use a particular road)

☞ 3.4 Activity 3

Choose a topic of interest and identify the statistical data which you would like to collect.

Plan a method of collecting the data taking into consideration:

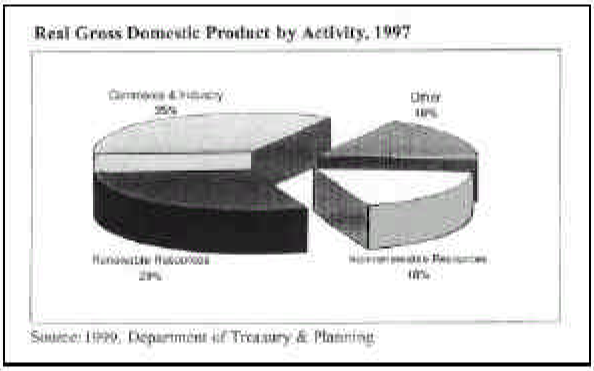
- *the type of data required (nominal or ordinal)*
- *who you will be collecting the data from*
- *the purpose of the data*

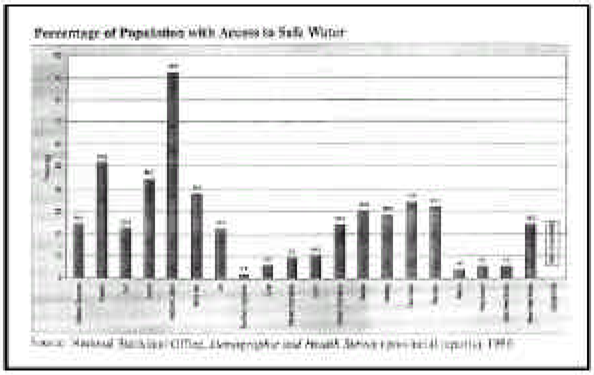
Form a small group to work together to collect the data

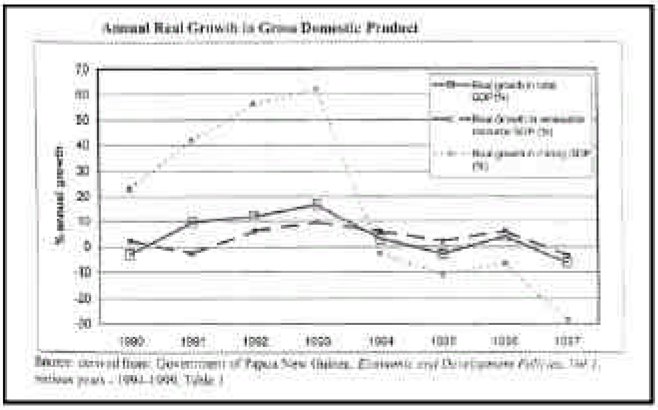
Topic 2 – Representing Data

Once collected, statistical data can be organised in a range of different ways. For example, the information collated can be represented through tables, graphs (picture, line, bar, pie/section) or diagrams.

Below are examples of different ways of representing statistical data.







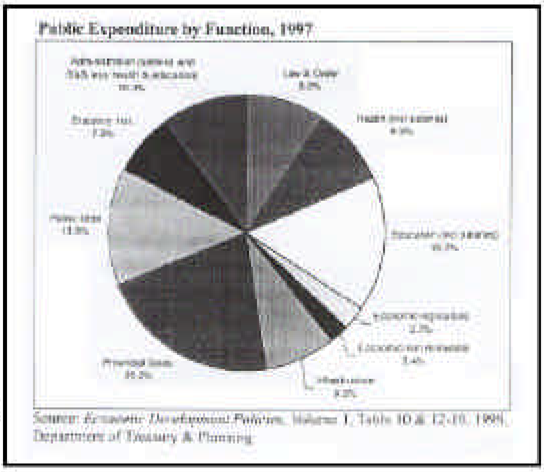
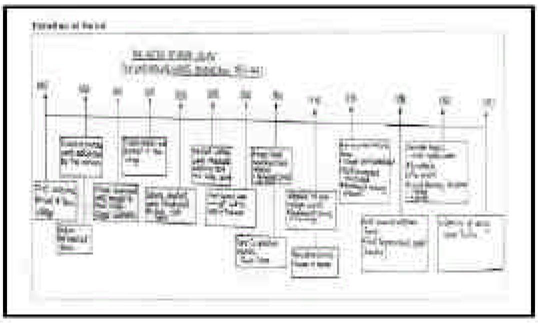
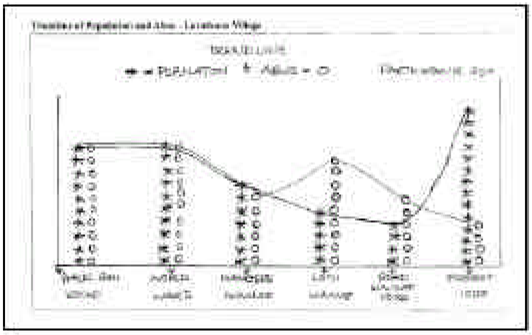


Table-Levels of Education Attained in Yau

Grade Completed	Youth		Adults	
	Male	Female	Male	Female
10	1	-	10	6
9	1	-	2	-
8	2	3	9	9
7	2	-	4	-
6	45	21	68	57
5	3	5	3	2
4	3	6	2	5
3	8	4	5	8
2	4	8	7	13
1	2	3	-	4
Not Stated	6	5		
TOTAL	77	55	110	104





The best statistical graph

It is difficult to say that one type of statistical graph is always better than another. It depends on the type of information that needs to be graphed. Sometimes one of them is definitely the best choice. Other times two or three types are equally effective.

The following are good general guidelines:

- Use line graphs if you want to see how something changes over time e.g. average monthly salary of a primary school teacher from 1980 to 2000
- Use pie graphs or section graphs if you want to compare different parts of a whole and there's no time element involved e.g. provision break down of the student body at the college
- Use bar graphs if you are not looking at changes over time and your categories are not parts of a whole e.g. favourite recreational activities of a group of people.

☞ 3.4 Activity 4

Represent the data collected in 3.4 Activity 3. Consider the type of data you have collected when making the decision about how best to represent it. You may wish to present your data in a number of different ways e.g. a table and a pie graph.

Measures of central tendency

When there is a large amount of data, statisticians need a way of finding a typical value that represents this information.

There are three different measures that describe the centre of a series of numbers. These measures of central tendency are:

- the mean or the average
- the median or middle number
- the mode or the most frequent number

Each of these measures of central tendency is calculated in a different ways and provides us with a way of finding a typical value.

For example:

For the first 6 basketball games played this year Margaret scored the following number of goals:

22, 6, 19, 18, 6, 16

Using this data we can calculate the mean, the median and the mode.

$$\text{Mean} = \frac{\text{sum of numbers}}{\text{number of scores}}$$

$$= \frac{22 + 6 + 19 + 18 + 6 + 16}{6}$$

$$= 14.5$$

The formula is $\bar{x} = \frac{?x}{n}$ where $\bar{x}$ stands for the arithmetic mean

? (a Greek capital letter pronounced sigma) stands for the summation of values

x refers to the value of the observation

n is the number of observations

Median: Put the scores in order of size first

6, 6, 16, 18, 19, 22

The middle score would be between 9 and 16 so we take the average or arithmetic mean

$$\text{Median} = \frac{16 + 18}{2} = 17$$

Mode: The mode is the most frequent occurring number. In this situation the mode is **6.**

Note: if a sample has two measures which occur with the same frequency then the distribution is said to be bi-modal. However, if more than two values share the highest frequency, then the distribution is said not to have a modal.

Each of these measures provides us with a different result and tells us a different story.

Measures of spread or dispersion

Statistical information can also be analysised according to measures of spread. Measures of spread consider the differences between the values within a sample.

For example, in a Year 2 Teachers College class, there are 20 students. Their ages are:

18, 21, 23, 22, 22, 30, 25, 19, 19, 25, 23, 22, 24, 22, 21, 20, 23, 25, 22, 23

Range

The range is the difference between the highest and lowest value of the data.

The range of the Year 2 student ages is the difference between 18 (the youngest student) and 30 (the oldest student). The range is therefore 12 years.

The interquartile range

To calculate the interquartile range the distribution of scores is divided into quarters and the interquartile range is defined as the difference between the upper and lower quartiles.

18, 19, 19, 20, 21, 21, 22, 22, 22, 22, 22, 23, 23, 23, 23, 24, 25, 25, 25, 30

lower quartile upper quartile

The lower quartile is the point of division of the lower-value half e.g. 21 and the upper quartile is the point of division of the high-value half e.g. $\frac{23 + 24}{2} = 23.5$

The interquartile range is therefore 23.5 - 21 = 2.5

The advantage of finding the interquartile range is that it removes the effects of the values at the extremities. It represents the range of the middle 50% of numbers.

☞ 3.4 Activity 5

Consider the data collected in 3.4 Activity 3 and represented in 3.4 Activity 4. Find the measures of central tendency for this data and calculate the range and interquartile range. Consider:

- *which measure of central tendency gives you the most accurate picture*
- *what these calculations are telling you*

Be prepared to discuss your findings and to explain what you have found out by calculating measures of central tendency and measures of spread.

Topic 3 – Interpreting Data

Once statistician have collected data, represented it through graphs, tables and by calculating measures of central tendencies and spread where appropriate, an interpretation is necessary. The interpretation tells us what information is provided by the data that will help us with to predict, plan and evaluate the need for change.

For example consider the table below which looks at the number of schools and colleges in Papua New Guinea and their student enrolments. (Statistics Digest 1999, p 43)

Table-Levels of Education Attained in Yau

Grade Completed	Youth		Adults	
	Male	Female	Male	Female
10	1	-	10	6
9	1	-	2	-
8	2	3	9	9
7	2	-	4	-
6	45	21	68	57
5	3	5	3	2
4	3	6	2	5
3	8	4	5	8
2	4	8	7	13
1	2	3	-	4
Not Stated	6	5		
TOTAL	**77**	**55**	**110**	**104**

By studying the data provided by the table you can see that there has been a marked increase (27.8%) in the number of Primary Schools over the three years covered by this data (1995 – 1997). On the other hand, the number of Vocational Schools has decreased (by 10.5%) over the same period. The number of universities in PNG has increased from 2 to 3, and although it is only one additional university it represents an increase of 50%. Other educational institutions such as Teachers Colleges and Technical Colleges have had minimal changes in their numbers.

When we examine the changes in student enrolments over the three years you can see that all institutions, apart from Technical Colleges, have experienced an increase in student enrolments. The size of this increase varies across institutions with Vocational Colleges having the biggest increase of 52% while the student numbers at Universities only increased by 4.2%. Enrolments in Technical Colleges decreased by 7%.

When we compare the two sets of data, that is the number of institutions and their student enrolments over the three-year period, we can identify a number of significant trends. For example if we look at the data for Teachers Colleges we can see a disproportional rise in enrolments compared to the increase in the number of institutions. Teachers Colleges enrolment numbers have increase by 25.3%, where as the number of colleges available to cater for these students has not increased at all.

A comparison of the data dealing with Vocational Schools show that while the enrolment of students attending Vocational Schools increased by 52%, the actually number of institutions available for these students to attend decreased by 10.5%.

These two examples suggest the need for additional Teachers Colleges and Vocational Colleges if the increases in student enrolments are to be catered for.

The data shows that the number of Primary Schools and Secondary Schools available to students has increased over the three-year period at a rate greater than the rate of enrolment increase. That is, the number of Primary Schools increase by 27.8% over this period when their enrolment figures only increased by 17.6%. In the case of Secondary Schools the number o f institutions increased of 12.40%, where as student numbers increased by only 6.7%. These figures suggest that the Education Department is in a better position in 1997 than it was in 1995 to cater for students at both the primary and secondary level.

✍ 3.4 Activity 6

Write an interpretation for the data you have collected and represented in the previous activities. Your interpretation should:

- *explain your results*
- *identify patterns and relationships*
- *conclusions which can be made.*

Topic 4 – Analysing Statistical Data

When examining statistical data we need to be sure that the information we are interpreting is conveying a true picture of what is happening. We need to be sure that the statistical information has not been misinterpreted or misrepresented and ensure that someone is not trying to make things look good or bad depending on their particular point of view.

3.4 Activity 7

Read the following article 'Abusing Statistics'. After reading the article review the statistical data above which looked at the number of Educational Institutions in PNG and their enrolments. Together with a peer write down a list of questions you would need to ask to confirm that the data and the interpretation provides a 'true picture' of the particular situation e.g. the validity of the data.

Consider what questions you would ask about:

- the sample
- the questions
- the answers

Abusing statistics

*(Adapted from **Family Numbers –Using Statistics** by Zbar, V., Cropley, M., & Rowland, M. Curriculum Corporation 1998)*

When analysing statistical information there are three matters which need to be carefully considered to ensure that the results have provided an accurate and true picture of the particular situation.

Sample

It is very expensive and time consuming to survey everyone about their behaviours, beliefs and practices. Every 5 to 10 years Governments generally organise a national census to count all the people in the country and to find out about a wide range of issues relating to where and how people live. Usually, however, statisticians tend to use a sample of the population to derive their data. A sample is a specially chosen group of people who are assumed to be typical of all of the people in the country. For a sample to be of any use, however, it has to be both large enough and representative of the population as a whole. Examples of surveys that do not meet these criteria are:

- Television polls where viewers are invited to ring in with their view on a topic and then the results are reported in the news. The problem with this sort of sample is that the people who choose to ring in are self-selected. They may not represent the general population and it is likely that those with strong views on the issue (who may be a small minority) will be the ones to ring in, thereby distorting the result.
- Polls where relatively few are questioned, their responses being reported as if representing a significant proportion of the population.

There are, in fact, statistical techniques to help ensure that any sample selected is both large enough and appropriately representative of the population that is the subject of the survey. In general, surveying organisations will report on both the size of their sample and how it has been selected.

The question

The way in which a survey question is framed can have a significant impact on the responses that people give. For example, a survey into capital punishment could ask either of these two questions:

- Are you in favour of capital punishment for murder?
- If someone murdered one of your relatives or a close friend would you want them to be hanged or just get a gaol sentence?

It is likely that the responses to the two questions would give entirely different results purely as a consequence of how the question was phrased. In general, those who conduct surveys will try to frame their questions as neutrally as possible. However, this can be quite difficult to achieve, especially if the issue is contentious.

The answers

An important assumption we make about surveys is that people answer the questions honestly. However, this is not always the case.

In some instances people are too embarrassed to give an entirely truthful answer. For example, how many children will acknowledge that they have stolen money from their parents' wallet or purse?

In other cases, people will occasionally give the answer they believe is wanted by the questioner. For example, when asked if they prefer Sprite or Coca Cola when being asked by a representative from Coca Cola, there are many who will select Coca Cola, simply out of politeness.

For the most part people will answer as honestly as they can. But we do need to recognise that this is not always the case. Surveyors are, of course, well aware of these potential pitfalls, and they have developed various controls to overcome them. Nonetheless, we do need to keep them in mind, and should always treat statistical data with a degree of scepticism depending on how it has been collected and the rate of response. (See 'Five helpful questions' below.)

'There are three kinds of lies: lies, damned lies, and statistics'.

Benjamin Diarach

Another issue to consider in relation to the use of statistics relates to how the information is reported, in particular, whether or not it has been reported in such a way as to increase its dramatic effect. Two examples can help to illustrate the point.

Student scores in a mathematics test

These are the results of 21 students for a Year 7 mathematics test. In this case, the median (or the middle) score of the students is 55 because there are ten students who scored higher than this and ten students who scored lower.

TEST SCORE	***NUMBER OF STUDENTS***
100	1
98	1
86	2
78	1
70	3
67	2
55	1
53	6
50	4

By contrast, the mean score (that is, the total of all scores divided by 21) is 65. Both of these scores could be called the average and may be reported in that way. For instance, a school that is trying to encourage extra enrolment from its feeder schools may choose to focus on the mean (65) rather than the median (55). Alternatively, a school that is initiating a campaign to increase the number of mathematics teachers in the school may be more interested in the median than the mean.

The point is that either measure on its own may be misleading. Graphs and charts are similarly open to manipulation, as in the next example.

Reporting on business results

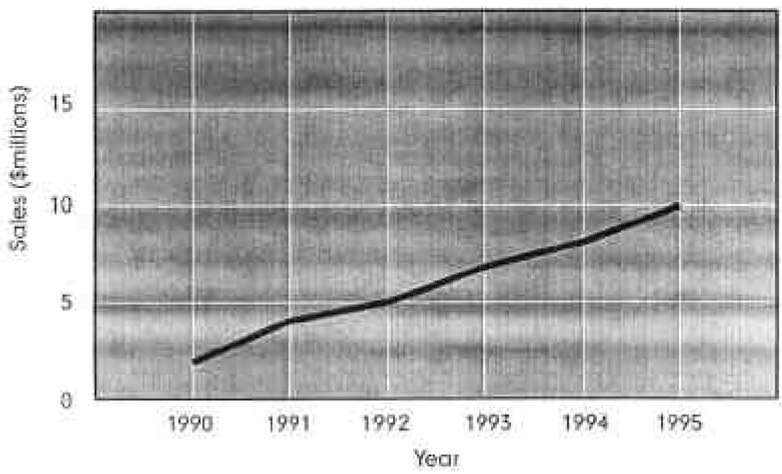
Between 1990 and 1995 a growing new business generated sales of:

1990	$2,000,000
1991	$4,000,000
1992	$5,000,000
1993	$7,000,000

1994	$8,000,000
1995	$10,000,000

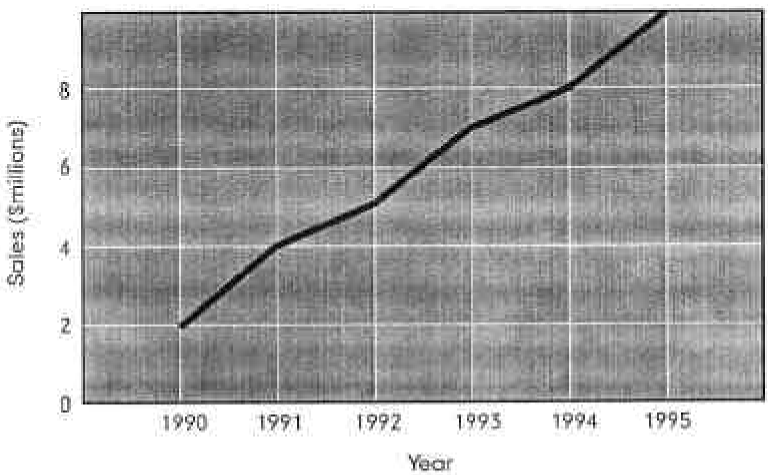
Proud of this success, the Managing Director decided to prepare a graph to show the Board.

Figure 1 Sales results 1990–1995



While relatively pleased with what Figure 1 showed, the Managing Director was concerned that it was not sufficiently dramatic to make the Board really sit up and take notice. Redrawing the graph with a different scale for sales would certainly help. Figure 2 is a much more exciting graph.

Figure 2 Sales results 1990–1995



This is not to suggest that this use of statistics is dishonest. People will always report the data in a way that best suits their purposes. Rather, it is to remind us to take care when we examine and interpret the information that is presented, and in particular, to avoid rushing to judgment on the basis of insufficient information.

We should be wary of the temptation to draw conclusions from different sets of statistics when they may not be related or when other factors are also relevant. For example, it is possible to collect data which suggests that people are more likely to have car accidents near their homes than at a distance from them. Does this mean that we should avoid short trips? Or is it just a reflection of the fact that more of our driving occurs within our neighbourhood than over vast distances?

The point is that the explanations for what we observe are often more complicated than simply relating two sets of data together. After all, it is probably the case that most of the people who have car accidents also eat bread. Can we therefore conclude that bread is a cause of car accidents?

Five helpful questions

So where does this leave us? On the one hand, we have said that statistics and data are essential for informed decision making. At the same time, however, we have sounded several notes of caution about using and interpreting statistics.

In this context, the message is to gather and use statistics as required, but to do so with care. There are five questions we should ask to help us in this task.

1. *Who says so?*
   Does the person/organisation have any interest, and therefore potential bias, in relation to the information being reported?

2. *How do they know?*
   How have they gathered the data and what is the basis of their interpretation? For example, if they used a sample of respondents, how big was the sample and was it representative of the population?

3. *What's missing?*
   Are we being told all of the relevant information? For example, when a washing powder advertisement suggests it washes 30 per cent brighter, does it tell us 'brighter than what'?

4. *Has the subject changed in any way?*
   Is it possible that circumstances have changed in a way that influences the results? For example, is an increase in reported tax avoidance cases due to a greater level of cheating or has the tax office just instituted a crack-down on the practice?

5. *Does it make sense?*
   Do the reported statistics accord with what we see around us and plain old commonsense? For example, does it really make sense to link car accidents to the eating of bread?

☞✍ 3.4 Activity 8

Review the statistical information you have collated in 3.4 Activities 3 to 6. Does your data give a 'true picture' of the situation? What factors may effect the validity of your data? Why do you believe this?

Write a paragraph about your findings.

References – for further reading

Department of Education Papua New Guinea (1998). *Lower Primary Mathematics Syllabus Grade 3-5*

Department of Education Papua New Guinea (draft 1999). *Upper Primary Mathematics Syllabus Grade 6-8*

Green, Wally (1998) 'Enrichment Topics in Number' Department of Education PNG

Government of PNG, (1999) *Papua New Guinea Human Development Report* 1998

Zbar, Vic. Cropley, Murray & Rowland, Mike. *Family of Numbers*, Curriculum Co-operation

Module 3.5 – Using Calculators in Primary Schools

Module 3.5 Using Calculators in Primary Schools is a recommended module within the ***'Enrichment Topics in Number'*** unit. Students of all ages generally enjoy using calculators and the ability to competently use a calculator is an essential and expected skill in many workplaces. Calculators allow students the opportunity to work with large numbers which are generally found when exploring real world data, and support the teaching of computation skills for children with learning disabilities. During this module you will use a calculator to explore numbers, as well as consider how calculators can be used effectively in primary schools.

Objectives

By the end of this module you will be able to:

- use a calculator to solve a range of problems
- identify the advantages and disadvantages associated with the use of calculators in primary schools
- identify the skills children need to be able to use a calculator effectively.

Concepts and skills to be developed

- Number properties (communicative, associative, and distributive)
- Recurring and terminating decimals
- Negative numbers
- Estimation skills
- Rounding numbers
- Basic number facts
- Writing skills

Introduction

Calculator history

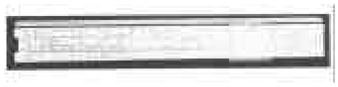
(adapted from Strasser, D., Phillips, G. & Nolan, J., Heinemann Mathematics 7, 1995)

Counting machines go back at least 20 000 years. The first ones were very simple and were nothing like the calculators we have today, but they served the same function: to speed up calculations.

One of the earliest type of calculator was the abacus. Throughout history and traders around the world have used the beads on their abacuses to help them with their work.

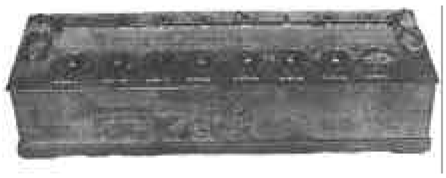
An abacus

Another type of calculator, the slide rule, was invented by William Oughtred in the seventeenth century. It was based on two lines of numbers which could slide alongside one another. The slide rule was still used in some Australian schools up to the early 1970s.



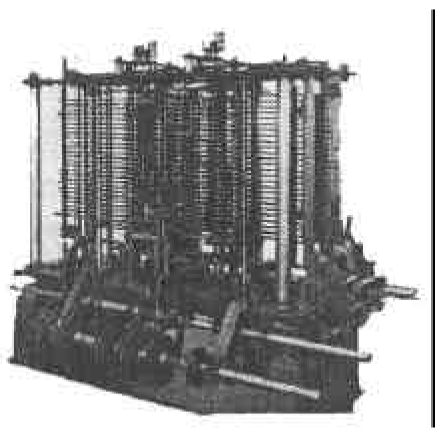
A modern slide rule

Blaise Pascal invented the first digital calculator in 1642. Pascal's calculator used numbered dials driven by cog wheels, a bit like the odometer that reads mileage in a car.



Pascal's calculator

The English mathematician Charles Babbage constructed another type of calculator, called a difference engine, in 1823. In 1833 he started building a more complicated calculator, called an analytical engine, which worked on punched cards. He didn't manage to complete this machine.



Part of the analytical engine

Calculators nowadays are much quicker than any of the previous types and can perform much more complicated tasks. Older models of twentieth century electronic calculators used large and expensive parts called valves. In the 1960s these were replaced by smaller and cheaper parts called transistors. The invention of the silicon chip has made it possible to make tiny parts which are very cheap and can contain millions of transistors. This has made the small, cheap and efficient pocket calculators possible.

Topic 1 – Using the Calculator

When using the calculator it is necessary to keep in mind the properties of various operations and the order in which we complete the operations.

Commutative operations

Commutative means that the order of the numbers in the operation can be reversed but the outcome will be the same. For example:

$2 + 5 = 7$

$5 + 2 = 7$

The 2 and the 5 can be reversed but the operation of adding still gives 7. So addition is a commutative operation.

Associative operations

Two operations are associative with each other if the order in which they are done can be changed but the result remains the same. For example:

$5 + 2 – 1 = 7 – 1 = 6$, with the order being 5 add 2 subtract 1

$5 – 1 + 2 = 4 + 2 = 6$, with the order being 5 subtract 1 add 2

so it does not matter in which order the operations ‘add 2’ and ‘subtract 1’ are done. So addition and subtraction are associative operations.

Distributive operations

One operation is distributive over another if it can be done either on individual numbers in a group, or on the result after the group is combined and the same answer is obtained. For example:

$6 \text{ x } (3 + 2) = 6 \text{ x } 3 + 6 \text{ x } 2 = 18 + 12 = 30$

$6 \text{ x } (3 + 2) = 6 \text{ x } 5 = 30$

so multiplication is distributive over addition.

BODMAS rules

When doing calculations you need to consider the order in which you carry out the calculations. The **BODMAS** rule is one way to remember the order of operations.

BODMAS stands for:

B for **B**rackets

O for **O**perators

D for **D**ivision

M for **M**ultiplication

A for **A**ddition

S for **S**ubtraction

'Brackets' applies not only to brackets ( ) but also to other grouping symbols. The bar between numerator and denominator and the bar on the top of a square root are also grouping symbols. For example

$3(5+2)$	adding 2 to 5 is done before multiplying by 3
$\frac{5+2}{3}$	adding 2 to 5 is done before dividing by 3
$\sqrt{15-2}$	subtracting 2 from 15 is done before finding the square root

'Operators' means symbols for operation. There are only a few of these that apply to ordinary mathematics, like power symbols and square root symbols. For example

$\sqrt{5}-3$	find the square root of 5 before subtracting 3
5^3+2	the cubic power of 5 is found before you add 2
$\frac{16^2}{2}$	square 16 before dividing by 2

By apply the BODMAS rule we can write the calculation steps for different problems, starting with the number followed by the operations in order. For example:

$3(5+7)$	5, add 7, multiply 3
$\frac{6-2}{5}$	6, subtract 2, divide by 5
$7+\sqrt{2}$	7, add 2, square root
$\sqrt{\frac{4^2-3}{2}}$	4 square, subtract 3, divide by 2, square root

☞ 3.5 Activity 1

Demonstrate that addition and multiplication are commutative but subtraction and division are not.

Demonstrate that multiplication is associative with division

Demonstrate that division is distributive over addition or subtraction

Use BODMAS rule to write the order in which you would complete the following calculations:

(i) $3 - 2 + 7$

(ii) $4 + 2 \times 8$

(iii) $5 \div 2 + 3$

(iv) $3\,(2 + 1)$

(v) $5^2 + 4$

(vi) $2 - 3$

(vii) $5 - 2$

(viii) $\frac{4 + 3}{2}$

(ix) $\frac{5^2 - 3}{4}$

(x) $2 + 3 \times 5$

(xi) $\frac{5 - 2 \times 3}{7}$

(xii) $\frac{6}{2} - 1$

Topic 2 – Skills for the Effective Use of the Calculator

Calculators offer a speedy way of doing calculations and as such are a very valuable tool that should be used. However, calculators in themselves do not teach understanding. It is important that people develop skills in using calculators and become competent and confident in their use of calculators.

3.5 Activity 2 – Using calculators effectively

Read the article 'Calculators' and identify the skills which are important for the effective use of calculators. Choose 4 of these skills and develop a problem to be solved using a calculator which illustrates the importance of the skill. For example, the following exercise uses estimation skills.

> If buying 2.5 metres of fabric at K7.95 a metre students should be looking for an answer between 2 x K8 and 3 x K8, that is between K16 and K24, or about K20. If the calculator answer is significantly different from K20 they know that they have made an error such as pressing the wrong key and need to repeat the calculation.

Calculators

(adapted from Marr, B. and Helme, S. (1991) ***Calculators and adult numeracy in breaking the maths barrier)***

A lot of adults, when faced with a computation at work reach for a calculator. They are a cheap, readily available and a reliable tool, so why not teach students how to use them?

Many people do not accept the widespread use of calculators, believing that using them in class is a form of cheating. There is also fear that people become too dependent on calculators and use them mindlessly for calculations which should be done mentally. But extensive research with school age children has demonstrated that using calculators is more likely to improve the average student's basic skills. Moreover, students using calculators tend to possess more positive attitudes to mathematics than students who do not use calculators do (McIntosh, 1990).

Why teach calculator skills?

There are a number of good reasons for teaching calculator skills:

- Using a calculator is an essential and expected skill in the workplace, therefore students should be taught this skill.
- Learning to use a calculator breaks down the fear of modern technology often experienced by students. Once students become familiar and comfortable with calculators, other technology, such as computers, becomes more accessible.

- Used in the classroom, calculators provide immediate and neutral feedback. A student can test an answer immediately without embarrassment or loss of self-respect. Errors and misunderstandings can then be rapidly diagnosed and remedied.
- Calculators allow students to work with real-world data, which usually involves large numbers which are difficult to work with using pencil and paper methods and standard algorithms. Using current and relevant real- world data rather than contrived data found in standard texts allows students to confront real issues and solve genuine problems.
- Calculators bring computation within the reach of intellectually disabled students. Students who are unable to remember basic number facts or the times tables, once taught to use a calculator, are greatly empowered in their everyday activities, such as working out how much money is needed to pay for things or checking their change.

How should calculators be used?

The question for mathematics educators is no longer whether calculators should be used along with basic skills instruction, but how. In the numeracy classroom, calculators can be used in a number of ways, including the following:

- As a **number – cruncher,** to deal with computations which are complex, tedious or whose performance by pen and paper methods would interfere with the main purpose of the activity. This is very true for low confidence adult students who feel more secure knowing there is a calculator handy if needed.
- To **check the results of calculations** done using standard algorithms or mental arithmetic...
- To **develop and reinforce basic concepts and skills**, such as operations, place value and multiplication tables. For example to reinforce the seven times table, students can use the constant function available on the calculators as follows:

7 + =	Answer displayed:	14
(press equals again)	Answer displayed:	21
(and again)	Answer displayed:	28

  Students can predict the next result and check their prediction using the calculator. Many calculator games have now been devised. These are a valuable way of practising basic skills without the tedium of drilled exercises.
- To explore **patterns and relationships** in numbers. For example using a calculator to investigate and predict what happens when numbers are multiplied and divided by 10, 100, 1000 and so on, students can discover the rules themselves.
- To enhance students' **understanding of large numbers** by estimating and then calculating. For example, estimating how long it would take for one million seconds to pass. (The result surprisingly enough is only eleven and a half days.)

- To encourage **experimentation in problem solving**. When a student has a challenging problem to solve, for example converting K100 into US dollars and is given the conversion rate of K1 = 36.4 US cents, they may not know where to start, whether to multiply or divide and by what: 0.364 or 36.4? What they probably do know from the information given or from their own experience is that the PNG Kina is worth less than the US dollar and that they should therefore get less than $US 100 for their PNG K 100. A calculator allows students to experiment with alternative approaches until they feel confident that their answer is reasonable (the answer is $US36.40).

Using calculators effectively

In order to use calculators sensibly, students need certain skills, many of which were not taught in the mathematics classes of the past.

> *We use technology every day and, as the use of a new technology becomes more widespread, old skills are lost and new skills are developed.* *(Willis, 1990, p. 13)*

What skills are important for the effective use of calculators?

- **An understanding of the value of numbers.** For example, to add K1.37 and 13 toea on a calculator requires understanding of the data we are feeding in, in particular, place value. In order to produce the correct result the student has to enter the 13 toea as 0.13, not simply 13. This requires an awareness of what the numbers are worth in this particular context, and how they should be entered on the calculator.
- **Making sense of the calculator answer.** In the previous example of K1.37 + 0.13, the calculator result is 1.5 because the calculator drops off the right hand zeroes. This result has to be interpreted by the student as equivalent to K1.50 and not one kina and five toea.
- **Estimation skills.** To be aware of whether or not a calculator result is acceptable, students need to have a rough idea of what result they are likely to get. For example, if buying 2.5 metres of fabric at K7.95 a metre students should be looking for an answer between 2 x K8 and 3 x K8, that is between K16 and K24, or about K20. If the calculator answer is significantly different from K20 they know that they have made an error such as pressing the wrong key and need to repeat the calculation.
- **Rounding skills.** The previous example illustrates the importance of rounding numbers to an appropriate degree of accuracy, first at the initial approximation (i.e. K7.95 rounded to K8) and second to make sense of the calculator result of multiplying 2.5 by 7.95 (19.875). Rounding up is required in this case, to obtain a final charge of K19.88.
- **Basic number facts.** In the previous example, a knowledge of tables was required to mentally calculate 2 x 8 and 3 x 8. Basic number facts are thus an essential skill for the effective use of calculators. However, students who cannot recall their tables could do the estimates 2 x 8 and 3 x 8 on the calculator.

- **Accuracy in entering data.** It is inevitable that students hit the wrong button occasionally, and thus need strategies for knowing whether or not they have entered the correct data. Such strategies include repeating the calculation until they obtain the same answer twice and/or using their estimation skills to detect unlikely answers and always checking the display after each entry.
- **Selecting the appropriate operation.** Students need to learn which calculation is necessary. Perhaps it is more important these days to know when to divide, rather than how to. For example, to work out how many PMV's are needed to transport 250 children on a picnic when each PMV holds 36 children, the student has to first decide that division is necessary, then enter the numbers in the right order and finally give meaning to the result. The calculator answer of 6.94 does not tell us how many PMV's; students need to round up to the nearest whole number in order to produce a meaningful answer.

What sort of calculators?

For basic numeracy, we recommend a simple calculator with well spaced single- function keys, a positive 'click' action, a percentage key, a memory key and preferably solar powered (so you don't have to worry about replacing the batteries). Large desktop calculators are recommended for students with impaired motor skills.

References

Australian Association of Mathematics Teachers and the Curriculum Development Centre (1988), *A National Statement on the use of Calculators for Mathematics in Australian Schools*.

McIntosh, A. (1990), Becoming Numerate: Developing Number Sense. In Willis, S. (1990), *Being Numerate: What Counts?* Australian Council for Educational Research Ltd, Hawthorn, Vic.

Willis, S. (1990), Numeracy and Society: the Shifting Ground. In Willis, S. (1990), *Being Numerate: What Counts?* Australian Council for Educational Research Ltd, Hawthorn, Vic.

Topic 3 – Calculators in the Primary School

For many years there has been an ongoing debate around the world about the place of calculators in the mathematics classroom amongst teachers, parents and the community. The attitude of individuals towards calculators varies enormously. While some people are in favour of their use and see them as a great asset to solving mathematical problems, others believe that the widespread use of calculators by children will be harmful to the development of children's mathematical thinking and that using a calculator is cheating.

📖✍ 3.5 Activity 3 – Challenging the myths

Read the following article 'Calculators in the Primary School, Challenging the Myths' by Paul Swan and Len Sparrow. Discuss each of the myths with a peer and develop your own point of view about each of these.

Write a journal which reflects your personal beliefs about the use of calculators in the primary school and give your reasons.

Calculators in the Primary School

Challenging the myths

If children are allowed to use calculators they will not be able to do mathematics

Do you believe this?

Paul Swan and Len sparrow begin to explore this and other myths surrounding calculator use in the primary school.

Introduction

There are very few issues in mathematics over the past twenty-five years that have caused more debate among teachers, parents and the community than the use of calculators in schools. The transition to using calculators in secondary schools has been a relatively smooth one with their use being accepted in final exams for over a decade. Unfortunately, the same cannot be said of calculator use in the primary school, particularly in the early years.

Numerous research studies (Hembree and Dessart 1986, 1992; Suydam 1982) have clearly indicated that using calculators at the primary levels has no detrimental effect on students' ability with arithmetic, and yet calculators are still not being used in some classrooms and for trivial things in others. One possible reason is parental objections to the use of a calculator. The purpose of this article is to explore some of the myths surrounding the use of calculators in the primary school and to answer the critics.

Myth 1: Calculators rot the brain

Proponents of this argument suggest that handing a child a calculator means they will lose their ability to calculate with pen and paper and that mental skills will decline. They believe that without practice and repetition of these skills children will forget what to do.

Counter-argument

Granted there is evidence to suggest that when first given a calculator, students and older students especially, suffer from the 'novelty effect' where even simple calculations are performed with the aid of a calculator. -Several research studies (Hembree and Dessart, 1986, 1992; Shuard, 1992, Groves and Cheeseman, 1995), however, could be cited to show that there are no detrimental effects to children's ability to calculate as a result of long term exposure to calculators. The 'novelty effect' soon wears off. Children who participated in the *Calculator Aware Number Project* (CAN) and *Calculators in Primary Mathematics (CPM)* and grew up using calculators from an early age prided themselves on being able to beat the calculator.

Experience suggests quoting research finding does little to quell the fears of anxious parents, rather, why not present them with some calculator activities that illustrate calculators do not rot the brain and require considerable thinking. Some examples are given below.

Task: blanks

Parts of the following questions have been blanked out. Try to discover which digits are missing.

$$\begin{array}{r} \square\;8 \\ +\;3\;\square \\ \hline 6\;4 \end{array} \quad ?$$

$$\square\;\square - \square\;8 = 24$$

Myth 2: The calculator will become a crutch

The children will come to rely on the calculator and become mentally lazy.

The suggestion here is that the children will come to rely upon the calculator and will not be able to do without it. Given a choice, children will tend to reach for a calculator because it is the 'easy' way out.

Counter-argument

A study by Price (1995) indicated that given a choice children choose paper and pencil methods over calculator and mental computation. Ruthven (1998) also noted that children who were previously involved with the CAN Project were more inclined to use mental methods of calculating than non-CAN children. Children from the aforementioned long-term studies prided themselves on being able to calculate mentally (Shuard, 1992, Groves and Cheeseman, 1995). Children who are given free access to calculators often choose to use mental computation instead of calculators. When they do use a calculator, children are freed from the tedium of calculation and are able to think about the problem. Coburn discusses the issue of dependency and states the following:

Dependence on a device like a calculator is inevitable to some degree. We become by nature dependent on things we use regularly, this in itself is not bad. The fact that many children are overly dependent on written computation is often overlooked. (A child who multiplies 300 by 122 using the traditional paper-and-pencil algorithm is dependent on written computation. The child who receives good instruction should decide to do this type of computation mentally, or at least take a written shortcut to the conventional algorithm.) The term 'crutch' implies a dependency without understanding. We need to examine this issue carefully because it is a common belief that if children use calculators, they will not understand what they are doing. It is as if understanding always enters the brain on a pathway from a pencil through the fingers (Coburn 1989, p.45).

Children who come to rely on the calculator are often those who have not been taught any useful mental computation strategies. They have nothing else to use apart from the calculator.

Myth 3: The children will no longer learn their basic number facts

Basic number facts, especially speedy recall of the tables are the sacred cow of mathematics. Parents can relate to the tables and while the may not have fond memories of chanting the tables, tables competitions and tests, they consider them to be part of the rigour associated with mathematics.

Counter-argument

Calculators are not designed to replace the learning of basic number facts. The basic number facts are more essential than ever.

When they do use a calculator children are freed from the tedium of calculation and are able to think about the problem.

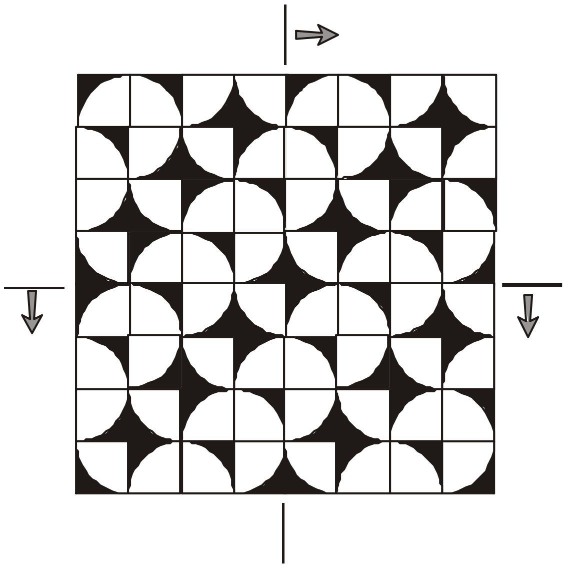
References – for further reading

Green, Wally (May 1998). *Report on mathematics education in community teachers' colleges in PNG*, Fourth Phase, Activities Observations and Recommendations. OHE-ADB-IDP Papua New Guinea Higher Education Project (Asian Development Bank Loan 1224).

Marr, Beth and Helme, Sue (1991). *Calculators and adult numeracy*, in 'Breaking the Maths Barrier'.

Swan, Paul & Sparrow, Len (1999). *Calculators in the primary school*, Challenging the Myth in Australian Primary Mathematics Classroom, Vol. 3 no. 4.

Module 3.6 – Clock Modulo Arithmetic



Module 3.6 Clock Modulo Arithmetic is a recommended module within the ***'Enrichment Topics in Number'*** unit. During this module you will develop your number sense and ability to think logically. You will have the opportunity to solve number problems, explore the relationship between operations and consider various ways in which numbers can be represented.

Objectives

By the end of this module you will be able to:

- explore the relationship between the four operations (addition, subtraction, multiplication and division) when solving a range of problems based around different modulo e.g. modulo-7, -12
- create patterns and designs using modulo numbers.

Concepts and skills to be developed

- The concepts of addition, subtraction, multiplication and division
- Relationship between operations, e.g. division is the inverse of multiplication
- Problem solving
- Interpreting
- Logical thinking
- Bases
- Euclidean transformation (translate, reflection or rotation)
- Rules of divisibility

Topic 1 – Clock Arithmetic

Clock Arithmetic is a special branch of mathematics called 'Modulo Arithmetic'. A clock is like a number line except the line is circular and after reading the number 12 it starts again. To work out how to do mathematical operations on the clock, remember how they are done using a number line. To make the comparison complete we replace the 12 with zero. Therefore in 'clock arithmetic'

$$6 + 9 = 3$$

$$6 – 9 = 9$$

$$5 \times 4 = 8$$

The mathematical name used to describe this situation is 'arithmetic modulo-12'. Mathematicians use the idea of clock arithmetic to explore number patterns and investigate different 'clocks' e.g. a 5-hour clock or a 7-hour clock. These are also referred to as modulo 5 or modulo 7.

☞ 3.6 Activity 1

Draw a 7-hour clock, replace the 7 with a zero. Use this to calculate the following problems:

4 + 6 =

4 x 6 =

5 + 3 =

3 + 4 =

4 – 3 =

3- 4 =

3 x 4 =

3 ÷ 4 =

In standard arithmetic we often use a table to show the basic facts. For example an addition table would be

+	**0**	**1**	**2**	**3**	**4**	**….**
0	0	1	2	3	4	….
1	1	2	3	4	5	….
2	2	3	4	5	6	….
3	3	4	5	6	7	….
4	4	5	6	7	8	….
….						

☞ 3.6 Activity 2

Draw a standard arithmetic table to show the basis addition facts for 0 to 6. Now use your 7-hour clock to complete an addition table for this system. Remember to change the 7 to a zero

+	***0***	***1***	***2***	***3***	***4***	***5***	***6***
0							
1							
2							
3							
4							
5							
6							

List

(i) the similarities between the two tables

(ii) the difference between the two tables

Construct the two tables(standard arithmetic and clock-7) for subtraction and again list the similarities and differences. Also consider how the subtraction tables differ from the addition tables.

By constructing these tables and investigating the number patterns important mathematical concepts can be explored. For example,

- Identity element
- Inverse element

- Closure
- Commutative property
- Congruent numbers

We can define these concepts listed above by considering a set of elements, 'S' and the different operations.

Closure

The operation is said to be closed in the set 'S' if as a result of carrying out the operation on elements in the set, the result is also in 'S'.

For example, addition modulo 7 is closed in the set 'S', which consists of the elements '0, 1, 2, 3, 4, 5, 6' because the result is always one of these elements.

Division on the set of counting numbers is not closed because the result is not always a counting number, that is $7 \div 4$ is not a counting number.

Identity (or neutral) element

The set 'S' has an identity element 'i', if combining 'i' with any other element either first or second, does not change the other element.

For example, if 'S' is the set of integers and the operation is multiplication, then 1 is the identity element because 1 x 7 = 7 and 7 x 1 = 7. However 1 is not an identity element for division because even though $7 \div 1 = 7$, it is not the case if the 1 is first because $1 \div 7$? 7

Inverse element

Each element in the set 'S' has its own inverse element if when combined with that element either first or second, the result is the identity.

For example, in addition the identity element is 0. The inverse of 6 is –6 because 6 + (-6) = 0

Subtraction cannot have an inverse element because like division, it does not have an identity element.

Commutative property

An operation is said to be commutative if when two elements in 'S' are combined the result is the same if they are combined in the opposite order.

For example, 49 + 73 = 73 + 49, therefore addition is commutative.

However, 49 – 73 ? 73 – 49, therefore subtraction is not commutative.

Congruent numbers

Take an infinite number line and imagine it wound round a 12-hour clock. The numbers that fall in the same position form a set of numbers that are congruent clock-12.

For example, in the case of clock-12 the numbers which fall in the same position as the number 1, are 13 , 25, 37 etc. These numbers are congruent clock-12.

☞ 3.6 Activity 3

Work out tables for multiplication and division for a five hour clock and compare them.

Consider the tables to investigate the following questions:

- *(i) Is multiplication modulo-5 closed?*
- *(ii) Is division modulo-5 closed?*
- *(iii) If 'S' is the set of integers modulo 5, and the operation is multiplication, is there an identity element?*
- *(iv) In multiplication the identity element is 1. What is the inverse element for 3 in multiplication modulo-5?*
- *(v) Which numbers would be congruent to 2 in modulo-5*

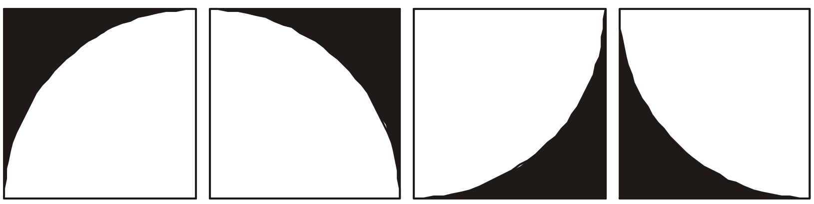
Topic 2 – Modulo Art

By using the number patterns found in modulo arithmetic and incorporating Euclidean transformations, artistic designs can be created.

For example, if you construct a table for addition modulo-4 you get the following information:

+	**0**	**1**	**2**	**3**
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

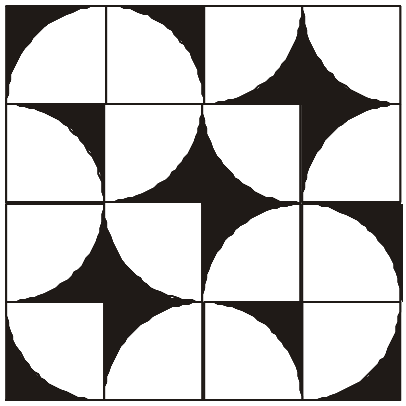
In each of the squares we can draw a pattern which will represent the number below it.



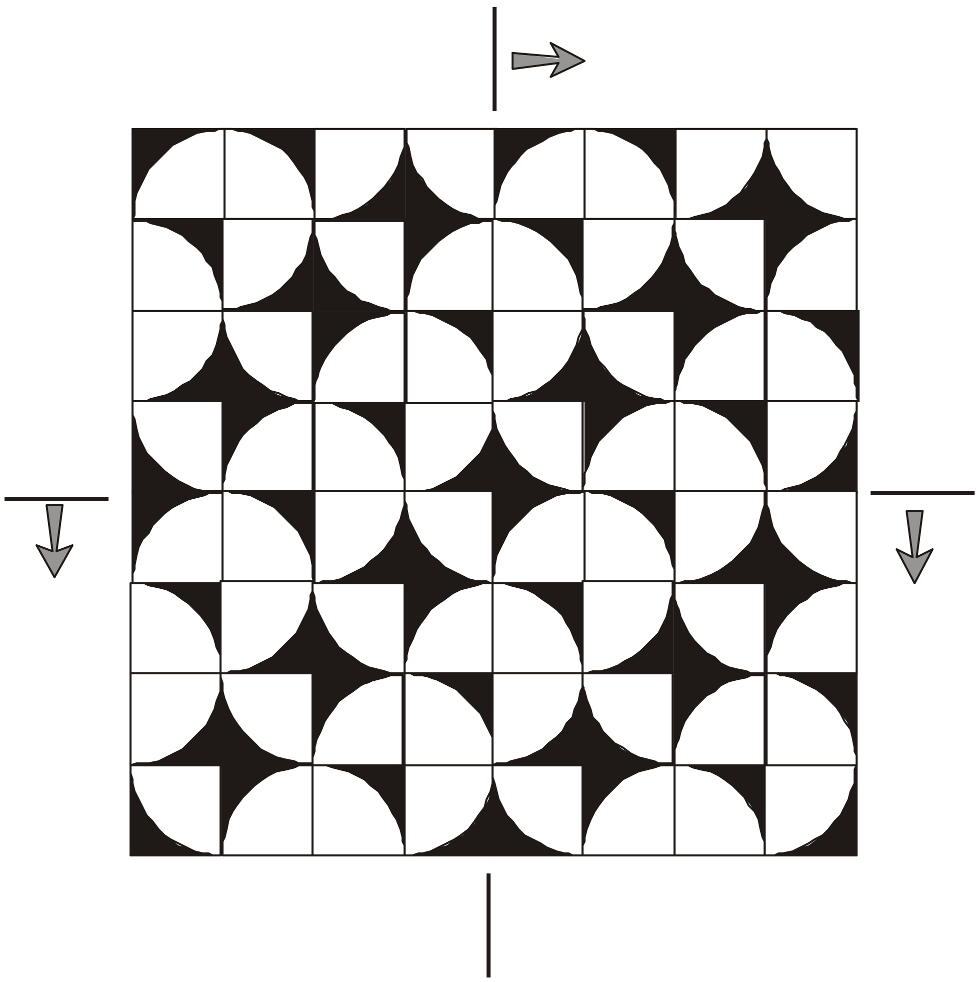
0 1 2 3

We can then copy this pattern onto the 4 x 4 grid so that each corresponds in position to its number in the addition table.

0	1	2	3
1	2	3	0
2	3	0	1
3	0	1	2



We then can copy this into the first quadrant of an 8 x 8 grid. Using and Euclidean transformation (translation, reflection or rotation) we can place the pattern into the other three quadrants. It is best to choose a particular transformation and use it for each of the three quadrants. The following example is of a translated pattern.



☞ 3.6 Activity 4

Construct a table for multiplication modulo-5, but omit the zero

For example:

X	***1***	***2***	***3***	***4***
1				
2				
3	*3*	*1*	*4*	*2*
4				

Use the data from your table to construct your own modulo art design using the steps outlined above. You can use the grid provided

Experiment using different modulo, different operations and different transformations.

☞ 3.6 Activity 5 - Extension activity

In 1997 the 1st September was a Monday. Use clock arithmetic to work out on what day of the week -

1. *The 1st September will be in the year 2005*
2. *The 1st January will be in the year 2000*
3. *The 16th October will be in the year 2010*
4. *Your birthday will be in the year 2012*

References – for further reading

Green, Wally, *Enrichment Topics in Number*

Green, Wally (May 1998) *Report on Mathematics Education in Community Teachers' Colleges in PNG*, Fourth Phase, Activities Observations and Recommendations. OHE-ADB-IDP Papua New Guinea Higher Education Project (Asian Development Bank Loan 1224)

Unit Glossary

Hindu Arabic System	The numeration system widely used around the world and currently used in PNG for education and business purposes. The symbols of the Hindu Arabic system are 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9
Additive System of Numeration	An additive system of numeration is one in which the overall value of the number represented is determined by adding the value of each symbol together. Each symbol has only one value
Multiplicative System of Numeration	A multiplicative system of numeration is one in which the value of the number represented is determined by the multiplication of some pairs of symbols as well as addition e.g. in the number 26, the value of the number is determined by multiplying 2 x 10 and 6 x 1 and then adding these values together.
Place Value System of Numeration	A place value system of numeration is one in which the value of the number represented is determined by the position of the numeral. For example in the number '26' the 2 represents 2 tens and has the value of 20 and the 6 represents 6 ones with the value of 6
Factors	All the whole numbers that can be divided exactly into another number. For example 6÷1 = 6 6÷ 2 = 3 6÷ 3 = 2 6÷ 6 = 1 so 1, 2, 3, and 6 are factors of 6
Proper factors	All the factors of a numbers except the number itself For example, the proper factors of 6 are 1, 2 and 3.
Prime Number	A counting number that can only be divided by one and itself. For example 2, 3, 5, 7,11. The factors of 2 are 2 and 1; the factors of 3 are 3 and 1. Note: 1 is not considered a prime number or a composite number.
Prime Factors	A prime number that will divide exactly into a given number; for example 2, 3 and 5 are prime factors of 30. (10 is a factors of 30 but not a prime factor)

Composite Number

A number with factors other than itself and one.

For example 12 = 12 x 1 = 3 x 4 = 3 x 2 x 2

33 = 33 x 1 = 3 x 11

but not 17 = 17 x 1

Both 12 and 33 are composite numbers but not 17. Numbers like 17 which have no other factors except themselves and 1 are called 'prime numbers'. Every whole number greater than one is either

1. a prime number or
2. a composite number

Multiple

A multiple of a given number is any number into which it will divide exactly.

For example multiples of 2 are 2, 4, 6, 8, 10, 12, …

Multiples of 5 are, 5, 10, 15, 20, 25, …

Lowest Common Multiple

(L.C. M.)

The lowest counting number that is a multiple of given numbers.

For example, What is the lowest common multiple of 2 and 3?

The multiples of 2 are 2, 4, ***6,*** 8, 10, ***12,*** 14, 16, ***18,*** …

The multiples of 3 are 3, ***6***, 9, ***12,*** 15, ***18***, …

The common multiples are 6, 12, 18, …

The lowest common multiple is 6.

Highest Common Factor

(H. C. F.)

The highest counting number that is a factor of given numbers. For example: What is the highest common factor of 18 and 24?

The factors of 18 are 1, 2, 3, ***6***, 9, 18

The factors of 24 are 1, 2, 3, 4, ***6***, 8, 12, 24

The greatest common factor is **6**.

Abundant Numbers

If the sum of a given numbers proper factors is greater than the number itself, the number is said to be abundant.

For example, the proper factors of 12 are 1, 2, 3, 4, 6. The sum of these proper factors is 1 + 2 + 3 + 4 + 6 = 16

As 16 is greater than the 12, then **12** is an abundant number.

Deficient Numbers

If the sum of a given numbers proper factors is less than the number itself, the number is said to be deficient.

For example, the proper factors of 8 are 1, 2, 4. The sum of these proper factors is 1 + 2 + 4 = 7.

As 7 is less than 8, then **8** is a deficient number.

Perfect Numbers

If the sum of a given numbers proper factors is equal to the number itself, the number is said to be perfect.

For example, the proper factors of 28 are 1, 2, 4, 7, 14. The sum of these proper factors is 1 + 2 + 4 + 7 + 14 = 28.

As the sum equals the number, **28** is a perfect number.

Amicable (friendly) Numbers

Two numbers are said to be Amicable or Friendly if the proper factor sum of both numbers are equal.

Sequence

An ordered set of number formed according to some pattern or rule.

For example, 1, 3, 5, 7 …… the rule for this sequence is 'add 2'.

Term

Each number in a sequences is referred to as a term of that particular sequence. The term refers to the position within the sequence

For example:

in the sequence 1, 3, 5, 7, 1 is the first term, 3 is the second term, 5 is the third term etc.

nth Term

The rule or formula which describes the sequence. The rule is expressed as a function of n where n is the number of the term (its position) in the sequence.

For example:

Find the nth term of the sequence 3, 5, 7, 9….

For the first term when n =1, 2n =1

For the second term when n=2, 2n = 4

For the third term, when n=3, 2n = 6

By identifying the pattern you can see that each term is one more than twice n. The nth term is therefore 2n + 1

Figurative Numbers

Different sequences of numbers associated with different geometrical figures, considered special by the Pythagoreans and other early mathematicians, are referred to as figurative numbers. For example triangular number, square numbers

Nominal Data

Data which cannot be ordered and can only be classified according to the category.

Ordinal Data

Data which can be ordered. Ordinal data can be divided into two types, discrete data and continuous data

Discrete Data	Ordinal data which can only take on particular distinct values e.g. the number of females/males in a class can only take the values 0, 1, 2, 3, ….. but it cannot be 2½ or 4¾
Continuous Data	Ordinal data which does not have a distinct value. For example the height of a student can be 158cm 0r 178.3cm or 164.345cm depending on accuracy of measurement.
Mean	A measure of central tendency. Also called the average. For example to calculate the mean for a set of scores you add all the scores together then divide them by the total number of scores.
Median	A measure of central tendency. It is the middle number in a series of numbers.
Mode	A measure of central tendency. It is the number which occurs with the most frequency in a series of numbers.
Range	The range is the difference between the highest and lowest values of the data.
Interquartile Range	This is the difference between the upper and lower quartiles in a set of data. The data is divided in to four quarters and the lower quartile is the division between the lower half values. The upper quartile is the division between the upper- half values.
BOMAS	BOMAS represents the order in which operations are carried out when completing calculations. 'B' is for brackets, 'O' is for Operators, 'D' is for division, 'M' is for multiplication, 'A' is for addition and 'S' for subtraction.
Commutative Operations	The order in which the numbers in an operation can be reversed without changing the result e.g. 7+3 = 10 and 3+ 7 = 10. Addition is therefore a commutative operation
Associative Operation	Two operations are associative with each other if the order in which they are done can be changes without the result changing. For example 4+3-2 = 5 if we add 4 and 3 together before taking away 2. If we take 2 from 3 and then add the 4 we will also get 5. Addition and subtraction are therefore associative operations.

Distributive Operations

An operation is distributive over another operation if it can be done on individual numbers in a group e.g. individual numbers inside brackets, or on the results after the group is combined e.g. when the brackets have been calculated, and final answer is the same. For example take 4 x (2 +5). If you complete the brackets first then multiple the answer would be 4 x 7 = 28. If we multiple each number inside the bracket and then add we will get 4 x 2 + 4 x 5 = 8 + 20 =28. The same answer is obtained. Therefore multiplication is distributive over addition.

Closure

An operation is closed in the set (S) if as a result of carrying out the operation on elements within a set, the answer is also in the set (S)

Identify Element

The set (S) has an identity element 'i' if combining 'i' with any other element either first or second does not change the other element. For example in the set of integers and the operation of addition if we combine 3 + 0 or 0 + 3 the element 3 does not change. Therefore 0 is the identity element 'i' for addition.

Inverse Element

Each element in a set has its own inverse element if when combined with that element either first or second, the result is the identity 'i'. For example, in addition the identity element is 0. The inverse of 5 is therefore –5 because 5 + -5 = 0