Unit 2: Teaching and Learning Mathematics

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Unit 2: Teaching and Learning Mathematics

Rationale

It is important that teachers develop an understanding of what is meant by mathematics so that informed decisions can be made about what will be taught in the classroom and how it will be delivered. Our philosophy of mathematics will influence these decisions and it is essential that future teachers spend some time considering these issues.

An understanding of how children learn mathematics is essential for all teachers. This should include an awareness of the important role language and culture play in the development of mathematical understandings. The use of concrete materials when introducing new concepts and the importance of establishing connections between the mathematics of the classroom and their application in the community are also factors which teachers need to develop an understanding of.

Teachers need to be familiar with a range of strategies for the teaching of mathematics and be able to cater all children, including those with special needs. Teachers require the ability to develop appropriate assessment strategies, which recognise a range of learning styles, relate to the full range of curriculum objectives and will provide information that can lead to an improved curriculum.

Many students come to the college with negative attitudes and feelings about their ability to succeed in mathematics. This in turn affects their attitude to mathematics and their approach to the teaching of mathematics. Developing enthusiastic, confident teachers with a positive attitude towards mathematics is essential.

Teachers’ beliefs about the learning process influences what occurs in the classroom. For this reason a thoughtful study and understanding of how mathematics is learnt, and therefore how it should be taught, needs to be a high priority for all future primary teachers.

Aims

This unit aims to produce beginning teachers who are:

- Confident and competent to teach mathematics in primary schools (Grades 3 to 8) and are familiar with a range of different teaching strategies
- Aware of individual differences and able to cater for all children, include those with special needs
- Familiar with the primary school mathematics curriculum documents and able to use these materials to plan mathematical activities
- Aware of issues which impact on the teaching and learning of mathematics
- Able to use a range of concrete materials to support children in the development of their mathematical ideas
- Able to assess children’s learning
- Reflective and able to evaluate their own practice in order to improve their teaching
Objectives

As a result of studying this unit you will be:

- Competent beginning teachers of mathematics, familiar with a variety of strategies that support the development of children’s mathematical thinking.
- Able to identify and discuss a range of factors that contribute to how children learn mathematics.
- Able to identify factors necessary to establish a learning environment that is conducive to the teaching and learning of mathematics.
- Able to devise appropriate mathematics activities that cater for all children, including children bridging from the vernacular to English, children who have special needs and children in a multigrade class.
- Be familiar with the primary school mathematics curriculum documents.

Unit outline

‘Teaching and Learning Mathematics’ can be offered as either a 2-credit point or 3-credit point unit.

To successfully complete this unit you must complete the following core modules:

- Module 2.1 Understanding How Children Learn Mathematics
- Module 2.2 Teaching Mathematics
- Module 2.3 Syllabus Studies

Each of these modules should take between 6 to 9 hours of lectures to complete. It is also expected that you will spend an equivalent number of non-contact time studying the ideas and concepts raised in this unit.

How to use this material

This material has been developed to support your studies in this unit. It aims to support the development of your own mathematical knowledge and skills, as well as prepare you to teach mathematics in primary schools.

The material for this unit has been set out according to the different modules. For each module the objectives and the concepts and skills to be developed within the module are stated. This information is followed by a series of topics. Each topic consists of readings as well as activities to complete. Extension activities have also been included. Your lecturer will guide you through the materials during the lecture program. At times your lecturer may ask you to complete activities and reading as homework. Sometimes you may work directly from the book during lectures. Your lecturer may also include additional information and topics.

A glossary can be found at the end of the unit to assist you in reading the material.
Assessment

Assessment details will be provided by your lecturer. A range of different assessment tasks will be given which require you to show your understanding of the concepts and skills covered by this unit.

Inclusive curriculum

In the delivery of this unit it is expected that every person will be provided with an opportunity to participate in and contribute to activities without fear or favour. Activities will be presented to cater for a range of abilities and will be gender inclusive. Assessment tasks will cater for a range of different learning styles.

You will be encouraged to plan activities for use in the primary school mathematics classroom which are gender inclusive and present positive and non-stereotypical representations of people.
Module 2.1 – Understanding How Children Learn Mathematics

Module 2.1 Understanding How Children Learn Mathematics is a core module in the unit ‘Teaching and Learning Mathematics’. During this module you will extend your understanding of what is meant by ‘mathematics’ and explore the relationship between culture and mathematics. The importance of language in the teaching of mathematics will be examined. Active participation, relating mathematics to real life situations and the use of concrete materials to support the development of mathematical concepts are issues which you will study. The understandings developed in this module will enable you to make informed decisions when planning and teaching mathematical activities.

Objectives
By the end of this module you will be able to:

- Develop a definition of what is meant by the term ‘mathematics’
- Explain how culture influences mathematical thinking
- Justify the use of concrete materials in the teaching of mathematics
- Discuss the relationship between language and mathematics
- Identify strategies for actively involving children in learning mathematics

Concepts and skills to be developed
During this module the following concepts and skills will be developed.

- Relationship between language and mathematics
- Teaching mathematics in context
- Active participation
- Oral presentation (debate, seminar)
- Research skills

Introduction: What is this thing called mathematics?
We have all spent many years ‘doing’ mathematics. It has been a subject we have studies at primary school, at high school and now again at a tertiary level. But have we ever really thought about what mathematics is all about and where all these mathematical ideas have come from? As teachers it is important that we develop an understanding of what is meant by mathematics as our teaching will be informed by our beliefs.
2.1 Activity 1

Write your own definition of what you think is meant by ‘this thing called mathematics’.

Read the following mathematical statements and consider how these ideas relate to your understanding of mathematics.

Statement One

“Mathematics is a creation of the mind, demonstrating powerfully the mind's capacity to seek order and pattern in the world of experience, to construct explanations and to intellectualise about this world, and to delight in challenge and in the resolution of problems posed to it by itself.”


Angles of polygons

Statement Two

Mathematics is an integral part of our existence. It is a powerful form of communication which enables us to represent, to interpret, to explain and to predict.

The study of mathematics involves the search for patterns and relationships, through which we are able to explore, explain, and interpret the world around us. The focus on problem solving and investigation, which has become a significant element of mathematics learning in recent years, ensures that these possibilities are realized.

We all use at least a fundamental knowledge of number, measurement, special relationships and statistics in our daily lives. At the same time, more sophisticated understandings in these areas form the basis for a myriad of vital activities in science, the humanities and the arts.

Beyond this specific value, the study of mathematics provides opportunities to develop logical reasoning and provides a powerful means of communication.

Mathematics is exciting, challenging and satisfying. It holds an inherent interest for young children, and this can be maintained through effective teaching which emphasises practical relevance.

Statement Three

Mathematics is a rapidly expanding experimental science. Students should be encouraged, through taking an active experimental approach to learning, to view mathematics as a developing body of knowledge rather than just a set of facts to be learned.

*Department of Education, Queensland (1987). 'Teaching, Curriculum and Assessment Guidelines', Queensland*

Measuring the height of a tree.

Statement Four

School mathematics is changing but mathematics is still often presented in schools as merely a body of knowledge rather than a way of knowing. Hence it is seen by many students to be objective and inflexible, linear and deductive, culture-free, dissociated from reality and as absolute truth. However, as professional mathematicians agree, mathematics is a product of an intuitive and creative process reflecting historical and social conditions and developments, and in its applications, presents models of reality, rather than universal truth.


Symmetry in PNG Sepik architecture.
Statement Five

Western mathematics is widely recognised as a human construct (Davis & Hersh, 1981). It is an agreed-upon way of thinking about the world which is continuing evolving. Human invention and reconceptualisation shapes the mathematics of the future. Despite this, Western schooling in colonial countries has enshrined mathematics as a static body of knowledge to be transmitted to the learner (Mousely 1990). It is not generally seen as a discipline where students’ exploration and wondering have a place. Whereas creative possibilities in studies such as science and English are apparent to the general public, original work is not often considered in mathematics.


Counting with fingers.

2.1 Activity 2 – Group discussion and writing

Choose one of the statements and work with a small group of peers to discuss the ideas raised. What do you think the statement means? Do you agree or disagree with the ideas? Prepare to share your findings with the rest of the class.

Together, write your own definition of ‘what we mean by this thing called mathematics’.
**Topic 1 – Mathematics and culture**

Each culture develops its own mathematical way of thinking. Societies construct their own way of interpreting the world around them and describing and explaining the patterns and relationships which represent their reality.

Below you will find examples of how different cultural groups have constructed the notion of ‘time’. Each example represents the way a particular cultural group has interpreted and explained their view of reality, seeking order and pattern in their world.

2.1 Activity 3

Discuss the examples provided and the different ways cultural groups have constructed their notion of time e.g.

- the Abelam Calendar
- the Gregorian Calendar
- the Chinese Calendar.

What similarities and differences do you see? What do you think are the reasons for the variations? Which interpretation is right? Prepare to share your ideas with the class

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**The Ambelam Seasonal Calendar**

The Ambelam people from the Maprik district have a seasonal calendar which people use in their day-to-day life to plan their activities. These include agricultural activities as well as customary obligations. It divides the year into two main seasonal periods which are known in the local Ambulas language as the Kwi and the Kuma season. If people do not follow the activities in a prescribed season, particularly during the Kwi season, then the whole family will go hungry and they will be unable to meet village obligations.
The Abelam Seasonal Calender  
(Maprik)

- Slash and burn  
- Planting of yams  
- Sago making

- Harvesting  
- Yam festival  
- Trade  
- Other customary duties

Gregorian Calendar

The modern calendar which we use today is referred to as the Gregorian calendar, named after Pope Gregory XIII. The calendar measures equal intervals of time and establishes dates from various events such as eclipses, full moons and Easter. The history of the calendar dates back to the Egyptians in 4241 BC, when the
Egyptian calendar of 12 months of 30 days each plus 5 additional days was introduced. The Romans later adapted the Egyptian calendar to introduce a 24-hour day, a fixed year, leap years and a 12-month year.

**Chinese Calendar**

A Chinese calendar consists of both the Gregorian and a lunar-solar calendrical systems. The lunar-solar calendar divides a year into twelve months, with each month equally divided into thirty-nine and a half days. The well-coordinated dual system calendar reflects the Chinese ingenuity.

Besides the two calendrical systems, a Chinese calendar also has twenty-four solar terms which are closely related to the changes of Nature. These terms are a very useful tool for farmers, providing information on the proper time for planting and harvesting.

**The Twenty-Four Terms**

The first solar term: *Beginning of Spring*, is used to refer to the first fifteen days of the Chinese lunar month, which usually starts from the fourth or fifth of February. The first day in the *Beginning of Spring* is the Chinese New Year’s Day and the beginning of the Spring Festival.

The second fifteen days are named: *Rain Water* from the nineteenth or twentieth of February, a time when rainy seasons are setting in.

In order come the following terms:

- *Waking of Insects* from the fifth or sixth of March, as the earth awakes from hibernation;
- *Spring Equinox* from the twentieth or twenty-first of March;
- *Pure Brightness* from the fourth or fifth of April;
- *Grain Rain* from the twentieth or twenty-first of April;
- *Beginning of Summer* from the fifth or sixth of May;
- *Grain Full* from the twentieth or twenty-first of May;
- *Grain in Ear* from the fifth or sixth of June;
- *Summer Solstice* from the twenty-first or second of June;
- *Slight Heat* from the sixth or seventh of July;
- *Great Heat* from the twenty-second or third of July;
- *Beginning of Autumn* from the seventh or eighth of August;
- *Limit of Heat* from the twenty-third or fourth of August;
- *White Dew* from the seventh or eighth of September;
- *Autumnal Equinox* from the twenty-third or fourth of September;
- *Cold Dew* from the eighth or ninth of October;
- *Frost’s Descent* from the twentieth-three or fourth of October;
- *Beginning of Winter* from the seventh or eighth of November;
Slight Snow from the twenty-second or third of November;
Great Snow from the seventh or eighth of December;
Winter Solstice from the twenty-second or third of December;
Slight Cold from the fifth or sixth of January; and lastly
Great Cold from the twentieth or twenty-first of January which brings the 24-term cycle to an end.

The 60-year Chinese Calendar Cycle

On the Chinese Calendar, you will also find terminology like Tian Gan and Di Zhi (Heavenly Stem and Earthly Branch), a peculiar Chinese way of marking the years in a sixty-year cycle. There is also a system that marks the years in a twelve-year cycle, naming each of them after an animal such as Rat, Ox, Tiger, Hare, Dragon, Snake, Horse, Sheep, Monkey, Rooster, Dog and Boar.
2.1 Activity 4 – Research

Carry out some research into one aspect of mathematics from your own cultural perspective. Think about the way in which your cultural group represents your worldview. Some of the topics which you may wish to explore are:

- traditional counting systems
- time (during the course of a day, seasons)
- sharing
- measuring
- kinship
- directions

You will need to:
1. Decide on the area you wish to research.
2. Collect relevant information.
3. Write down in your own words what you have found out.
4. Prepare to share your findings with others.

2.1 Activity 5 – Cultural mathematics in the school

Reflect on your own mathematical cultural knowledge. How has this been incorporated into your education? Is it valued? What effect has this had on your learning of mathematics? Write down a few notes to summarise your thinking. Prepare to discuss your reflections with others.
**Topic 2 – Language and mathematics**

Language plays a vital role in the teaching and learning of mathematics. Mathematics is part of our everyday life and it is through language that mathematical ideas are communicated. Opportunities for talking, explaining, describing, questioning, clarifying, writing, recording and sharing need to be provided in the mathematics classroom so that children are able to develop their ability to express their mathematical ideas.

The reading below discusses the role of language in the teaching of mathematics. It examines the relationship between the everyday language of mathematics, maths specific language and the symbols of mathematics. The use of English and Vernacular when teaching mathematics is also considered.

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**2.1 Activity 6**

Read the article and highlight any new ideas which you feel are important. Be prepared to discuss the article with the class.

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The role of language/s in the teaching of mathematics

*(adapted from Maths in Context, Primary units of work. Northern Territory Department of Education, Darwin 1993)*

Language is the most powerful tool which people use to make meaning of their world. If students are to access the mathematics of the wider society then teachers need to be conscious of how language is taught as students learn mathematics.

Language is central to the learning of mathematics. Therefore students need to be able to talk about mathematical concepts with understanding, and record their knowledge in everyday language.

As students become increasingly confident in expressing mathematical ideas in their own words, the specialist language of mathematics represented by symbols and specific terms will be more easily understood. Many young children can manipulate mathematical symbols, signs and numbers without fully comprehending the understandings they represent. It is not until students have experiences where they need to use the mathematics in real situations and talk about it that they will be able to develop an understanding of the mathematical symbols they are manipulating.

Students need to explore and learn about how mathematics is used in the real world. To do this they need to:

- Experience mathematics in real and realistic contexts
- Talk about mathematics with adults and other children
- Learn to use the special language that belongs to mathematics
- Use different ways to communicate, record, interpret and reflect on the mathematics they are learning
- Learn and use mathematical symbols to record what they are doing and finding out
- Learn new concepts and learn to talk about them.

Language will play the most vital role in determining how well students come to understand the culture of mathematics and how they are able to use mathematics in their lives as it is needed now and in the future.

The following diagram shows the three linguistic aspects involved in the teaching of mathematics. All three areas are crucial to mathematical understanding. Where students come from a culture which is reflected in the mathematics, the everyday language aspect of maths occurs as a natural part of teaching. Everyday language needs to be taught in context along with the symbols and maths specific language if students are to understand the role of mathematics and its relevance to their lives. This is especially important if the student comes from a culture which reflects a view of the world which differs from that of school maths.

It is important to remember that the symbolic maths language is only the ‘shorthand’ way of recording the thoughts and ideas that have resulted from lots of talking, writing, experimenting and interpreting real life mathematical situations. While it is a necessary part of maths, it is not the most important part of learning and understanding mathematics. Symbols and specific
terms will not be fully understood by students until they are able to express the ideas in their own words.

**What language to use in the teaching of maths?**

All students bring to the school context a wide range of linguistic skills and knowledge from their own cultural background. The worldview represented by these languages is often very different from the techno-mathematics culture of the wider society. Even aspects of the techno-mathematical culture that are in use in many villages may sometimes be used differently from the ways in which members of the majority culture use them. Similarly students may use English in different ways. They may use certain English terms but have interpretations or uses for those words different from those used by speakers of English as a first language.

Many of the children in our schools will be learning English as a language additional to those they use out of school (Tok Ples, Tok Pisin). They will be learning English as a second language. The methodology used to teach maths needs to reflect the language needs of the students. One effective way to teach the cultural understandings implicit in the language is to have students participate in social contexts where the maths of the wider society is integral to the context.

**The role of the vernacular**

When students who speak a language other than English are being taught in English, their first language plays a crucial role in accessing knowledge. Students who are learning another language for instructional purposes need to use their own language to think about and assimilate new ideas in the process of comparing and contrasting them with what they already know.

When teachers teach western maths concepts, a lot of explanation may occur in Tok Ples or Tok Pisin while the mathematical concepts are being introduced. Some languages have ways of describing or naming parts of the number system; other languages have ways of explaining space and measurement concepts. When it comes to difficult English vocabulary, teachers may use Tok Ples or Tok Pisin to explain what the terms mean. As students progress through the mathematical stages, the amount of English needed by students for mathematical learning will increase.

**The role of English**

When teachers consider the teaching of English in their maths program it is important that they think about the full range of language skills in everyday English that they need to teach to students so that they can understand and use school mathematics. Students need to learn more than just the maths specific vocabulary e.g. perpendicular, divisor etc. When English is used as the medium of instruction, teachers must be aware that they need to use English as Second Language (ESL) techniques. The following examples illustrate the attention to detail which is necessary in an ESL teaching context and the possible confusion in learning which can occur if the language of mathematics is not taught.

Mathematical meaning can be altered dramatically according to the prepositions used and understood:

- The temperature rose by ten degrees
- The temperature rose to ten degrees
• The temperature rose from ten degrees

Mathematical meaning can be altered depending how the word is used. For example consider how the word ‘left’ is used in these two different situations.

• Turn left at the third corner.
• Separate them into three sets. How many are left?

Often teachers of mathematics forget about teaching these small words because they don’t seem all that important. Yet these small words make a big difference to the meaning of the information. This example shows the importance of explicitly addressing all aspects of language (including grammar). The language teaching must be done repeatedly in real life or realistic contexts for students to understand the differences and meanings expressed in English. This is an important ESL strategy.

2.1 Activity 7

What are the important language considerations in the teaching of mathematics? Write down your ideas and prepare to share these with the class.
**Topic 3 – Concrete materials**

Research has shown that children learn to understand new concepts when they have the opportunity to use concrete materials at the same time as the new ideas are being introduced. A variety of concrete materials should be available for children to use whenever they feel they need for them. Some of the materials that teachers need can be collected by the children and teacher (e.g. shells, stones, containers and bundles of sticks) while other materials will need to be purchased or made (base ten blocks, rulers, geometric shapes).

The reading below discusses the value of using ‘hands-on’ materials when teaching mathematics to people of all ages to ensure that real understandings are developed. It presents a ‘four stage learning model’ in which the use of concrete materials is an essential component.

### 2.1 Activity 8

*Read the article ‘Using hands-on materials’ and highlight the important ideas. Be prepared to discuss the article with the class.*

**Using hands-on materials**

*(adapted from Marr, B., & Helme, S. Breaking the Maths Barrier (1991), Department of Employment, Education and Training, Canberra, Australia)*

Hands-on experience is fundamental to all learning, and the learning of mathematics is no exception. Using hands-on materials in teaching maths to students enriches the learning experience by encouraging visualisation and expanding the range of senses involved to include touch, sight and movement.

By manipulating concrete objects, students learn by exploration and discovery, becoming active participants in the learning process.

Hands-on materials are also useful as an aid in problem solving. Using pieces of paper, counters or objects they can move around encourages students to visualise the problem and take risks in trying out solutions.

In the past most students were taught maths using traditional paper and pencil methods, and were expected to memorise rules and formulas without really understanding them. A classic example is the rule of ‘borrow and pay back’ for subtraction, which tends to mystify rather than explain this basic arithmetical process.
Concrete materials are a powerful means enabling students to understand that such rules do make logical sense. British psychologist Richard Skemp (1986) warns against attempts to learn mathematics by rote, concluding that if basic concepts are not properly understood, later learning which builds on these concepts becomes unnecessarily difficult, if not an impossible task. Perhaps this is the reason that most adult students have little real understanding of maths and consider it to be a difficult subject.

Many teachers involved in education were the victims of similar educational methods as their students. Having learnt maths using traditional paper and pencil methods they may not feel very confident with alternative approaches. Teachers using hands-on materials for the first time may also fear the reaction from students when they produce materials which they may think belong in the kindergarten!

But it is important to overcome these initial reservations. Once you have used them and seen how effective they are, hands-on materials will become a valuable part of your teaching repertoire.

**Using hands-on materials for learning mathematics**

This approach to teaching mathematics owes its origins to the work of Piaget and Dienes. They argued that before a child acquires the ability to manipulate and abstract concepts mentally, he or she must pass through a stage of concretely manipulating objects. This is true also for adults: **no matter how old the student is, no verbal or written instruction can replace what can be learned through experience.**

Hands-on activities can give students the chance to experiment with many concepts, such as number, place value and fractions. In doing so they begin to conceptualise relationships and make the connections that are necessary before proceeding to more formal work. Thus, learning is not simply the transfer of knowledge from teacher to student, but is an active process:

> *Learners construct their own understanding of the world: their constructions are personal and necessarily linked to their own experiences. When the mind experiences something new, older perceptions and concepts are pushed around and shaken up so that the new information can be fitted in with what is already known.... Learning is a natural process, a process of making sense of things* (Mathematics Frameworks 1988, p.22).

**A four stage learning model**

A framework for mathematics learning which incorporates hands-on experience has been developed by Ballagh and Moore (1987). This model derives from research into experiential learning which draws together the ideas of Dewey, Lewin and Piaget (Kolb, 1984).

The process of learning maths can be understood as a four-stage cycle which involves four learning modes: concrete experience, reflection, abstraction and application.
**Stage 1: Concrete experience**

In this stage new concepts are presented in a concrete way through a practical activity or physical model. This gives students the opportunity to draw on their prior knowledge and experience together with the sensory information gained in the activity, to form an intuitive understanding of the concept.

It may also serve a motivational purpose by engaging students in an enjoyable and non-threatening activity. In exploring fractions for example, students may start by collecting pieces together to make circles. They may name pieces they already know, and name unfamiliar sizes according to how many pieces make a circle.

They may notice that two pinks (quarters) fit exactly on top of a red (half), which leads us to Stage 2.

**Stage 2: Reflection**

Students naturally use language to explore and reflect upon the information they are gaining at a concrete level. The role of the teacher is to encourage students to describe, discuss, and explain the relationships they are observing. It is important that they start by using informal language, expressing what they have observed in their own words. For example, students may express their observations initially by commenting that two pink pieces are the same as a red piece. The teacher then begins to encourage the use of more formal language and may introduce some new words or notation, such as the words numerator and denominator or the idea of equivalent fractions. This process of reflection allows students to build on their experience, discover important relationships for themselves, articulate these relationships and begin to formalise them in writing.

It is during this process of examining concepts anew that students may discover that rules or assumptions they made in the past do not work any more. For instance many students assume that when adding the fractions $\frac{3}{8}$ and $\frac{1}{4}$ you simply add the top and the bottom separately to give $\frac{4}{12}$. If students have the opportunity to work with fraction circles, they would find that this rule is inconsistent with what their experience tells them, because combining the fraction pieces would make a piece much bigger than $\frac{4}{12}$. If students continue exploring the problem and replace the $\frac{1}{4}$ with $\frac{2}{8}$, they would then have in front of them the fraction $\frac{5}{8}$, the result of adding $\frac{3}{8}$ and $\frac{3}{8}$. When students articulate a new and more appropriate explanation based on their own experience, they are developing real understanding and not mindlessly applying rules.

The importance of written and spoken language at this stage cannot be over-emphasised and strategies for developing these language skills need to be incorporated into a teacher’s planning.

**Stage 3: Abstraction**

Once students have discovered important relationships, they are ready to integrate their observations into logically sound procedures or rules, translating their own language into formal language and symbols. (For example, to multiply two fractions you multiply the top...
numbers together and the bottom numbers together.) It is important that students have access to the hands-on materials in order to check their predictions. For example having predicted that \(\frac{3}{8}\) of a cake is more than \(\frac{1}{4}\) of it, using their rule for equivalent fractions (i.e. \(\frac{1}{4}=\frac{2}{8}\), which is less than \(\frac{3}{8}\)), they can return to the fraction kit to check their results.

Stage 4: Application

The final stage in this process is applying their understanding of new concepts in unfamiliar situations, for example, applying their knowledge of equivalent fractions to finding percentages. Students able to generalise to fractions which are not in the kit, can convert \(\frac{25}{100}\) into \(\frac{1}{4}\) and thus solve a problem using a 25% discount.

References

Cobb, P. (1988). *The Tension Between Theories of Learning and Instruction in Mathematics Education.* Educational Psychologist, 23 (2) pp 87-103.


2.1 Activity 9

What are the advantages of using hands-on materials when teaching mathematics?

Give an example of a mathematics activity you have taught using hands-on materials and explain how these have helped develop children’s learning.
Topic 4 – Active involvement

Active involvement requires teachers to provide opportunities for children to participate in their own learning. Children need to be able to explore new mathematical concepts and construct their own understandings. Teachers need to be able to provide a classroom environment which motivates children to learn and allows children to engage in mathematical activities; where children manipulate materials, discuss their learning, solve problems and debate solutions, so that children can build their own mathematical understandings.

2.1 Activity 10

Review the illustrations below and discuss them with a peer. What do you see as the value of children participating in these activities? What problems do you see with implementing these types of activities in the classroom? Write down your ideas and prepare to share them with the class.

Children measuring outside the classroom

Playing maths games
Using concrete materials

Recording mathematical ideas

Sharing learning with the class

Talking together and discussing ideas

**Topic 5 – Teaching in context**

For mathematics to be seen as relevant to students,
teachers need to make connections between the mathematics of the classroom and the mathematics people use in the real world - outside of school. In the home, in the workforce, in completing day-to-day activities people need to apply their understanding of mathematics in a wide range of different contexts. When teaching mathematics it is important that children develop an understanding of the purpose for learning mathematics. This can be achieved by teaching mathematics through real life and realistic experiences.

The following reading discusses the need for teaching mathematics in context, particularly when students come from a background with a different worldview from that presented by the mathematical concepts included in the primary school curriculum.

2.1 Activity 11

Read the article and highlight new ideas and the ideas which you consider to be important. Be prepared to discuss the article with the class.
Context based mathematics

(adapted from 'Maths in Context, Primary units of work'. Northern Territory Department of Education, Darwin 1993)

Every socio-cultural group uses a variety of systems to organise and explain the world around them. What we teach as mathematics represents some ways in which the technological cultures organise and explain this particular worldview. Where the learners of mathematics come from a culture which has other explanations for the reality in which its students live, it is necessary to present mathematical knowledge within a realistic, socially-based context with which they are in contact.

Teaching maths in context enables the learner to gain mathematical knowledge and skills that are implicit in the worldview, expressed through the techno-mathematics culture. Teachers need to create a purpose for learning mathematics. The context provides the purpose for learning. The context needs to be real or realistic.

The context teachers will be able to provide for their students will sometimes be real-life experiences. Here are two examples:

- While on an excursion in an unfamiliar city or town, students use a map to get to a certain place
- Students estimate the number of pieces of bamboo they need to make a fence around a garden plot and then construct a fence

The context teachers will be able to provide for their students will sometimes be realistic experiences. Here are two examples:

- Students run a classroom shop, giving change to other students who buy goods
- Students use a model of their school to plan where to place a ‘new’ classroom, and make a model of the building using cardboard

There are many contexts which teachers will use where it would not be possible or practical to provide a real experience. Instead teachers will provide a context that tries to create a realistic (life-like) model of the real context. Both real and realistic contexts provide students and teachers with the opportunity to experience the purpose and culture of the mathematical knowledge and skills in real-life or life-like contexts.
2.1 Activity 12

Identify an example of how you would teach a mathematical concept through:

- a real life experience
- a realistic experience.

Why is it considered important to teach mathematics in context? Write down your ideas and be prepare to share these with the class.

References – for further reading


Department of Education Papua New Guinea (1998). Lower Primary Mathematics Syllabus Grade 3-5,

Department of Education Papua New Guinea (draft 1999). Upper Primary Mathematics Syllabus Grade 6-8


Mousley, J. and Sullivan, P (1996). Learning about Teaching, an interactive tutorial program to facilitate the study of teaching. AAMT, Deakin University and The Australian Catholic University
Module 2.2 – Teaching Mathematics

*Module 2.2: Teaching Mathematics* is a core module within the unit ‘Teaching and Learning Mathematics’. During this module you will consider how your beliefs about mathematics influence the approach you adopt when teaching the subject. Strategies for developing children’s ability to talk about and record their mathematical thinking will be explored. Co-operative learning, as a strategy for the teaching of mathematics, will be considered together with a range of other strategies appropriate for teaching mathematics in PNG primary schools.

**Objectives**

By the end of this module you will be able to:

- Discuss your personal philosophy on the teaching of mathematics
- Describe a range of strategies suitable for the teaching of mathematics
- Discuss the advantages and disadvantages of using a co-operative learning approach when teaching mathematics
- Identify a range of different ways children can demonstrate their mathematical thinking
- Construct open ended mathematics questions suitable for use in the primary mathematics classroom

**Concepts and skills to be developed**

During this module the following concepts and skills will be developed.

- Teaching skills
- Co-operative learning skills
- Reading and interpreting skills
- Writing skills
- Oral skills
- Mathematical skills
- Questioning skills
- Reasoning and thinking skills
Introduction – teaching a ‘good’ mathematics lesson

What makes a ‘good’ mathematics lesson? What do we need to do to be ‘good’ mathematics teachers? The different mathematics teachers we have had throughout our education have all had their own personal theories about how mathematics should be taught and this has influenced the approaches they have adopted when teaching. As a result we may have had both positive and negative experiences when learning mathematics. During this module we need to spend time develop our own theories about what ‘good’ mathematics teaching is all about. We need to identify the strategies that we can implement in the classroom to ensure that children develop their mathematical understandings as well as a positive attitude towards mathematics.

The following reading identifies a number of things teachers’ can do to improve their teaching of mathematics.

2.2 Activity 1

Read the article ‘The twelve most important things you can do to be a better maths teacher’. Which of the ideas suggested have you tried in your own teaching of mathematics? How did it go? Identify one idea that you would like to try next time you are teaching a mathematics lesson? How might you go about implementing this idea?

Be prepared to discuss your ideas with the class

The twelve most important things you can do to be a better maths teacher

(adapted from an article by Marilyn Burns in ‘Instructor’, April 1993)

Not too long ago, teachers saw the main goal of maths instruction as helping children become proficient in paper-and-pencil computation. Today, mathematics instruction is less about teaching basic computation and more about helping students becoming flexible thinkers who are comfortable with all areas of mathematics and able to apply mathematical ideas and skills to a range of problem solving situations.

Making these goals a daily part of maths instruction may require a shift in the way you think about mathematics and your role in teaching it. And so I offer the following suggestions and examples from actual classroom lessons to help you re-think your teaching practices.
Set the following expectation for your students: Do only what makes sense to you.

Too often, students see maths as a collection of steps and tricks that they must learn. And this misconception leads to common recurring errors in arithmetic calculations – when subtracting, students will subtract the smaller from the larger rather than borrowing; when dividing, they’ll omit a zero and wind up with an answer that is ten times too small; and when combining fractions, they’ll incorrectly add both the numerators and the denominators. In all instances students arrive at an answer that makes no sense, and they neither notice nor care.

There is no place for children doing mathematics by rote. Students must be able to explain the purpose for what they’re doing, the logic of their procedures, and the reasonableness of their solution.

1. **Have your students explain their reasoning in all instances**

It is insufficient and short sighted to rely on quick, right answers as indicators of students’ mathematical power. During maths lessons, probe children’s thinking when they respond. Ask; Why do you think that? Why does that make sense? Convince us? Prove it? Does anyone have a different way to think about the problem? Does anyone have another explanation?

When children are asked to explain their thinking, they are forced to organise their ideas. They have the opportunity to develop, cement, and extend their understanding. Teachers are accustomed to asking students to explain their thinking when their response is incorrect. It’s important, however, to ask children to explain their reasoning at all times, even when their answer is correct.
2. Encourage children to talk to one another during the maths class.

Communicating is essential for learning. Having students work quietly – and by themselves – limits their opportunities for learning. Interaction maximizes children’s opportunities to talk about their ideas, get feedback for their thinking, and hear other points of view. The challenge today is to have students engage in dialogue and work together to solve problems and bring meaning to mathematics ideas. Students can learn from one another, as well as from the teacher.

3. Make writing an integral part of maths learning.

Communication in the maths class should include writing as well as talking. In his book, *Writing to Learn*, William Zinsser states ‘Writing is how we think our way into a subject and make it our own.’ When children write in the maths class, they have to revisit their thinking and reflect on their ideas. Student writing gives teachers a way to assess how their students are thinking and what they understand.

Writing in the maths class best extends from children’s talking. When small group interaction or a whole class discussion precedes a writing assignment, students have a chance to formulate their ideas before they’re expected to write. It’s also helpful to write prompts on the board for students to get them started. For example ‘I think the answer is ______. I think this because ________’.

4. Embed the maths activities in contexts.

When connected to situations, mathematics comes alive. Contexts give students access to otherwise abstract ideas. They stimulate student interest and provide a purpose for learning mathematics. Contexts can draw on real-life examples, or they can be created from imaginary situations. Many children’s books for example offer starting points for classroom mathematics lessons.

5. Use manipulative materials whenever possible.

*Example of manipulative materials – Base 10.*

Manipulative materials provide a concrete context for thinking about mathematics. They give children hands-on experiences for grabbing onto mathematical ideas, turning them around, and viewing them in a different way. Manipulative materials can serve in several ways – to introduce concepts, to pose problems, and to serve as tools to figure out solutions. It’s important that manipulatives are not relegated only to young children, but that they are made available to students in every grade.
6. **Bring the quality and richness often apparent in student’s writing into their maths work.**

Typically in primary classrooms, children’s rich, varied, and creative art and writing fill the walls, while the maths work that is posted commonly consists of arithmetic worksheets or progress charts that tracks students’ learning of basic facts. And in language arts and art lessons, children’s imaginative ideas are invited and applauded; unique and unexpected results are common. In maths lessons, students often learn and practice specific prescribed methods, consistency among student papers is desired.

Find ways to make maths lessons and assignments as intriguing, rich, and motivating as they are in other areas of the curriculum. Give students the chance to use their creativity when thinking about mathematics. Encourage them to be inventive and trust that they will be.

7. **Make calculators available to all children at all times.**

Calculators are valuable tools for doing laborious mathematical computations. As stated in the NCTM *Curriculum and Evaluation Standards for School Mathematics* (page 8): ‘Contrary to the fear of many, the availability of calculators … has expanded students’ capability of performing calculations.’

Calculators are not a replacement for student thinking. In any problem situation, a child needs to know which buttons to push, whether the answer displayed makes sense, and what decisions to make with the results. Calculators eliminate the drudge of complex calculations. They also help children solve problems they might not otherwise be able to tackle.

8. **Let children push the curriculum rather than having the curriculum push the children.**

Choose depth over breadth. David Hawkins has said, ‘You don’t want to cover a subject, you want to uncover it’ (*The Having of Wonderful Ideas* by Eleanor Duckworth, Teachers College Press 1987). There are many pressures on teachers, and the school year passes very quickly. But students’ understanding is key and doesn’t always happen according to the schedule suggested in the text materials. Just as students should do only what makes sense to them, the same is true for teachers. There is value in staying with a topic children are interested in, pushing more deeply, and taking the time for a side investigation that can extend a lesson in a different direction.

9. **Keep an eye out for instructional activities that are accessible to students with different levels of interest and experience.**

A wonderful quality of good children’s books is that they delight adults as well. Of course, adults appreciate books for different reasons than children do, but enjoyment and learning can occur simultaneously at all levels. The same holds true for mathematics investigations. Search for activities that can engage children who have the least mathematical experience while challenging students with the most experience.
10. **Remember that confusion and partial understanding are natural to the learning process.**

Do not expect all children to learn everything at the same time, and do not expect all children to get the same message from every lesson. Although teachers want all their students to be successful, they rarely reach every student with any one lesson. Learning should be viewed as a long-range goal, not as a lesson objective. It’s important that children do not feel deficient, hopeless, or excluded from learning mathematics. The classroom culture should reinforce the belief that errors are opportunities for learning and should support children taking risks without fear of failure or embarrassment.

11. **Take delight in students’ thinking.**

There is no one way to think about any mathematical problem. Encourage students to think in different ways. After children respond to questions (and of course, have explained their thinking), ask; Does anyone have a different idea? Keep asking until all children who volunteer have offered their ideas. By encouraging participation, you’ll not only learn more about individual children’s thinking, but you’ll also send students the message that there is more than one way to look at any problem or situation.

### 2.2 Activity 2

*Working with a small group of peers, choose one of the suggested ideas outlined in the reading ‘The twelve most important things you can do to be a better maths teacher’.*

*Discuss the idea together and as a group prepare to share your understandings with the rest of the class. You will need to:*

- explain what the suggestion is and what it means (in your own words)
- describe how you could implement this suggestion to teach mathematics in primary schools.*
**Topic 1 – Philosophies of mathematics**

Our personal beliefs about mathematics will influence how we approach the teaching of mathematics. Our philosophy of mathematics will influence the decisions we make about what to teach during our mathematics lessons as well as impacting on the way we teach mathematics.

Outlined below are a number of different philosophies of mathematics, which have influenced how mathematics has been taught over recent years. When reading the article consider each of the philosophies and think about your own personal believes about how mathematics should be taught.

### 2.2 Activity 3

*Read the article ‘Philosophies of Mathematics’ and highlight the ideas which you consider to be important.*

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**Philosophies of mathematics**


Teachers aim to educate the children in their care. As responsible professionals, teachers make decisions about what they will teach and how they will teach it. Belief systems and values will strongly influence the decisions teachers make as they do those things that they judge are right. When making these judgements teachers are acting out their philosophy of teaching.

In developing your own philosophy for the teaching of mathematics you are required to develop ‘a well considered view of how mathematics ought to be taught’. To help us develop our own philosophy we will consider the philosophies of mathematics which have been the most influential during the last half-century. These are formalism, logicism and intuitionism.

**Formalism**

There are people who look at mathematics as a whole and observe that mathematical statements and arguments are presented in characters and that the characters are strung together in an organised way. People who believe that mathematics is the study of how characters are put together to make well formed formulas are known as formalists.

Formalists believe that mathematics is the collection of all consistent well-formed formulas together with the rules for their formation. These rules are known to all mathematicians and are
considered ‘correct’ or ‘proved’. Formalists consider the main job of the teacher is to present children with the formulas and the rules for their combination. The students would then practice these so that they master the particular rules.

In primary school terms, the arithmetic part of mathematics is the collection of number sentences such as

\[2 + 3 = 5, \quad 11 \times 23 = 253\]

and the doing of mathematics is the deriving of statements such as

\[11 \times 23 = 253\]

from other statements, namely

\[1 \times 3 = 3, \quad 1 \times 20 = 20, \quad 10 \times 3 = 30, \quad 10 \times 20 = 200 \quad \text{and} \quad 20 + 30 = 50\]

The formalist would agree that young children need a lot of motivation to learn mathematics and that a good form of motivation is the use of games. As the child grows older less motivation is needed and therefore the formalist would believe that the need for concrete materials decreases until none are used at all. The formalist would also agree that when a person has learned some mathematics it is desirable to use that mathematics, that is, apply it to the real world.

**Logicism**

There are people who look at mathematics and observe that the essential thing about mathematics is that the results (the well-formed formulas of the formalist) are arrived at by the process of pure reasoning. These philosophers see mathematics as a branch of logic and are known as logicists. For logicists, mathematical statements can be true or false. True statements are true not because of empirical evidence (experiences) but because they are derivable from and consistent with, other logically true or assumed to be true statements.

The aim of a logicist teacher will be to present a concise argument, with each step in the proof carefully laid out and explained in terms of previously known principles.

For example, where the formalist considers ‘2 + 3 = 5’ as a consistent well-formed formula, the logicist consider it as a logical theorem. A logicist would say that ‘2’ is name given to \((1 + 1)\) and that ‘3’ is the name given to \((1 + 1 + 1)\) and ‘5’ is \((1 + 1 + 1 + 1 + 1)\) so that the theorem

\[2 + 3 = 5\]

is simply a convenient way of writing

\[(1 + 1) + (1 + 1 + 1) = (1 + 1 + 1 + 1 + 1)\]

Logicist use a process of deductive reasoning to explain the algorithm, presenting each step clearly.

**Intuitionism**

There are those people who look at mathematics and claim that the characters used in mathematics do have meaning and that they have the status of being symbols. The symbols stand for ideas, concepts and abstractions and this is what is considered important. True statements are true because they are intuitively obvious and it can be related to reality.
Symbols are used to model certain aspects of reality. People who stress the meaning of symbols are called intuitionists.

Taking an intuitionist approach means that the learner is actively constructing their own knowledge, rather than passively waiting for clues to trigger already present knowledge. A good intuitionist teacher will not be a teller of things mathematical to their students. They will provide the very best environment relevant to the aspect of mathematics they wish students to construct, providing materials for students to manipulate. Real world problems will be presented and natural language used to explain the ideas being developed. Symbols will be introduced only after the relevant concepts have been developed.

The intuitionist view ‘2+3=5’ as being a symbolic statement which says something about the real world: that is if you take a collection of two things and combine it with a collection of three things then you have as many things as there are in a collection of five things. ‘2+3=6’ is false because it does not relate to reality.

2.2 Activity 4 – Discussion

Prepare to discuss the following questions:

- What do you see as the main differences between these three philosophies?
- Consider your own experiences in learning mathematics. What philosophical approach have your various teacher taken?
- What is your own philosophy for the teaching of mathematics?

A constructivist view of mathematics learning

(adapted from ‘Teaching Primary Mathematics’, p6-9)

One of the current views influencing the way in which mathematics is now being taught is the belief that mathematics is learnt by individuals constructing their own ideas. If you have a constructivist view of mathematics, you believe that each person develops their own mathematical understandings, ideas and processes for themselves. These mathematical understandings are constructed through having a range of experiences and activities, through having opportunities to manipulate materials, talk and record mathematical ideas, and through making mistakes, asking questions and building on past experiences and existing knowledge. Children build up their own view of mathematics, reflecting how they see the world and how it is organised. Cultural background will influence how a child constructs the world around them and therefore how their mathematical understandings will develop.

The role of a teacher following a constructivist approach to teaching mathematics is to support students to create more powerful constructs and appropriate ways of thinking. A teacher
guides students towards understanding, and helps people interpret any new information or situation which they experience. The teacher needs to ask questions which challenge ill-informed constructions, ideas or generalisations and pose new problems which require people to revise old ways of thinking. The teacher’s role is to acts as a guide, leading students to new learning. Children who acquire their knowledge through this process of constructing their own ideas tend to be able to apply their learning in new and different situations.

In guiding students to construct their own knowledge there are three main phases

1. Identify any prior learning which may be used as a foundation for building the new concepts or skills

2. Ensure that all children have the necessary prior learning

3. Allow an opportunity for each individual to participate in the process of constructing new concepts or skills.

It is important to match the meaning of the mathematics being taught with a suitable context for learning and at the same time provide appropriate materials for children to explore, and opportunities for people to talk about what they are doing and to record their ideas, using symbols when appropriate. Consideration must also be given to the child’s level of development and their ability to take in information, to generalise, and to construct reasonable views of the underlying mathematics.
2.2 Activity 5 – Constructing learning

Consider the following situation

A child reads the number 3005 as three hundred and five, rather than as three thousand and five.

What misconception does the child have e.g. what concept does the child not understand?

Following a constructivist approach to teaching what activities would you plan to guide this child towards re-constructing their mathematical ideas in this area? Thinks about:

- the types of questions you might ask
- the activities you might do with the child
- the materials you might use

Share your ideas with a peer and be prepared to discuss these with the class.
Topic 2 – Strategies for teaching mathematics

The Primary School Mathematics curriculum documents are made up of a range of topics and cover a wide range of learning outcomes which are taught across a number of years. There is no one single suitable method for the teaching of mathematics, so it is important that teachers develop a range of strategies which they can implement to support children in the development of their learning and understandings. As stated in Herrington, Sparrow, Herrington, Oliver (1997)

Successful student learning occurs in situations where the students are motivated to learn, where tasks build upon previous experiences, where students can construct their own understandings. Tasks need to be challenging, provide multiple perspectives and representations, and allow avenues for reflection and communication that are developed in social and supportive environments. Some or all of these general principles may be reflected in a variety of teaching approaches. (p. 2)

Herrington, Sparrow, Herrington, Oliver (1997) in ‘Investigating teaching strategies in mathematics classrooms’ identify a range of strategies suitable for use in primary schools. Outlined below are a number of these strategies.

2.2 Activity 6

Read through the strategies outline below. Identify the strategies you have used before in your own teaching. What did you teach and how did it go? Which of the strategies outlined do you see as being useful to you when teaching future mathematics lessons?

Peer tutoring

The general practice here is to pair one student with another of greater ability and knowledge in the subject. The pairings may be from within the class which presents less of an organisational concern for the teacher. At other times, older students are paired with younger ones thus requiring the cooperation of more than one class. The system provides advantages for both the participants, not just the less able or younger child. Obviously the child being tutored gains from having constant and immediate attention, instant feedback to their question or method as well as having the idea or task explained in ‘childspake’. The tutor needs to understand the material or idea well but will gain further insight and understanding from trying to help the partner. Training of the tutors is important, as they need to help rather than tell their partner how to do the task. Some teachers use the pause, prompt and praise reminders to help the tutors with their interactions. By freeing themselves from the actual teaching task the teacher is now able to move around the classroom to monitor and assess students’ understandings by listening and observing the conversations.
Guided discovery

Whilst there is a designated endpoint or learning envisaged by the teacher, a guided discovery approach will see the teacher working in a very different way to the exposition method often practised. The teacher plans and sets up a situation and task that has embedded within it the desired learning outcome. For example, the teacher presents groups of students a set of 2D shapes and asks them to examine them and note any patterns and relationships they find.

The students have to use their problem solving techniques to help them find the patterns and relationships. During the work the role of the teacher is to ask questions that probe students’ understandings and direct them to useful processes such as making a table or chart. By using a class reporting session towards the end of the activity a relationship, for example, Euler’s rule:

\[ \text{Faces (F)} + \text{Vertices (V)} = \text{Edges (E)} + 2 \]

can be clarified or students helped to see it.

Generally a guided discovery lesson consists of three phases:

- the teacher sets the problem or activity
- students explore the situation
- teacher and students discuss the situation and draw their conclusions.

Students in a guided discovery activity have to take much more of the initiative acting like mathematicians and solving problems rather than as passive receivers of knowledge. The teacher on the other hand is more of a guide and questioner rather than a provider of knowledge.
Projects
Projects can take a number of forms ranging from finding out about squares and building up a file of drawings, relationships, and photographs, to researching famous mathematicians or designing, costing and building a swimming pool. Many projects can relate mathematics to the real world and can involve techniques needed in the later lives of the students. Some of the more effective projects exploit the integrated nature of their subject matter.

Most projects would involve the students in investigating, solving problems, researching and applying mathematics. Projects are usually completed with the help of a group of students involved in negotiation, group work skills, identification and collection of resources, and presentation skills (both oral and written).

From a teaching point of view, such work also gives an insight into how the students select and apply their various mathematical tools.

Modelling
Mathematical modelling is the term used to describe processes involved in using mathematics to solve real-world problems. Many problems that students face in schools are chosen by the teacher (or the textbook) as practice for recently learned procedures. Mathematical modelling is quite different in that once students have understood the nature of the problem, part of the task is to select the appropriate mathematics that will lead towards a solution. Generally, mathematical modelling problems involve a range of suitable solutions and for this reason alone they are particularly useful to use with mixed-ability classes. An example of a mathematical modelling problem would be to ask students to plan and cost a family trip. Such an activity requires students to gather information on hotel accommodation, the costs of meals, the costs of transport, the availability of transport and so on. Alternatively, this can be provided by the teacher. Students then need to select and represent mathematically the necessary costs and compare alternatives leading to possible solutions. Different schedules can be considered depending on the family budget and the time of the year when the trip is taken. The results can then form the basis of a report for the class (and family) to discuss. In this way students develop important modelling processes that include constructing a mathematical model, solving the model for mathematical solutions, interpreting these solutions in the context of the real situation and refining the model to produce better solutions. As with other problem solving activities, the use of group work is a valuable way for students to learn about mathematical modelling.
One way to show the relevance of a mathematical idea is to apply it in a realistic situation. Alternatively, a different teaching approach is to begin with the context or theme and investigate the related mathematical ideas. Making connections is an important process in learning and understanding. The connections that can be made include mathematical ideas across different content areas like measurement and geometry as well as across other disciplines such as music and art. Themes can be chosen that have significance for the children being taught. For example, a theme such as ‘the sea’ or ‘dinosaurs’ may be relevant for some communities and age groups and not others.

Once a theme has been chosen the teacher can determine suitable mathematical ideas that are related and prepare activities for investigation. It is a good idea to include student questions about the theme as these may lead to mathematical ideas that had not been considered by the teacher.

Often the theme is a realistic situation such as ‘the school fete’ or ‘PMV’s’ but the context could just as easily be mathematical such as ‘the cube.’ The mathematical ideas that can be linked to a cube theme might include the ideas such as nets, tessellations, rotational symmetry, planes of symmetry, Euler’s law, volume, and surface area.

A thematic approach can be motivating for many students and can result in classroom displays and project work that show the relevance and connectedness of mathematics.
Problem solving

Working with students in problem solving ways has become one of the main recommendations of current curriculum documents. A problem in this context is a situation that does not have an immediate and obvious method of solution for the solver. Someone has suggested that problem solving is what you do when you don't know what to do. Thus many of the things that have in the past been considered problems are in fact, according to this definition, not a problem.

Problem solving is a fundamental way of working in mathematics and should be part of the mathematical program of all students not just the more able ones or those who finish early. Recommendations from research studies indicate that problem solving should be an integral approach through which all areas of mathematics can be learnt. It should not be seen as an 'add on' to traditional approaches.

By using problem solving as a teaching and learning strategy, there is a change in teaching style that emphasises students working mathematically, with the teacher as a helper and questioner rather than as a transmitter of facts and skills. Students have to bring into play mathematical thinking in order to solve the problem presented. Wrong methods and dead ends are accepted, evaluated and used on the way to a solution. In fact there may be more than one acceptable solution and more than one route to its discovery.

Students will usually, though not always, be organised into groups so that the power of talk and discussion as well as cooperative learning may be used as a vehicle for others to learn important mathematical processes necessary in solving problems.

Problem posing

Problem posing is an important strategy to utilise in the classroom particularly one that has within it a range of abilities and interests. Initially this may come directly from the instigation of the teacher but eventually the intention is that students will pose their own problems to be solved. The teacher-question starter: 'What happens if ... ?' is a useful one in that it allows aspects of a problem situation to be changed easily. For example one may ask 'What happens if I change the whole numbers to fractions?' This then develops the original situation into a new one to be explored.

This will develop the inquiry approach and frame of mind that is important in mathematical exploration. The students' work becomes self-generating from their questions which in turn adds the intrinsic motivation and develops the idea of ownership.

Other problems worth exploring come from incidental questions posed by the students in everyday situations. Rich data handling activities can arise from the problems posed by the students rather than imposed by the teacher or the textbook.
2.2 Activity 7

Working with a small group of peers, choose one of the strategies outlined above. Plan a mathematics activity which demonstrates the use of the strategy you have chosen. Use the Primary School mathematics curriculum document to choose a grade level and a topic to teach. Prepare to teach the lesson and to explain your strategy to the class.
Topic 3 – Communicating mathematical ideas

If children are to develop their mathematical understandings it is vital that they be given opportunities to talk about their learning and to record their ideas. Talking and writing about mathematics gives people an opportunity to clarify their thinking, to explain their ideas and to demonstrate their understandings. Also when listening to children talking about their learning, or by reading their work, teachers can gain valuable insights into any misconceptions which the children may have.

When planning our teaching we need to consider how we can incorporate opportunities for children to:

- Talk
  - with the teacher
  - together in small groups
  - when presenting findings to the whole class

- Record their ideas by
  - writing journals explaining what they have learnt
  - drawing tables
  - creating graphs
  - drawing diagrams
  - making charts
  - writing word problems for their peers to solve

Group work on Charting.

Illustration by Murray Kaiyage
Year 1 Student
MTC 2000
2.2 Activity 8 – Recording mathematical ideas

Consider the mathematics activity you planned in 2.2 Activity 7. What opportunities did you provide for children to:

- talk about what they were learning?
- record their mathematical thinking?

What additional opportunities could you provide? Share your ideas with your peers.
Topic 4 – Co-operative learning

Throughout the course of our life there are many situations in which we are required to work co-operatively with others, such as in our workplaces, in our homes, in our villages, and in the wider community. Co-operative learning is a strategy that can be successfully used in the teaching of mathematics. Through co-operative learning students can increase their mathematical learning as well as develop important personal skills.

The reading that follows highlights the advantages of working co-operatively in the mathematics classroom.

2.2 Activity 9

Read the article and highlight the ideas which you consider to be important. Be prepared to discuss the article with the class.

Students at Madang Teachers College working co-operatively to solve a problem
Interaction and co-operation

(adapted from: Marr, B & Helme, S. Interaction and Co-operation in Breaking the Maths Barrier (1990) Department of employment Education and Training, Canberra)

The advent of cooperative learning is probably the most innovative change in mathematics teaching and learning in the past decade. Cooperative learning activities can be described as those performed by a small group of two or more people working together to reach a common product. This could be a solution of a problem, the collection of information and production of a graph, an estimation and measurement activity, or exploratory work with hands-on materials such as fraction circles or place value equipment in order to understand a concept.

Why co-operative learning?
The benefits gained through the use of co-operative learning techniques fall into two categories: increase of mathematical learning; and development of students' personal skills. Both of these aspects will be discussed below.

Advantages for learning mathematics

Learning through communication
Co-operative learning is used because it offers opportunities for students to learn new concepts and skills through communication with each other. In groups small enough to break down the barriers of shyness and lack of confidence, students learn through talking and listening to each other. They can clarify their own ideas by expressing them out loud to other students, gain by listening to the alternative approaches of fellow students, and consequently developing each others ideas further. Students can also assist each other to articulate their reasoning by asking clarifying questions.

Reduction of anxiety
Being part of a group working towards a common product can remove the feelings of isolation and panic so often experienced by students learning mathematics. From experience we know that students associate the study of maths with classes in which they are required to sit quietly and to passively listen, and where to look at another person's work is considered cheating. Students sit alone with paper and pencil struggling to understand the problem, or worse, to get the right answers without really understanding the concepts and processes being used.

Group work removes this isolation and replaces competition with co-operation. It engages students actively in the learning process, completely changes the atmosphere of maths learning and creates a more relaxed and informal classroom atmosphere.

Reduction of student anonymity
Small groups also help to break down the feelings of anonymity felt by members of larger classes. When the teacher interacts with a small group she/he is able to make eye contact with each individual and concentrate on each of their responses. This way each group member has a feeling of having received individual attention.
Development of mathematical language

Group work is particularly valuable for the development of the language associated with mathematical concepts. For example in cooperative logic activities a great range of basic vocabulary and mathematics concepts can be introduced or reinforced. Logical problem solving activities are structured so that students need to read the words on the clue cards out loud and understand their meanings in order to reach a solution. Thus they will explore and explain to each other new concepts and language out of a genuine and immediate desire to know, and in a specific context. This is much more effective than a blackboard vocabulary lesson where the teacher does most of the talking and explaining.

Such activities are, of course, extremely useful in the teaching of students from non-English speaking backgrounds. Research by educational researcher, Richard Skemp (1987) supports the observations of teachers of both adults and young students that students learn more effectively when they have attempted to convey their reasoning in words to another person (p. 61).

Increased student verbalisation and participation

Once we accept that verbalisation assists learning, it is easy to appreciate the advantages of having a number of small groups in the room as opposed to one large group. The amount of verbalisation taking place is increased and the shy, more reserved class members are less afraid to contribute.

Increased level of mathematics

Another advantage of groups is that students in groups can often handle more challenging tasks than they would be able to do on their own. Individuals working alone may give up when a problem looks a bit complex. However, with group support and mutual encouragement they will persist with the task.

Increased feedback

Having students explaining to each other also allows the teacher to move around and listen to the verbalisation of students' thought processes. This is a most effective way of picking up misconceptions which the teacher can then address, either on the spot, or later with the entire class, if such misunderstandings appear to be common.

Other advantages of co-operative learning

There are other advantages to be found in co-operative group work besides the actual mathematical learning and language development taking place. Group work can also help to:

- **Develop social skills** and effective interactive behaviour in the students, because they are learning to work with other people. These include skills such as attentive listening, explaining ideas to others, encouraging others to participate and group decision making.

- **Foster interaction between class members** by changing groups so that all class members work with people they would not normally mix with.

  Both these points can be especially useful when integrating students with disabilities into the class and/or encouraging groups of boys and girls to work together.

- **Encourage the idea of completing a task**: support and energy is available where one person alone might have given up.
• Develop the confidence of shy students who may be more inclined to speak in a smaller group.
• Make students take more responsibility for their learning and that of other group members and encourage them to trust their own judgement. Thus they become less dependent on the teacher.

References

2.2 Activity 10

Write a reflective journal which discusses how you feel a co-operative learning approach could be applied in the teaching of mathematics in PNG schools.
**Topic 5 – Questioning**

When teaching mathematics it is important to provide students with opportunities to talk together, to discuss their learning, explain their understandings and clarify their ideas. Through this talk, teachers are able to gain valuable insight into students thinking. The type of questions asked will influence the quality of the talking which takes place in the classroom.

The reading below looks at the different types of questions used by teachers in the mathematics classroom and the type of responses they encourage from students.

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### 2.2 Activity 11

Read the article below. Write down a factual, reasoning and an open question you could use in a mathematics class.

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**Questioning in the mathematics classroom**

(adapted from Vacc, N. (1993) Questioning in the mathematics classroom. Arithmetic Teacher (October), 88-91)

**Question types**

If we as teachers are to find out what our students already know so that we can help them use that understanding to construct new knowledge, we need to focus on asking questions that will assist us in achieving that goal. Three categories of teacher questioning that relate directly to classroom instruction have been identified in research conducted by Barnes (1990). These are factual, reasoning and open.
Factual questions

A large majority of the questions asked by teachers seek factual information, that is, a name or specific information. My experience at various times during the past year with a total of approximately 175 mathematics educators (i.e. pre-service and in-service elementary school teachers and teacher educators) illustrates this situation well. Each group was given the task presented in Figure I. With the exception of one teacher who taught in a combined second- and third-grade classroom, all the questions generated by these mathematics educators sought factual information. Students responding to their questions would be required to identify a specific phenomenon but not show insight into its use. Some of their suggested questions are as follows:

- What is the name of figure 0?
- What do we call figures B, D, and S?
- How many figures have an acute angle?
- What do we call angle X in figure F?
- Which figure is congruent with figure G?
- Which figure has five sides?
Teachers asking questions in this category will find out whether their students know specific mathematical facts, but they will gain little, if any, information about whether their students actually understand the given concept. Likewise, students responding to these questions will be challenged only to recall previously learned data. They will not be encouraged to make comparisons between or among acquired facts and new observations, nor will they have much opportunity to question previously learned facts. Also, students responding to factual questions are usually unable to agree or disagree with another student or the teacher as a means of confirming or questioning what they already know; they have no opportunity to reconstruct what they have previously learned. To give students this opportunity, non-fact-seeking question need to be a major part of classroom discourse. These include a variety of reasoning questions, as well as open questions that do not call for reasoning.

Reasoning questions
Reasoning questions require students to construct, or reconstruct from memory, logically organised information. They are divided into four main types: closed reasoning - recalled sequences, closed reasoning - not recalled, open reasoning, and observation (Barnes, 1971). A brief overview and representative questions for each, based on Figure 1, are presented subsequently.

Closed reasoning - recalled sequences
Questions in this category require respondents to develop one acceptable, logically organised response based on previously acquired knowledge. Sample questions of this type include the following:

- Using figure G as a unit of measure, how many mats would you need to cover the floor in a room the size of figure U?
- How many of these figures can be divided into three equal parts?

Closed reasoning - not recalled
Teachers asking questions in this category will expect one correct answer only, but it is not based on previously learned knowledge. The following, using Figure 1, illustrates this type of questioning:

- If you put a fence around a field shaped like figure J, and the distance between each fence post is the same size as the shortest side of figure L, how many fence posts will you need to buy?
- Which five figures, when put together so only one side of one figure is touching one side of another figure, will form a new figure that is the same shape and size as figure U?

Open reasoning
Questions in this category have more than one acceptable answer. Because answers will vary, these questions afford valuable opportunities for students to listen to, and question and learn from, each other. The following are representative of this type of reasoning questions:

- In what ways are figure K and figures S similar?
- Why aren't figures C, J, and K called triangles?
• Which figures can be divided into two equal parts?
• How many different ways can you make figure U using other figures on this page?

Observations
Reasoning questions in this category require students to interpret what they perceive concerning given data. Examples of questions that could be used, based on Figure 1, include the following:

• Which is larger, figure P or Q, and how do you know?
• Of figure L, T, and U, which two are most alike?
• In what ways are figures C and J different?

Open questions not calling for reasoning
The second–third grade teacher cited earlier indicated that she would ask the students the following questions about Figure 1:

• What do you notice about these shapes?

Although this question could be considered factual because it elicits previously learned knowledge, it is an open question, since a wide range of acceptable answers exists. Most importantly, a question of this nature presents an opportunity for students to describe observed phenomena for which they have not yet learnt a name. Additional questions in this category that could be asked, using Figure 1, include:

• What can you tell me about figures B, D, and S?
• What is the difference between figures E and M?
• Which figure is a polygon?
• How is figure E like figure O?

Teachers who ask questions in this category gain specific information about their students’ cognition’s that can be used in introducing new concepts and in planning instruction that specifically meets students’ individual needs.

Summary
Learning is affected by the opportunities students have to relate incoming information to what they already know and then restructure their existing knowledge or construct new ideas were appropriate. As the Professional Standards for Teaching Mathematics (NCTM, 1991) indicates, classroom discourse, or the ways of representing, thinking, talking, agreeing, and disagreeing, is central to helping students develop mathematical understandings and skills (p 34). This development, however cannot be achieved without teachers asking a variety of questions that challenge students; thinking – questions that require much more than factual recall.
References


2.2 Activity 12 – Extension Activity

Read the article ‘Improving the quality of learning by asking ‘good’ questions’. What makes a ‘good’ mathematics question? How will asking ‘good’ questions improve children’s mathematical understandings? Prepare to share your ideas with the class.

Improving the quality of learning by asking 'good' questions


Teachers communicate with students mainly through the questions they ask. A direct way to improve the quality of teaching and learning is to improve the quality of the classroom questions. The goal is to find questions which:

- Can engage the students in creative and active thinking;
- Are suitable for learners at different stages of development;
- Use practical situations to link tasks with the learner's experience;
- Provide opportunities for communicating about their learning; and
- Reveal the way that individual students are thinking when solving problems.

Consider the following example. One particular Grade 6 student, Jane, had just completed a unit on measurement, the final aspect of which was on calculating perimeter and area. The class had been set tasks which included diagrams of rectangles with dimensions given. The students were asked to calculate the perimeter and area. Jane was able to complete these items correctly. However, in a subsequent discussion, when asked the question: ‘I want to make a vegetable garden in the shape of a rectangle. I have 60 metres of fence for my garden. What might be the area of the garden?’ Jane could not answer and in fact claimed that there was insufficient information. The question clearly requires a different level of thinking from standard text exercises on perimeter and area. To find one or more answers which satisfy the conditions a student must think about the constraints which the perimeter of 60 metres places on the lengths of the sides of the rectangle, as well as thinking about the area.
Now this particular question might be tricky in a number of ways. First the language could have created a problem. In this case, the teacher was able to discuss the question with Jane and she appeared to be able to restate the question in her own words. Second, it is framed in an unfamiliar way. This may well have caused some confusion. In particular, students sometimes find it difficult to cope with mathematical questions which have more than one correct answer. This might have been the basis of Jane’s difficulties. After experience with such questions this would not be a serious problem.

Nevertheless, the question did allow the teacher to assess the learning of Jane on this topic in a different way. It seemed from the test that Jane had mastered the topic since she could accurately complete the exercises on perimeter and area. After further probing it was revealed that Jane had little appreciation of perimeter as the distance around, and she had no concept of area as covering. In other words, she had learned to answer routine exercises but had not fully understood the concepts. This question enabled the teacher to become aware of the depth of Jane’s understanding on this topic. Another feature of the question was that it became clear to Jane that her understanding of the concepts was inadequate.

It is possible to create questions like this for most topics and grade levels. We call them ‘good’ questions. ‘Good’ questions are defined as possessing three features:

1. They require more than recall of a fact or reproduction of a skill.
2. Students can learn by answering the question, and the teacher learns about the student from the attempt.
3. There may be several acceptable answers.

Each of these is discussed in the following sections.

**More than recall**

Bloom, Englehart, Furst, Hill, and Krathwohl (1969) claimed that levels of thinking include knowledge, comprehension, application, analysis, synthesis and evaluation. While there has been much debate about the application of these to classroom teaching, they serve as a useful guide in this instance. It is unfortunate that much teaching simply requires students to recall some knowledge or to reproduce a skill. Few of the exercises which appear in the major primary texts require much analysis, synthesis or evaluation. Even exercises which allow the possibility of the levels termed comprehension and application are often completed by recalling a set procedure.

The above question on perimeter and area required comprehension of the task, application of the concepts and appropriate skills, and some analysis and synthesis of the two major components involved. Yet it must be stressed that the question was not a trick, and it required processing of the same information needed for conventional perimeter and area questions.

Another example of this type of question is from the topic of averages. A common question in textbooks is like "Find the average of 7, 8, 5, 16 and 4". This requires mainly recall of a technique. Consider the question is asked as follows:

*After five games, the Vipers have averaged 3 goals per game. What might have been their scores in each of the five games?*
It requires a different level of thinking and a different type of understanding of the topic of averages to be able to give an answer which satisfies both the teacher and the student. Students need both to comprehend and to analyse the task. In this case, they must have a clear indication of the concept of average and must either use the principle that the scores are evenly placed about the average or that the total of the scores is 15 as the basis of their response. It is more than recall.

Students can learn by answering the question

Consider the following question:

Michael and Monica each measured the length of the basketball court. Michael said that it was 20 rulers long, and Monica said that it was 19½ rulers long. How could this happen?

We asked some Grade 5 students to discuss the question in-groups. The students suggested a variety of plausible explanations. Based on their explanations, the students were invited to suggest what they need to think about when measuring length. Their list included:

- The need to agree on levels of accuracy
- The need to agree on where to start and finish; and the importance of starting at the zero
- The need to avoid overlap at the ends of the rulers, or spaces between the ends
- The need to measure in a straight line, and
- The need to measure the shortest (perpendicular) distance

Clearly the students have learnt by doing the task. In this example, the students were able to establish for themselves these essential aspects of measurement from the act of answering the question.

Several acceptable answers

Most questions in mathematics classes have only one correct answer. There is nothing wrong with such questions, but there are also many mathematical situations where a range of possible alternatives exists. Students should experience such questions at times. Consider the following example:

A basketballer scored 11 points in three games. What might be her scores in each of the games?

Clearly there are many possible answers to the question. It entails no more knowledge than an addition task of the type $3 + 4 + 4 = ?$, yet it requires a different approach and a different level of thinking, as well as revealing to the students that multiple correct answers are possible. It also appears that there is a link between such questions and the fostering of higher level thinking. Sweller, Mawer and Ward (1983) showed that the use of open-ended questions develops students' problem solving expertise while still meeting the conventional goals of mathematical skill acquisition. Under Sweller's instructional model the conventional task, "Find
the sum of these two numbers", would be modified by the new requirement "Find out everything you can". For example,

Write down everything you can know and everything you can find out about the number 180.

Note that there are different levels of sophistication at which individual students might respond.

It is characteristic of such 'good' questions that each student can make a valid response which reflects the extent of the student's understanding. A student with a broad understanding of the topic can be expected to make more general statements. Since correct answers can be provided at a number of levels, such tasks are particularly appropriate for mixed ability classes. For example, a student who responds quickly at a superficial level can be asked to search for alternative or more general solutions. Able students will typically recognise the possibility of such alternatives for themselves and devote the same time and effort in the search for a general solution that less able students expend on a less sophisticated solution.

In the case of the earlier question on the area of the garden, there is a range of acceptable answers, even using only whole number lengths (29 x 1, 28 x 2, 27 x 3, ..., 15 x 15). Some students might be asked to find the largest possible garden. It might be suggested to others that they draw a graph. Some students will recognise that an adequate answer might require the identification of a class of suitable rectangles. Students can even be asked to describe all possible rectangles. It is the openness of the task which provides this richness. It is the existence of several acceptable answers, which stimulates the higher level thinking and the problem solving.

In summary, the quality of the learning of students is related to the tasks set and the quality of questions which the teacher asks. Students can learn mathematics better if they work on questions which require more than recall of information, from which they can learn by the act of answering the question, and which allow for a range of possible answers. These are called 'good' questions.

References


2.2 Activity 13

Choose a topic from the Primary Mathematics School Syllabus. Develop a series of good questions you could use to teach the topic.
References – for further reading


Mousley, J. and Sullivan, P (1996). Learning about Teaching, an interactive tutorial program to facilitate the study of teaching. AAMT, Deakin University and The Australian Catholic University
Module 2.3 – Syllabus Studies

Module 2.3 Syllabus Studies is a core module in the ‘Teaching and Learning Mathematics’ unit. During this module you will examine the ‘Mathematics Syllabus for Lower Primary, Grade 3-5’ and the ‘Mathematics Syllabus for Upper Primary Grades 6-8’. You will further develop your skills in programming, planning, teaching and evaluating mathematics activities which are gender inclusive and give consideration to catering for children:

- Bridging to English
- With special needs
- In multigrade classes.

Objectives

By the end of this module you will be able to:

- Interpret the primary school mathematics syllabus
- Use the primary school mathematics syllabus and Teacher’s Resource Books to develop a mathematics program
- Identify a range of strategies which are gender inclusive and suitable for teaching children who are:
  - Bridging to English
  - Have special needs
  - Are in a multigrade class
- Plan, teach and evaluate a series of mathematics activities
- Identify appropriate assessment procedures suitable for use in the primary school

Concepts and skills to be developed

During this module the following concepts and skills will be developed.

- Programming skills
- Teaching skills
- Assessment skills
- Evaluative skills
- Interpretation skills
**Topic 1 – Syllabus documents**

The Department of Education Papua New Guinea has developed curriculum documents to support the teaching of mathematics in all Primary schools in PNG. As future teachers it is important that you are familiar with these documents and able to use them to guide your teaching.

Below is a diagram which identifies the curriculum resources available to teachers and the relationship between these documents.

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**2.3 Activity 1**

Study the diagram below. Locate an example of these resources and consider how the different documents support the teaching of mathematics. Be prepared to discuss your findings with others.
Primary School Mathematics Curriculum and Support Materials

<table>
<thead>
<tr>
<th>Lower Primary Mathematics Syllabus</th>
<th>Upper Primary Mathematics Syllabus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade 3 - 5</td>
<td>Grade 6 - 8</td>
</tr>
</tbody>
</table>

Teachers Resource Book

<table>
<thead>
<tr>
<th>Grade 3</th>
<th>Grade 4</th>
<th>Grade 5</th>
<th>Grade 6</th>
<th>Grade 7</th>
<th>Grade 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>e.g. Grade 4 Teachers' Resource Book</td>
<td></td>
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</tbody>
</table>

Pupil Books

| e.g. Grade 4 Mathematics Pupil Book 4A | e.g. Grade 4 Mathematics Pupil Book 4B |

Note: As a result of the current education reform process the mathematics curriculum materials are currently being reviewed and updated.
**Topic 2 – Planning to teach mathematics**

Below is a suggested model for planning mathematics activities. When using this model you will need to consider how you can teach the ideas outlined in the syllabus documents in context. It also emphasises the need for teachers to consider language issues when planning to teach mathematics lesson.

### 2.3 Activity 2

*Read the article ‘The planning model’. Discuss it with your peers, talking through each of the steps. What you see as its advantages and disadvantages of using this model? Write down any questions you have about the Planning Model.*

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**The planning model**

(adapted from *The Planning Model, a step by step guide to planning for Maths in context in Aboriginal Schools, Northern Territory Department of Education 1993*)

**Step 1: Identify the maths you need to teach**

Using the curriculum documents:

- Identify the level you need to teach (e.g. Grade 5)
- Decide on the strand(s) you want to cover (e.g. number and measurement)

**Step 2: Decide on a context for teaching and learning**

The context is created by the area of interest or the topic around which you will develop teaching/learning activities. By having a context for learning Maths, a meaningful link is established between the activities allowing students to:

- Apply basic maths skills in a real life or life-like situation with a purpose
- Solve problems which are identifiable in the context and for which there is a realistic need to find a solution
- Extend their mathematical understandings by talking about, reading about and writing about what they are doing, why they are doing it and how they are learning
Examples of contexts which you could use to teach mathematics are:

- The garden
- Trade stores
- The environment
- Family
- Media
- Transport

**Step 3: Decide on the objectives you are going to teach**

Choose objectives from the strand you want to cover e.g. Grade 4 Measurement: ‘Establish the meaning of perimeter/girth and measure perimeter’.

Choose objectives for which you can write realistic activities for the chosen context e.g. the above objective would be appropriate for the context of ‘the garden’.

If an objective doesn’t easily fit into the context you have decided on, leave it out. It can be taught as part of another unit or as one of your regular maths lessons e.g. the Grade 4 objective ‘make fraction walls’ would not be appropriate for the context ‘the garden’ and would best be left to a regular maths lesson.

**Step 4: Write the learning activities**

Decide how long you want this topic to last and estimate how many objectives you can teach in this time. It could be work which takes place every day for four weeks or every Thursday and Friday for ten weeks.

As teachers we need to keep asking ourselves as we plan:

- What are we teaching?
- How are we teaching?
- How do we know if the students are learning?

**Step 5: Look at the big picture**

As you are write your objectives and activities, keep a checklist of all the objectives you plan to cover. Many objectives will need to be taught again and again in different contexts providing for lots of practice throughout the year. Therefore a numbering system on the checklist which shows how many times you have taught an objective may be appropriate.
Step 6: Identify the English language being taught

Identify the language functions and outcomes which will occur during the planned activities. As you read the learning activities, think and discuss how you will be using English, and what aspects of English you will be teaching in that activity. Write details of the language teaching under the maths activity. Keep a record.

Remember to ask the following questions:

- How am I teaching language?
- What aspects of language are the students learning?
- What good ESL strategies should I be using? e.g. Do Talk and Record

For example an English objective from the Grade 4 Language syllabus such as ‘Use English to describe simple problems related to their experiences and with support from the teacher"
discuss possible causes and solutions and arrive at a simple decision’ could be covered while teaching mathematics activities.

**Step 7: Identify the vernacular language being taught**

Think about, discuss and decide which activities would be best taught in the Tok Ples or Tok Pisin. You may need to identify the following:

- Activities which need to be taught only in the Tok Ples or Tok Pisin
- Activities which need to be taught first in the Tok Ples or Tok Pisin and then in English
- Activities which need to be taught only in English

Identify objectives for the vernacular language covered according to the activities. Keep a record.

An example of a vernacular objective from the Grade 4 Language syllabus which could be taught through mathematics activities could be ‘write independently and creatively and at an acceptable level of accuracy in a wide range of forms including …expository forms requiring basic selecting, ordering and connecting of information, such as a report on an event or a set of instructions’.

**Step 8: Organise resources needed to teach the unit**

You need to prepare your resources. You need to decide:

- What type of resources you need, e.g. Books, art and craft materials, people
- Where you will find what you need
- What is most suitable for your students
- What you need to make
- When you need it

**Step 9: Assessing students’ learning**

Decide which objectives/learning outcomes you need to assess. Consider how you will assess the students’ understandings of the objectives/learning outcomes. You can keep a record of what students know by using:

- Checklists (based on the objectives)
- Work samples with dates and comments (see examples next page)
- Teacher’s notes
- Photos and comments
- Audio/video tapes

Assessment is continuous and we need to remember that:

- Some objectives/outcomes can be assessed during the topic or at the end of the topic
- You should try to plan assessment tasks which students can demonstrate if they have achieved the objective
- You may need to do other activities to reinforce the same objective/outcome before assessing it
• You need not assess all objectives taught if you know you are going to deal with the same objectives again in other contexts. It may be fairer to assess these objectives the third/fourth time they are taught
• You need to keep all this information in each student’s file

Sample of children’s’ work. By looking at each of these examples the teacher can gain valuable information about the child’s mathematical understandings.

Step 10: Evaluating what has been taught
These questions may help you evaluate a unit of work and to identify what needs to happen in future units.
• Were all the objectives covered that you planned to teach? If not, which ones need to be planned for again in future units?
• Did all the students achieve the objectives? If not, which objectives need to be planned for again in future units?
• Did all students understand and cope with the learning activities? If not, which learning activities would be good to repeat, modify, eliminate?
• Was the unit enjoyable?
Remember:

- Some students learn things quickly while others take longer
- Some things can be learned quickly. Others take more time to learn
- Some things are easy to forget unless they are covered over and over in a variety of ways
- The way students learn and the time it takes them to learn varies as much as the students themselves
**Topic 3 – Gender awareness**

As a teacher it is important that we ensure that all children in our care are provided with the same opportunities and experiences. We need to be aware of our own practices and when we teach we:

- Provide a gender inclusive curriculum which values girls and women as well as boys and men
- Encourage girls to speak out and to participate during class activities and discussions
- Encourages children to listen to one another and to value the contributions of others, both girls and boys
- Use gender inclusive language, content and pictures
- Ensure that our teaching time and classroom interactions are shared equally between boys and girls
- Ensure that creativity, enquiry and risk taking is encouraged just as much in girls as it is in boys
- Provide equal access to resources
- Use a variety of assessment strategies and methods of reporting.

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2.3 Activity 3 – Classroom observations

Next time you are in a school, observe a mathematics lesson. During the lesson observation note down:

- how often girls and boys answer questions (note whether the teacher chooses children to respond or if the children volunteer an answer)
- how the boys and girls interact with one another
- the language the teacher uses and the examples used (e.g. are they gender inclusive?)
- the time the teacher spends working with and interacting with girls and boys.

From your observations what comments can you make about how gender inclusive this lesson was? Write a journal reflecting on your observations and give reasons to explain your opinion.

**NOTE:** You may also like to consider the above questions when participating in your lectures at the college. How gender inclusive are the college’s practices?
**Topic 4 – Bridging from vernacular to English**

Supporting children bridging from the vernacular to English is an issue we need to consider when teaching across all areas of the curriculum. When planning mathematics activities teachers need to think about which language they will use as well as the strategies they will adopt to support children learning the language of mathematics. All mathematics lessons should also be considered language lessons.

### 2.3 Activity 4

Read the article ‘The language of mathematics’ and summarize the important language issues you need to take into consideration when teaching mathematics. How would you use the vernacular to support the teaching of mathematics?

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**The language of maths**

(adapted from: Jenkinson L, (1999) Bridging material, Curriculum Development Division, Department of Education, PNG)

1. **Everyday words with precise maths meanings**

In Maths we use a lot of words and phrases that are used in our everyday language. When we use these same words and phrases to do Maths, they take on a different or more precise meaning. When we do Maths we learn to use the Maths attitude of caring about ‘precision’. When we use Maths language we must use it in a way that reflects that attitude. Students need to learn that when they are doing Maths, the language used has a precise Maths meaning.

For example, if you have a bucket full of fruit, and a family member asks you to give him/her half, you would probably just divide your fruit using approximation – maybe tipping about half of the fruit into another container. But in Maths if we take half of the fruit, we would have to count the total and find half, (or if the fruit were very small, we would weigh or measure the volume to calculate half). So the word ‘half’ can be used generally in everyday life, but has a precise meaning in Maths language.

**Bilingual Considerations:** Sometimes it is difficult to use the vernacular language in a way that makes the precise Maths meaning clear - because that style of precision may not exist or be important in the vernacular culture. When the precise Maths meaning is not held when using the vernacular Maths language, it is better to teach the English for this precise meaning.
2. Maths words

Some words and phrases are Maths language, with only a precise Maths meaning. They are used in everyday life for mathematical purposes. Words or phrases such as ‘subtract’, ‘count how many’, ‘triangle’ and ‘centimetre’ are these kinds of words. Students need to learn and use this Maths vocabulary.

**Bilingual Considerations:** Sometimes specific Maths actions and things are not named in the vernacular. Some vernacular Maths actions and things are not named in English. For example, there might not be a word with the same meaning as ‘triangle’ in a vernacular. At the same time, there might be many shapes important to that vernacular culture, that have no names in English. When a teacher is teaching about shapes, they should include vernacular shapes using vernacular names and English shapes using English names.

If the students are learning English names for Maths concepts, they will learn these as words borrowed into their vernacular at the start of the Grade 3. By the end of Grade 3, they should be trying to talk about the English concepts/words using simple English sentences (e.g. ‘A triangle has 3 corners and 3 edges.’). By the end of Grade 5 they should be able to use English to explain complex ideas (e.g. These two triangles are different, because one has 3 equal angles, while the other has one right angle and two different angles).

3. Written maths language

Maths language is spoken and written language, and students need a lot of experiences that require them to use Maths language for communication. Written maths language uses many symbols (e.g. +, =, %, >, π, √), and representations (e.g. sums, graphs, calendars, geometric and other diagrams, scales).

**Bilingual Considerations:** When students are writing about and recording their Maths experiences in the vernacular language, they should incorporate these symbols and representations into their vernacular texts. When students are writing about and recording their Maths experiences in English, they should be incorporated into the English text.

**Teaching the language of maths**

New Maths concepts and the new language for the concepts need to be learnt together. The concepts of Maths, and the ways to apply them, are learnt through activities in which students investigate and problem solve. Learning processes help teachers’ plan for good Maths learning activities. These learning processes also offer opportunities for good language learning.

When learning a new Maths concept, students need to start at the concrete level. They need to see the concept in the real life context. Students will use their everyday language to understand the context, but will need to be taught the Maths language of the concept, and how to apply it in that context.

*For example, if students are learning modelling/mapping of the school grounds, they will already know how to talk about and write about things in the school ground. They will need to learn the concept of making a model/map of the grounds though, and of representing relative size and relative position- and the language used for talking about this, (e.g. the tree north of the school office, half way between the gate and the first classroom).*
As the students become familiar with the concept, their representations become more and more abstract. So does their language

“From the school office, go N, 10 metres, W, 15m then S, 25 m to find the treasure.”

or “Find what is located at A3 and D 5 on your map.”

Bilingual Considerations: When first introduced to a new concept, the students should learn about it in whatever language is most effective for this purpose. Usually this will be the vernacular language, as the concept will be introduced in the context of how it is used in everyday life. Some English words might be borrowed into the vernacular to name the concept.

Early in Grade 3, the students may just learn key words in English, and very little English text. As they go from concrete to abstract, they will use the Maths symbols and representations in their vernacular texts. They will use vernacular talk to help them reach understandings and solve problems.

By the end of Grade 3, they should have started to use English for naming things, describing the things they discover through problem solving. This text would be a recount of what they did to solve the problem, and a description of the problem and/or solution.

For example: My group used 100 sticks. We took away 56 and we counted all the sticks left. There are 44 sticks left

or

The man had $100, and he spent $56. Then he had $44 left

By the end of grade 4, students should still be using vernacular for new concepts, but should be using a lot more English in explaining the processes, giving their ideas and opinions during problem solving, comparing approaches to problem solving, etc. They would also be further developing their recounts of their work and descriptions of problems and solutions.

For example: Our problem was; there are 84 lollies, and they are shared among 7 people, how many would each person get. We got 84 sticks, and put them out in 7 groups - one group for each person. We put them out one at a time. When the sticks were finished, we counted how many in each group. There were 12.

or

We measured the length of the school room. I measured it as 25 steps. Dulcie measured it as 29 steps. I think they are different because our steps are different sizes. I had the biggest steps.

By the end of Grade 5 they should be introduced to some new concepts in English first, with vernacular explanations afterwards. The students should develop their use of the text types learnt in Grade 4, but should also learn to argue their approaches and points of view about their problem solving.
For example: The best way to find the answer is to measure with the tape measure. It’s more accurate. If you measure with a short ruler, you might make a mistake, and get the wrong measurement. The tape measure is better.

When planning your Maths program, always think about how students are learning not just the Maths, but also the language of Maths.

2.3 Activity 5

Read through the information describing the teaching strategy ‘Do, Talk, Record’. Consider how this approach could be applied to teaching a mathematics lesson. Develop a lesson plan for a Grade 3 mathematics class which uses this strategy.

A Teaching Strategy

When teaching new language the following principles are important to keep in mind:

1. Active learning: Teach the skills of speaking, listening, writing and reading English in an integrated, active and meaningful way
2. Meaningful context: Select contexts which are relevant to the children
3. Multi-sense learning: Using more than one of the five senses reinforces and aids learning e.g. listening, feeling, touching, speaking, doing
4. Feedback and correction: Pupils need to know when they make errors so they can correct them. But you must give corrections wisely. There must be an encouraging and supportive learning environment where pupils feel free to participate in all activities
5. Overloading: Don’t try to teach too much at once.
6. Positive reinforcement: Praising pupils when they are successful will motivate them for additional learning.

The following teaching strategy, ‘Do, Talk, Record’, is an approach used by many teachers when introducing new ideas and concepts to students. It incorporates the principles of language learning outlined above and provides a model for teaching new language.
Do Talk Record

1. Talk – before the activity
   - Introduce the topic to the students
   - Discuss what the students already know about the topic
   - Outline/discuss what the students will find/do/learn
   - Discuss any new words e.g. Teach English words using the vernacular
   - Discuss what the pupils want to learn about the topic

2. Do – the activity
   - Guide the students in the activity (whole group, small groups, pairs, individual)
   - Indoor or outdoor activity
   - Use concrete materials

3. Talk – after the activity
   - Discuss the findings
   - Ask open-ended questions e.g. what, when, where, why, who, and how
   - Discuss how to record and display the findings
   - Discuss follow up activities

4. Record – a way to recall
   - By drawing as individuals, pairs or small groups
   - By writing different types of texts
   - By making a model
   - Create a chart
   - Construct a graph
**Topic 5 – Catering for children with special needs**

As teachers we need to be able to cater for all children within our class. Some of these children will have special needs e.g. children who are visually impaired, hearing impaired, have learning difficulties, intellectual disabilities or physical disabilities. It is important that all children are given an opportunity to participate and contribute to society to the best of their ability and that each individual’s contribution is valued. As educators we need to remember that all children can learn regardless of their disability.

Once a student has been identified as having special needs, it is important not only to identify appropriate curriculum materials to support the child’s learning, but also to decide on useful teaching approaches. A teacher needs to plan how they are best able to support the child, given their particular disability, and prepare their teaching accordingly.

For example, a student with hearing loss may appear not to understand mathematical concepts when in fact they are not understanding the language that is being used. When ways are found to reach these students, often through visual or tactile approaches and modelling, they often show a good understanding and good ability to apply their understanding.

For a student with a learning disability, who is having difficulty with understanding a particular concept, it may be best to pair the student with a peer to construct a model that shows an application of the concept. If, for example you are developing the concept of ‘perimeter’ have the pair construct a model of a garden and then calculate how long a fence you would need to be to go around the garden.

One of the main goals when working with students with physical disabilities is to develop self-sufficiency. Whenever possible, students with physical disabilities should be encouraged to do their own work, even if it takes much longer, so that self esteem and independence can be fostered. In order to sustain independence it may be necessary to allow these students to complete a shorter assignment or to be given the assignment in advance so they can plan and use their time in the best way for them.

If you treat students in the way you would want to be treated if you had the same disability, you will be off to a good start.

Some suggestions for supporting children with special needs are outlined in the following sample mathematics lesson.

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**2.3 Activity 6**

*Review the lesson outlined below. Identify the strategies used to support children with special needs. Discuss these with your peer.*
Measurement lesson  Grade 5

(adapted from Winn, S. PASTEP Special Education 1999)

The scene: A class of 45 children average of 11 years of age. Within the class there are children with special needs

- One child has impaired vision
- One child has a hearing loss
- One child has an intellectual disability of a mild nature

Objectives: The mathematics lesson is a measurement lesson with the following objectives

- Review prior knowledge of shapes
- Recognition of shapes
- Finding the perimeters of squares and rectangles

Lesson activity: Children will complete an exercise from a textbook

Child with impaired vision:

Requirements: Ruler with tactile markers e.g. notches marked on the ruler to indicate each centimetre

A variety of shapes

Establish a ‘buddy’ system where another student is prepared to help with note taking.
Provide concrete objects, e.g. squares and rectangles. Ensure the shapes are of different sizes. Provide the child with a ruler with raised increments (e.g. notches highlighting each cm). Review with the child that they know what the increments mean. Child to work with ‘buddy’ to complete the task, with the ‘buddy’ recording the answers.

*Child with hearing impairment*

Requirements: Establish a ‘buddy’ system

- A chart showing basic signs
- A variety of shapes

Provide concrete objects, e.g. squares and rectangles. Ensure the shapes are of different sizes. The student may be able to read the text (provide clearly written questions about the task) or it may be necessary to use a signing system to communicate. Child to work with ‘buddy’ to complete the task. The student’s speech may be slower than other students’ or not as clear, but it is important that you allow them to finish their sentences. The student can provide written answers or a signed answer.

*Child with an intellectual disability*

Requirements: Establish a ‘buddy system’

- A variety of shapes

Provide concrete objects, e.g. squares and rectangles. Ensure the shapes are of different sizes. The student needs to know what the task is. Break the task down into simple steps. When providing a worksheet make sure it is simple and ensure it is clear. Try to keep the amount of text to a minimum. Ensure that responses can be performed by either ticking a box or circling an object. Revise the use of a ruler with the student by asking a few simple questions e.g. how many cm is this? Which is more, 5 cm or 8 cm? Allow students to complete the task and to communicate in a way most suitable to them e.g. signing, speaking (remember they may not speak as quickly or as clearly as other students). Use the buddy system to support the student to make notes and scribe if necessary.
**Topic 6 – Multigrade teaching**

As a result of recent educational reform it is expected that some schools will be organised in a way that requires multigrade teaching. Multigrade classes are where a class may be made up of children from two or more different age groups, for example a multigrade 3 – 4 class, in which the Grade 3 students will be expected to do different work from the Grade 4 students.

It is important to consider the learning needs of all children in a multigrade class and organise your teaching in such a way that all children have an opportunity to learn at their own level. In a multigrade class the students need to be organised into groups and it important that the teacher shares their time equally among each groups.

Teaching in a multigrade class requires the teacher to be well organised. Unless the teacher is very clear about the learning tasks for each group and has strong management skills, the lesson can be very difficult and very little learning will take place. When planning your teaching to cater for children in a multigrade class there are a number of approaches that a teacher can use.

**Small group organisation**

Small group work provides a way of catering for the varying abilities of students. This involves small groups of children sitting together and being led through their instruction by their teacher. This approach is one of the most effective learning models available to teachers.

In a multigrade class the teacher would need to divide the children into different groups (based on their grade or ability level) and then develop a timetable to teach groups on a rotational basis. The teacher would plan so that each group has at least one teaching lesson each week where new work is taught and the other lessons would involve assignments where students work independently of the teacher either in small groups or individually.

For example a teachers mathematics timetable for a multigrade 3 – 4 may look like this:

<table>
<thead>
<tr>
<th></th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics</td>
<td>Gr 3</td>
<td>Gr 4</td>
<td>Gr 3</td>
<td>Gr 4</td>
<td>Gr 3</td>
</tr>
<tr>
<td></td>
<td>T</td>
<td>I</td>
<td>I</td>
<td>T</td>
<td>I</td>
</tr>
</tbody>
</table>

This means that on Monday Grade 3 would be taught directly by the teacher and that Grade 4 would have mainly assignment work based on previous learning. On Tuesday Grade 4 would be taught directly by the teacher and the Grade 3 students would have mainly assignment work based on the previous day’s lesson.

**Learning centres**

Learning centres are usually an arrangement of desks, tables, wall space and learning materials with a focus on a particular topic or subject area. They are a carefully constructed learning component of the classroom in which the materials and resources are arranged to allow for students to learn knowledge, understandings and skills in an independent mode of learning.
Learning centres can cater for special needs of students and allow for students to learn at their own rate.

In the case of a multigrade classroom, learning centres can be established to support students with different learning requirements. Each learning centre could evolve around self-instructional workcards. The function of self-instructional workcards is to teach students new knowledge and skills without the direct presence of the teacher. The teaching is built into the workcards so it is important to construct these workcards carefully so that the self-instructional function is included. The following features should be included in self-instructional workcards:

- Each workcard should be attractive to students in terms of layout, brightness, and in its general presentation. This should encourage student motivation.
- Each workcard should be graded in a justifiable sequential manner. The progression through the cards ought to be developmental or logical, moving from the known to the unknown.
- Each workcard should contain one or more teaching components in which the new learning is introduced or explained, then followed by practice examples and opportunities for the students to complete.
- Every workcard, to be independent, should contain a self-marking component so that the students can contain immediate feedback. This can be included on the reverse side of each workcard or on a separate answer card.

**Co-operative group work**

Co-operative group work involves students working together, interacting in a task related way with each other and some students supporting those who need or ask for help. Tasks and responsibilities are shared between group members as students work and learn together, providing one another with feedback and support.

This approach allows for students in a multigrade classroom to work together on activities and provides an opportunity for all students to work at their own level. Through co-operative learning students can develop their mathematical language as well as increase their level of mathematics.

**Peeling off**

This strategy begins with direct teaching of the whole class, with all students participating in the same activity. Prior to the lesson the teacher plans a number of different activities which will follow on from this initial teaching that cater for the range of ability levels of the students.

Students begin by working together and at different points of the lesson, a group of students ‘peel off’ to complete one of the activities planned. The remained of the children continue working with the teacher receiving further instructions. The next group then ‘peels off’ to complete an activity set by the teacher. The remaining children continue with working with the teacher to further developing their understandings and then ‘peels off’. This process continues to all groups are working independently. The teacher starts with a common input and provides layered outcomes. An example can be found below.
TOPIC – AREA

CONTEXT: GARDENING

COMMOM INPUT

- Visit to Garden
- Discuss what area is.
- Demonstrate using common units to find area.

TEACHER

- Visit to Garden
- Discuss what area is.
- Demonstrate using common units to find area.

PEEL OFF

Find area of plots

LAYERED OUTCOMES

Find area of plots

TEACHER

- Discuss need for standard unit – square metre, demonstrate use.

PEEL OFF

Find area of plots using square metres.

TEACHER

- Discuss need for accurate measure. Demonstrate using square metres / part squares.

PEEL OFF

Find area of plots using square metres and part square metres.

TEACHER

- Teacher supports children to complete activities.
Class discussion – sharing of findings.

**Ability groups**

Similar strategies to those suggested for teaching in a multigrade class can also be used to organise one grade where you have a range of ability groups. For example if you have a grade 6 class, for mathematics you might have three ability groups (the above-average, the average and the below-average), and you could establish the groups accordingly. When planning your teaching you will need to think about how you can organise your teaching to cater for each of these three groups using the strategies suggested above.

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**2.3 Activity 7**

Prepare to discuss

- What do you see as the advantages and disadvantages of teaching in a multigrade class?
- What strategies would you adopt to teach mathematics in a multigrade class?
Topic 7 – Assessment issues

Traditionally children’s mathematical understandings have been assessed through paper and pencil tests. The following article suggests that there are alternative ways to finding out what children know and understand, and that this information can be collected during out day to day teaching.

2.3 Activity 8

Read the following article and highlight the ideas which you feel are important. Discuss ways in which you can assess children’s learning during your teaching. How would you record this information? Provide an example.

Assessment

(from Grade 4 Mathematics Teacher’s Resource Book, Department of Education Papua New Guinea)

Assessment is the process of finding out what children know and what they have learnt. It is important that you know what mathematical concepts the children understand, how they learn best, and what concepts they are ready to learn. It is not necessary (or appropriate) to use only tests to find out what the children know. Good assessment information can be gathered during everyday teaching by listening to what the children say, by watching what the children do, and by looking closely at what the children make, write and draw.

When and how to evaluate your pupils

You should continually build up your knowledge of what the children know. This starts on the first day at school and continues through the year. The ongoing assessment of learning is a natural process which all teachers can do. Some important ways for you to learn about what the children can do are:

• By listening to the children’s answers when they respond to questions
• By watching carefully what the children do during classroom tasks
• By listening to what the children say when they are talking to each other and to you

It is important that you do not expect children to have mastered all the concepts at this stage. If you provide them with the necessary experiences they will begin to use the concepts more often.
There are some teaching strategies which will help you to get meaningful information about the children. One important way is to give open, rather than closed tasks. For example:

\[ \_ + \_ = 43 \]

is an open task. There are many possible correct answers. The children can find some different number combinations which can be added to make 43. In so doing, they may become aware of the pattern involved when writing answers, e.g. $1 + 42$, $2 + 41$, $3 + 40$, etc. You can learn about their understanding from their response.

On the other hand: \[ 25 + 18 = \_ \] is a closed task. The children only need to perform the addition process to find the answer. Such questions are useful at times, but teachers do not learn much about the children from such questions.

It is also important that you give children the opportunity to talk about mathematics to each other and to the class. This is a good way to learn, since language is one of the first steps in forming mathematical concepts. Listening to children also tells you much about what the children know. Another way to find out more about your pupils is to look closely at their drawings and written work.

Try to collect some information during each mathematics lesson. It is better to collect a small amount of information each time rather than trying to collect a lot all at once. One way to do this is to select three or four children in a particular lesson and observe them. Listen to their answers and look carefully at their work.

**How to record assessment information**

Teachers who watch and listen to children soon learn a lot about what the children know. It helps if there is a systematic way for recording this information about the children. You should choose at least one method and stick to it. You may decide to use a combination of two methods. The most important part is to keep your records up to date. There should be some change almost every week.

Some teachers use a collection of each child's work as a record of that child's performance. You keep a folder of work for each child. Every time a child produces a new piece of work they discuss with you whether or not it should go into their file. Only pieces of work which show new understanding or improvement on old skills belong in the file. If it is an improvement on older work, then the older work is removed from the file to stop it getting too full. This only works well if:

- The children are actively involved in choosing what is in the file
- The files are kept up to date
- Enough suitable student work is produced (if you are short of paper, much of the children's work will be on blackboards which cannot be put on file)

Make a checklist of the main objectives from the scope and sequence. You should make entries in the checklists progressively. As each bit of information is found out, a note should be made on the sheet. It is best that a consistent method is used for recording the information.
A system such as the following is helpful. Use a ‘/’ if a child shows you that he or she understands the concept. Use a ‘X’ if the child can use the concept to do something else or solve a problem. If you use this method you must keep it up to date.

**Grade 5 Mathematics Assessment Checklist**

<table>
<thead>
<tr>
<th>Strand and Numeration</th>
<th>Topic</th>
<th>Objective</th>
<th>Student Names</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number and Numeration</td>
<td>Number</td>
<td>Use the terms odd and even numbers</td>
<td>AM</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Recognise the first ten square numbers</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Recognise triangle numbers</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Use triangle and square numbers in given calculations</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Be familiar with and identify prime and composite numbers and factors</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Recognise 1,2,3,5,7,11 as prime numbers</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Determine the prime factors of composites with primes 1-11 as factors</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Explore number patterns and determine rules</td>
<td>/</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Round numbers to the nearest tens, hundreds and thousands</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Represent ten thousand</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Represent hundred thousands</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Write numbers including hundreds of thousands in words and figures</td>
<td>/</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Visually recognise numbers in groups</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Estimate the number of a group of objects</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Represent millions in concrete materials</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Write numbers including million in words and figures</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Solve problems relating to numbers</td>
<td></td>
</tr>
</tbody>
</table>

**Assessment checklist**

Another strategy is for teachers to have a blank class list on which they can record information. It is best if this class list is used only for things which the teachers see which they do not expect. For example, a child who usually does not answer questions might give a good response. This would be worth recording on the class list.

The knowledge about the children in the teacher's mind is then combined with the information on the class list to produce a better picture of what the children have learned. It is very important to be consistent and record all unusual events. An example of an annotated class list is as follows:

<table>
<thead>
<tr>
<th>Name</th>
<th>Comment (unexpected events)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abat</td>
<td>Worked well in his group. Gave a good report.</td>
</tr>
<tr>
<td>Julie</td>
<td></td>
</tr>
<tr>
<td>Ben</td>
<td>Did not know his 3x table.</td>
</tr>
<tr>
<td>Carrie</td>
<td>Understands fractions well.</td>
</tr>
</tbody>
</table>
Using assessments

Teachers assess children so that they can help the children to learn better, especially in the early grades. All assessment information should be used for teaching purposes. In other words, if a teacher finds through assessment that a child or a group of children cannot do something, then they should plan to re-teach this particular idea.

If a child does not understand a concept then it is necessary to try a different explanation. It is not worth collecting assessment information if it is not used. It is only when a child understands a particular concept that he or she can be asked more difficult questions.

References — for further reading

Booker, George; Bond, Denise; Briggs, Jack; and Davey, Geoff (1997). *Teaching primary mathematics*, Longman

Department of Education Papua New Guinea (1998). *Lower Primary Mathematics Syllabus Grade 3-5*,

Department of Education Papua New Guinea (draft 1999). *Upper Primary Mathematics Syllabus Grade 6-8*


Unit Glossary

**concrete materials**
These are materials which students can physically hold onto and move round. Examples of concrete materials include such things as blocks, sticks, shells, money and counters. Concrete materials are very useful when introducing new concepts and assist students in solving a range of problems. Concrete materials can be used with students of all ages e.g. from Elementary to Tertiary. Concrete materials are also referred to as manipulative materials.

**co-operative learning**
Co-operative learning is a strategy which involves a group of students working together to reach a common goal.

**Ethno-Mathematics**
Ethnomathematics refers to the mathematical ideas belonging to a particular ‘culture’ or ‘race’. ‘Ethno’ means ‘race’ or ‘nation’. PNG mathematical counting systems are an example of ethnomathematics.

**Formalism**
Formalism is a philosophical approach to teaching mathematics. People who have a formalist approach to teaching mathematics believe that mathematics is a set of characteristics (e.g. 1, 2, 3, 4, +, =) which are joined together in an organised way to form formulas. These formulas have certain rules and to learn mathematics you need to learn all these rules and formulas.

**Intuitionism**
Intuitionism is a philosophical approach to teaching mathematics. People who are intuitionist believe that mathematics is made up of a series of symbols which have meaning and represent ideas (e.g. 1 which represents one object, + which means add a number of objects together, = which is the symbol which means equals). To learn mathematics intuitionist believe you need to develop your own understanding of what the mathematics symbols mean through constructing your own knowledge. This means using everyday language to talk about mathematical ideas, using concrete materials to develop mathematical concepts and then learning about how to use the symbols of mathematics to express these ideas.

**Logicism**
Logicism is a philosophical approach to teaching mathematics. People who are logicsists believe that learning mathematics involves logic and reasoning. To learn mathematics logicist believe you need to learn the logic and reasoning behind the formulas.
real life experiences
This refers to when you have first hand experience of something. For example rather than hearing or reading about what life it like in a particular place, you actually go there yourself and experience the place.

realistic experiences
This refers to a situation which you set up to imitate as much as possible a real situation. For example in a classroom you may set up a class shop which is similar to a trade store.

teaching in context
When you relate your teaching to real life situations and make connections between what you are teaching in the classroom and what happens in the real world outside the classroom, this is referred to as teaching in context.

techno-mathematics
This refers to the mathematical understandings which are shared by people around the world, and are reflected in technological developments. This is the type of mathematics which we teach through the syllabus documents in our schools and represents some ways in which technological cultures view the world.