Mathematics

Teacher Guide

Grade 7

Standards Based

'FREE ISSUE NOT FOR SALE'

Papua New Guinea
Department of Education

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Contents

Secretary's Message

This Mathematics Teacher Guide for Grade 7 was developed as a support document for the implementation of Mathematics Syllabus for Grades 6, 7 and 8. It contains sample guided lessons and assessment tasks and rubrics with suggested teaching and learning strategies that teachers can use to work towards the achievement of content standards and benchmarks in the syllabus.

The importance of mathematics curriculum is to ensure that all students will achieve mathematical skills and competencies of the 21st century that will serve them well in their lives and help them to compete locally and globally. The curriculum will engage learners, who are mathematically literate and can think differently and creatively. Therefore mathematics curriculum is vital to support every learner to reach their full potential.

The Teacher Guide reflects the essential knowledge; skills and values that students are expected to know and be able to do at the end of Grade 7. It is designed to promote a firm understanding of practical everyday mathematical concepts, thus raising the standards in mathematics. It also provides an excellent vehicle to train the mind, and to develop its capacity to think logically, abstractly, critically and creatively.

This Teacher Guide is to be used with the Grades 6, 7 and 8 syllabus for the teaching of Mathematics in Grade 7. It is a guide for teachers to deliver the Mathematics content outlined in the syllabus. There are lessons for each day and teachers are expected to follow these throughout the year to ensure students meet the required standards.

I encourage teachers to read each section of the guide carefully and become familiar with the content of the subject specified in the teaching and learning and other sections of the guide. I also encourage teachers to try out your own ideas and strategies that you believe will effectively work in your schools for your students.

Teachers are encouraged to make reference to the National Mathematics Text books to effectively plan and teach their lessons.

I commend and approve this Grade 7 Mathematics Teacher Guide to be used in all schools throughout Papua New Guinea.

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DR. UKE W. KOMBRA, PhD
Secretary for Education

Introduction

Purpose

This Teacher Guide must be used in conjunction with the Grades 6, 7 & 8 Syllabus. The main purpose is to implement the syllabus in the classroom.

The Teacher Guide provides you with guidelines and directions to help you plan and develop teaching and learning activities for the achievement of Content Standards and Benchmarks. It provides you with information and processes to:

- understand and expand on the relevant Knowledge, Skills, Attitudes and Values (KSAVs) provided in this guide
- develop teaching programs based on your school contexts
- plan and develop daily lesson activities
- plan and conduct assessments to monitor students' achievements.

Teachers are required to read carefully and use the guidelines in the Teacher Guide to plan and develop teaching and learning programs. The guide contains the following main components:

- yearly and termly overview which consists of all strands, units, topics and lesson titles
- sample weekly program or timetable
- suggested daily plans which consists of guided lessons and KSAVs
- assessment tasks and rubrics
- support resources for use when planning and programming.

How to use this Teacher's Guide

You are encouraged to use this Teacher Guide to help you design your teaching programs, lesson and assessment plans. Therefore, you need to:

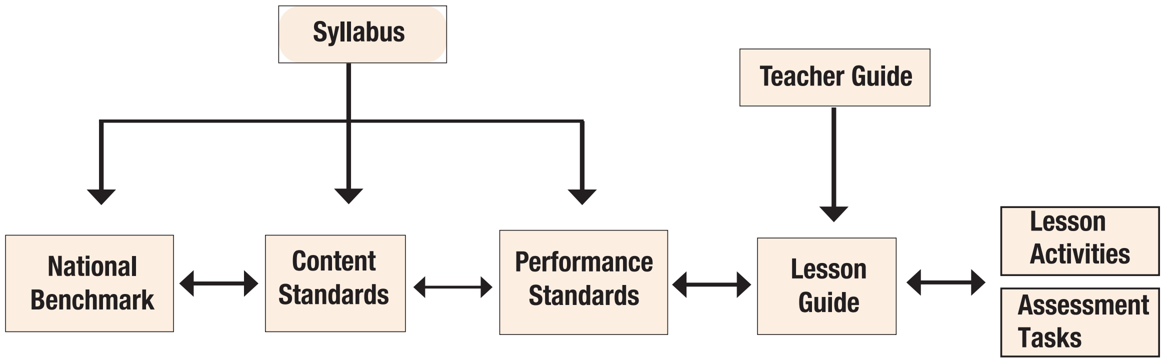
- read this teacher guide and the syllabus carefully
- to understand the content and what you will require for your classroom teaching
- become familiar with the syllabus strands, units, topics and lesson topics
- read and understand the content standards and benchmarks
- read and understand how the assessment plans and tasks are structured so that you can design appropriate assessment plans
- read and understand the structure and content of sample guided lessons and background information to support you in modification of your lesson.

Link to the syllabus

The teacher guide illustrates key parts of the mathematics syllabus. It provides practical ideas about how to use the syllabus and why the teacher guide and syllabus should be used together.

The Teacher Guide explains ways you can plan and develop teaching, learning and assessment programs. It includes recommended knowledge, processes, skills and attitudes for each of the content standards in the syllabus and examples of assessment tasks and how to record and report students' achievements.

You are encouraged to select and adapt the strategies and processes illustrated in the guide to meet the needs of your students and their communities.



Time Allocation

Mathematics week for Grade 7 is to be timetabled for 240 minutes per. Teachers can use the time allocation to do their timetable or programme according to their school program. Topics and activities may vary in length however; you can plan for double periods of more than 40 minutes (Refer to Appendix for Sample Timetable)

Key Features

The key features found in this Mathematics teacher guide are unique and important in planning, development and implementation of this subject.

Mathematical Process Skills

The five Mathematical process skills that can help the students improve their mathematical thinking.

1. Mathematical Problem Solving

- Understand the meaning of the problem and look for entry points to its solution
- Analyse information (givens, constrains, relationships, goals)
- Make conjectures and plan a solution pathway
- Monitor and evaluate the progress and change course as necessary
- Check answers to problems and ask, “Does this make sense?

2. Mathematical Communication

- Use definitions and previously established causes/effects (results) in constructing arguments
- Make conjectures and use counter examples to build a logical progression of statements to explore and support their ideas
- Communicate and defend mathematical reasoning using objects, drawings, diagrams, actions
- Listen to or read the arguments of others
- Decide if the arguments of others make sense and ask probing questions to clarify or improve the arguments

3. Mathematical Reasoning

- Make sense of quantities and relationships in problem situations
- Represent abstract situations symbolically and understand the meaning of quantities
- Create a coherent representation of the problem at hand
- Consider the units involved
- Flexibly use properties of operations

4. Mathematical Connections

- Look for patterns or structure, recognizing that quantities can be represented in different ways
- Recognize the significance in concepts and models and use the patterns or structure for solving related problems

- View complicated quantities both as single objects or compositions of several objects and use operations to make sense of problems
- Notice repeated calculations and look for general methods and short cuts
- Continually evaluate the reasonableness of intermediate results (comparing estimates) while attending to details and make generalizations based on finding

5. Mathematical Representation

- Apply prior knowledge to solve real world problems
- Identify important quantities and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas
- Make assumptions and approximations to make a problem simpler
- Check to see if an answer makes sense within the context of a situation and change a model when necessary.

Mathematical Activities

Mathematical activities are various activities related to mathematics where students are actively engage in by and for themselves to discover the properties of Number and geometrical figures based on what they have learned and apply them in their life and further studies.

Mathematical activities are usually done through problem solving with rich mathematical thinking which includes various questioning on problem situations such as for finding methods and better ideas in solutions. it also includes explanations for sharing ideas with various representations such as changing / translating representations to find the beautiful and reasonable pattern.

Mathematical activities are easily done if students acquire the fluency for operations and reasoning. They are necessary for developing mathematical thinking and proficiency, providing opportunities for students to feel the joy of thinking and learning, utilizing and appreciating mathematics in their lives.

Through the reflection of mathematical activities student are possible to appreciate the value of mathematics such as simple, easier, reasonable, general, and beautiful and harmony.

You can incorporate these activities into your lessons to have the mathematics lessons become;

- More students centred activities and more proactive with rich content.
- More fun to students.
- Easier to understand by students.
- More compelling and elaborative.
- More innovative with various discussions
- Creative and exploratory.
- Connected to daily life and natural phenomena.
- Easier to think about activities that relate to other subjects and Integrated Study.

Grade 7 Mathematical Activities

Activities / Experience	In learning each content of "Number, Operation and Computation , Geometry, Measurement and Transformation, Patterns and Algebra and Statistics and Probability", and the connections of these contents students should be provided opportunities doing mathematical activities for;
Performance Activities	(a) Finding out the properties of numbers and geometrical figures based on previously learned mathematics. (b) Making use of mathematics in daily life. (c) Explaining and communicating each other in one's own way by using mathematical representations.

Teaching and Learning Strategies

Teaching Strategies

Teaching strategies guide the teacher to teach the lesson content with appropriate learning strategies. Effective learning and acquisition of knowledge, skills, attitudes and values by students in a lesson is achieved through demonstrating appropriate teaching strategies.

It is therefore required that teachers identify and apply the best teaching strategies to deliver the content in the classrooms

KWL Chart

To help students to build on what they already know, ask them to use a KWL (Know, Want, Learned) strategy when they work on a topic or theme. An example is given below for the theme Traditions, Customs and Festivals.

K (what I already know)	W (what I want to know)	L (what I have learned)
What I know about Relative Amount	What I want to know about Base Amount	What I have learned about Comparative

Apply the following steps when using the KWL strategy:

1. Organize the students into small groups.
2. Tell the students the lesson topic.
3. In small groups ask the students to list what they already know about the topic.
4. Get the groups to share their ideas with the class as a whole.
5. Ask the students to list what they want to know about the topic.
6. Students complete the first two columns before they start the topic and the third column is completed at the closure of the unit, topic or lesson.

Way of teaching and delivery of Mathematics lessons

The new curriculum promotes the competencies in Interest/Motivation/Attitude, Knowledge and Understanding, Skills and Mathematical Thinking that is specified clearly each guided lessons.

The method of delivery of Mathematics is student oriented through the process of problem-based approach. In this SBC teaching method of mathematics the students are given the opportunity to think and derive ideas by themselves about the task (problem) and be able to improve their way of thinking mathematically.

Delivery Methods of Mathematics Lessons

1. Situation oriented lesson conceptualizing is situation oriented or problem based

2. Encourage to deriving operations/ideas opportunity is given for students to derive ideas or operations by themselves

3. Allow for Thinking of ways to calculate
   Students to think of ways to calculate by themselves

4. Conclusion using students' ideas
   Lesson conclusion should come from student's viewpoints, and use these ideas for understanding new concepts

5. Practice for consolidation
   Practice similar exercises for affirmation of their understanding of the concept learned in the lesson.

Learning Strategies

The students should develop the ability to recognize and categorize situations critically, provide rationale reasoning, constructively solve problems, apply knowledge intelligently, and communicate effectively. Special consideration and more emphasizes must be given to identifying suitable learning strategies which encourage high student participatory learning.

Students' way of learning Mathematics

Steps of conceptual understanding through problem solving approach.

i. Emphasis on understanding meanings
   Students should look at task (problem) given, understand the task, and try to express what the task is.

ii. Thinking how to calculate
   Students think how to solve the tasks, and share ideas of how to solve the problem, and obtain the answer of the problem.

iii. Emphasis on expression and explanation (Reasoning)
   Students conclude and understand new concept from the way they solved the problem.

Planning and Programming

Yearly and termly overview

Teachers are encouraged to use this yearly and termly overview and yearly lesson overview to develop the weekly plans.

Term	Strand	Units	Topics
1	**Number, Operation and Computation**	Positive and negative Numbers	Positive and negative numbers
			Calculation with positive and negative numbers
			Unit check point
	Patterns and Algebra	Algebraic expression	Algebraic expression using letters
2			Algebraic expression using letters
			Calculating algebraic expressions
			Unit check point
3		Linear Equations with one unknown	Linear equations with one unknown
			Using equations
			Unit check point
		Proportional Function	Proportion
			Inverse proportion
			Applying proportion and inverse proportions
			Unit check point
4	**Geometrical, Measurement and Transformation**	Plane Figure	Rectilinear figures and transformation
			Figure transformation
			Circles and sectors
			Unit check point
		Space Figures	Solids and space figures
			Surface area and volume of solids
			Unit check point
	Statistics and Probability	Distribution of data and representative values	Using data trends
			Unit check point

Yearly Lesson Overview

Yearly and termly overview

Teachers are encouraged to use this yearly and termly overview and yearly lesson overview to develop the weekly plans.

Strand	Units	Topics	Lesson No.	Lesson titles
Number , operation and Computation	Positive and negative Numbers	Positive and Negative numbers	1	What kinds of numbers are there
			2	The number line
			3	Expressing amount with positive and negative numbers
			4	Absolute value
			5	Size of number
		Calculation with positive and negative numbers	6	Using a number line
			7	Addition with Positive and negative numbers (1)
			8	Addition with Positive and negative numbers (2)
			9	Addition with Positive and negative numbers (3)
			10	Laws of Addition
			11	Subtraction
			12	Calculation with both addition and subtraction
			13	Multiplying by a positive number
			14	Multiplying by a negative number
			15:	Dividing with positive and negative numbers
			16	Multiplying and dividing by both positive and negative numbers
			17	Multiplying and dividing with fractions.
			18	Laws of Multiplication
			19	Calculation with both multiplication and division
			20	Multiplying a number by it self
			21	Calculation with four arithmetic calculations
			22	Distributive laws
			23	Expanding numerical range and the four Arithmetic calculations
		Unit Check point		Review on positive and negative Numbers

Strand	Units	Topics	Lesson No.	Lesson titles
Patterns and Algebra	Algebraic expression	Algebraic expression using letters	24	Using letters to express quantity
			25	How to write algebraic Expressions
			26	Algebraic Expressions and quantity
			27	The Value of Expressions (1)
			28	The Value of Expressions (2)
		Calculating Algebraic Expressions	29	Terms and coefficients in algebraic expressions
			30	Simplifying Expression
			31	Adding Expressions and Subtracting Expressions
			32	Multiplying and dividing expressions with one term
			33	Multiplying and dividing expressions with two or more terms
			34	Expressions showing equal relationships
			35	Expressions showing relative sizes
		Unit Check point		Review on Algebraic Expressions
	Linear Equations with one unknown	Equations	36	Equations and their solutions.
			37	Properties of equalities
			38	Solving equations of addition and subtractions
			39	Solving equations of multiplication and division
			40	How to solve equations
			41	Various types of equations
			42	Properties of proportional expressions
		Using equations	43	Using equations to solve various problems
		Unit checkpoint		Review on linear equations with one Unknown
	Proportional Function	Functions	44	Understanding functions
			45	Quantities which change together
		Proportion	46	Proportional expressions
			47	Coordinates
			48	Graphing proportion
			49	Graphing with domains
		Inverse Proportion	50	Inversely proportional expression
			51	Graphing inverse proportion
		Applying Proportion and Inverse Proportions	52	Applying proportion
			53	Applying inverse proportion
		Unit Check point		Review on proportional function

Strand	Units	Topics	Lesson No.	Lesson titles
Geometrical, Measurement and Transformation	Plane Figure	Rectilinear figures and transformation	54	Lines and angles
			55	Angles formed at the intersection of two lines
			56	Perpendicular and parallel lines
		Figure Transformation	57	Transformation and translation
			58	Rotation and reflection of figures
			59	Perpendicular bisectors
			60	Angle bisectors
			61	Perpendicular lines
		Circles and Sectors	62	Properties of circles
			63	Measuring circles and sectors
			64	Central angle of a sector
		Unit Check point		Review on plane figures
	Space Figures	Solids and space figures	65	Classifying solids
			66	Nets of various solids
			67	Planes and lines in the space
			68	Positional relationships between two lines
			69	Positional relationship between lines and planes
			70	Positional relationships between two planes
			71	Various ways of looking at solids
			72	Projection of solids
			73	Surface area of prisms and cylinders
			74	Surface area of pyramids and cones
		Surface area and volume of Solids	75	Volume of prisms and cylinders
			76	Volume of pyramid and cones
			77	Volume and surface area of spheres
		Unit Check point		Review on space figure
Statistics and Probability	Distribution of data and representative values	Using data trends	78	Frequency distribution tables
			79	Histograms
			80	Frequency distribution polygons
			81	Relative frequency
			82	Mean
			83	Median and mode
			84	Data distribution and representative value
			85	Dispersion
			86	Approximate values
		Unit Checkpoint		Review on distribution of data and representative value

Content Background

Strand: Number, Operation and Computation

Positive and negative numbers

Based on the study in primary school, the range of numbers is expanded to include positive and negative numbers in grade7. Students are expected to understand the need for and the meaning of positive and negative numbers. In addition, students are to understand the meaning of the four arithmetic operations and to develop computational fluency with positive and negative numbers. Another aim is to enable students to use positive and negative numbers to represent and process concrete situations.

(a) Need for and meaning of positive and negative numbers

As for the need for positive and negative numbers, help students understand them by making connections to prior experiences or everyday situations where positive and negative numbers are used, such as showing the difference in the high temperature from yesterday. As students develop the understanding of positive and negative numbers, they should also use positive numbers and negative numbers so that they may know their merits such as the following:

- Opposite directions or characteristics may be represented by numbers
- Size may be compared
- Number lines may be used as a representation
- Subtraction is always possible
- Addition and subtraction may be unified

The range of numbers has been expanded gradually since elementary prep. The expansion of range of numbers to include positive and negative numbers, it is necessary to reconsider the set of numbers. For example, in elementary school mathematics, integers are referred to as 0 and positive integers. However, in lower secondary school mathematics, negative integers will be included in this set of numbers, and integers as a mathematical concept is defined. Then, we aim to deepen students' understanding of the concept of numbers by discussing the possibility of arithmetic operations with numbers in this set.

(b) Four arithmetic operations with positive and negative numbers and their meaning

The four arithmetic operations with positive and negative numbers become possible as their meaning is expanded from those of elementary school mathematics. For addition and multiplication, commutative and associative properties still hold true, as does the distributive law.

Here, the emphasis is to help students to understand the meaning of the four arithmetic operations and to develop the computational fluency with positive and negative numbers. To expand the range of numbers and think about the meaning of the four arithmetic operations will be important in grade 9 when square roots are taught.

Suppose we are given the calculation, 3-2. If we consider the symbol " -" as an operation symbol, it is subtraction. However, if we represent the expression as (+3)+ (-2), we can see it as addition. By seeing addition and subtraction from a unified perspective, we can consider an algebraic expression with both addition and subtraction as the sum of positive and negative terms. We can then calculate it more efficiently. In addition, if a>b, then the distance between the two points representing a and b on a number line can be represented as a-b.

Mastery of this way of seeing and calculating algebraic expressions is important as it also plays an important role in combining algebraic expressions or solving equations, which are to be studied later. The mastery should be developed not only in the study of positive and negative numbers but also in the study of algebraic expressions with letters and equations, which are to follow.

(c) Representing and processing with positive and negative numbers

Positive and negative numbers are not only the objects of calculation but are also helpful in representing various phenomena and changes easily and also make efficient processing possible. For example, we can represent how well we are achieving the previously set goal by representing the difference between the target and actual values using positive and negative numbers. We can also calculate the average (arithmetic mean) efficiently by using an estimated average. Help students to deepen their examination of phenomena by using positive and negative numbers in concrete situations, and enable them to understand the need for positive and negative numbers.

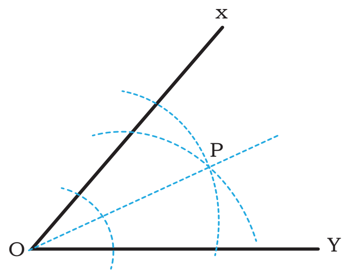
Strand: Geometrical, Measurements and Transformation

1. General content on geometrical figures

In Grades 7, 8 and up , students will learn to construct geometrical figures with a prospect in mind by focusing on the symmetry of geometrical figures. They are also expected to use methods of construction in concrete situations.
Through this study, students deepen understanding of geometrical figures and nurture their ability to perceive and think about geometrical figures empirically. At the same time, cultivate their ability to examine and represent logically. Students are also expected to understand about transformation of geometrical figures and by examining relationships between two geometrical figures, they are expected to enrich their ways of viewing geometrical figures.

Fundamental geometrical construction and their use.

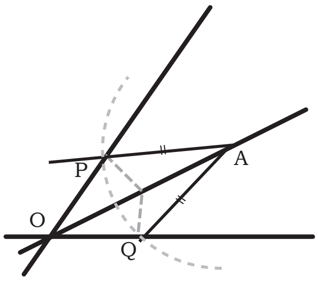
The act of drawing figures is an important fundamental skill in the study of geometrical figures. At the same time, it can increase student's interest and curiosity about geometrical figures. Because it can also deepen their ways of viewing and thinking about geometrical figures intuitively, it can also serve an important purpose for facilitating logical examination of geometrical figures.

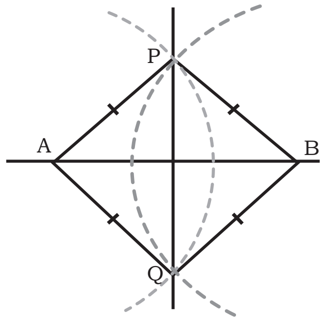


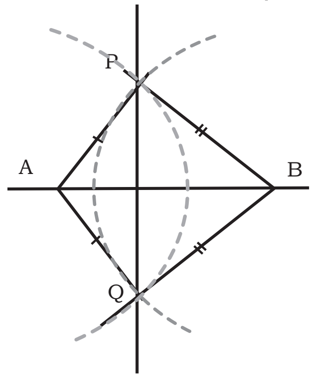
The following basic geometrical construction will be discussed based on the symmetry of plane figures studied in junior primary mathematics, angle bisector, perpendicular bisector of segments, perpendicular lines to given lines. In geometry construction, the rulers are used as tool to draw a line that goes through two points and compasses are used to draw circles and to copy lengths.
Instead of simply giving directions to complete particular constructions, students should focus on the symmetry or constituent parts of the given figures and think about the steps of construction on their own. It is also important that they can explain the steps of construction in an orderly fashion.

For example, let us consider the construction of the angle bisector of X O Y.
This construction requires us to determine the two points the angle bisector will pass through, and then draw a line (a ray) through those two points. Help students to recognize that the angle bisector is the line of symmetry for the given angle, perhaps by having students fold the paper so that the two sides of the angle, OX and OY, will coincide. Since the bisector will go through the vertex of < XOY , what we really need is another point, P, which will be on the angle bisector. If the construction of the angle bisector is considered from this perspective, students can have the prospect that what they need to do is find a point, P which is on the angle bisector of <XOY . In teaching this construction, cultivate students' abilities to examine and express their ideas logically by having them explain the prospect they developed or the steps of construction using their own words.

In the construction of the angle bisector, since OP=OQ and AP=AQ, the points of intersection of circles centred at O and A are points P and Q. In the construction of a perpendicular bisector, the points of intersection of circles with the same radius centred at A and B are points P and Q. In the construction of a perpendicular line to the given line the points of intersection of circles centred at A and B are points P and Q.







In all cases, the fact that the two circles are symmetric about the line connecting the two centres. Help students to see that symmetry of geometrical figures plays an important role in considering geometric construction from a unified perspective by reflecting on the methods of construction.

The method of constructing a tangent line to a circle can also be considered by focusing on the symmetry of a circle. By translating the line perpendicular to a line of symmetry, draw a tangent line. Using this idea, it is possible to understand the way to construct the tangent line through a point in the circle. From this one can

verify that a tangent line is perpendicular to the radius of the circle.
As for the rest of the geometric construction, discuss the fact that, unlike drawing lines and angles using a ruler (marked straight edge) or a protractor, it is possible to draw geometrical figures accurately without relying on their measurement.
For example, have students construct angles measuring 30° or 45° using only a straight edge and compass without using rulers or protractors. Or have them copy sides and angles of a triangle to draw a congruent triangle.

This idea behind congruent triangles by copying the constituent parts is that it is possible to do so by copying three sides, by copying two sides and in between them, or by copying a side and two angles on its end points. Understanding these methods can become the foundation for logical examination and similar activities are also included in primary mathematics to lay the groundwork.

Translations, reflections, and rotations

Although symmetry of geometrical figures as a relationship between two figures is discussed in grade 6,the examination of symmetry from the perspective of transformations of geometrical figures is discussed for the first time in grade 7. In grade 7, by overlapping two figures by transforming one of them or comparing the original figure and the transformed one, students are expected to identify properties of geometrical figures.

With respect to transformation of geometrical figures, students have examined their properties by “sliding,”“turning,” and “flipping,” and naturally have grasped that the shape and the size of a geometrical figure do not change through those motions. At the same the three rigid motions, translation, reflection, and rotation, are discussed.

Under a transformation of a geometrical figure, it is moved to a different position by moving all points on the figure following a specific rule. Under a translation, a figure is moved in a particular direction for a particular distance. Thus, translation is defined by that direction and distance. A reflection will move a geometrical figure around a line as the axis of reflection. Therefore, it is defined by the position of the axis of reflection. Under a rotation, a figure is moved around a given point for a specified angle, and it is defined by the position of the point, the centre of rotation, and the measure of the angle. When the angle of rotation is 180°, it is the point transformation (Under a point transformation, each point on a plane is moved in the direction of the centre of point transformation the distance equal to the distance between the point and the centre of transformation)

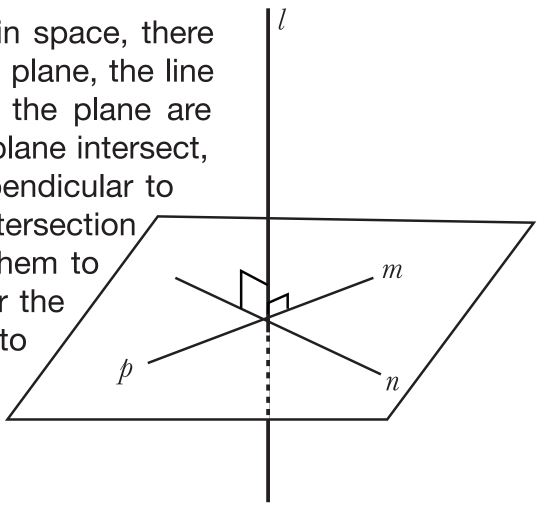
As these ideas are being taught, it is important to help students identify properties of geometrical figures or enrich their ways of viewing geometrical figures by having them compare geometrical figures before and after a transformation, for example, comparing the positions of line segments, measures of corresponding angles, and congruence of the given figure and its image. In addition, as a way to observe tessellations, it is possible to have students investigate what motion will overlap two geometrical figures in a tessellation or creating a tessellation by transforming a given geometrical figure as the starting point. As transformations of geometrical figures are discussed, students will be drawing the images of given figures under transformations. Through such activities, help students become used to the use of the straight edge and compass. Also, in order to help students understand the meaning of geometric constructions, it is important to verify the accuracy of the methods and the results of the basic construction from the perspective of geometric transformations.

Position relationships of lines and planes in space

In primary mathematics, students have learned about parallel and perpendicular relationships of lines and planes in conjunction with the study of solid figures such as squares. However, their examination was limited to the relationships of constituent parts of specific solid figures. In Grades 7, 8 and up the level, students are to examine space relationships of lines and planes as abstract objects while reflecting on their study of solid figures in primary and also while examining concrete space figures. A line in space is considered to be extending in both directions forever just as is the case in a plane. Similarly, students are to understand that a plane in space extends forever in every direction.

Students are to understand that 2 points in space will define a line, and 3 non-collinear points, a line and a point not on the line, and two intersecting lines all define a plane. Moreover, it is important that students can enrich their spatial sense by extending ideas from a plane to space by analogy. For example, just as a plane can be split into two regions by a line, space may be split into two parts by a plane. As for the positional relationships of lines and planes in space, students are to examine the positions of lines and planes and how they intersect each other.

There are two possible cases for the positional relationships of two lines in space: when the two lines intersect, and when the two lines do not intersect. When two lines do not intersect with each other, the lines may be parallel, or may not be parallel. When the lines are not parallel (yet do not intersect each other), we say that they are in a skewed position. When two lines intersect with each other, or when they are parallel to each other, the two lines will define a plane. In other words, those two lines will be on the same plane.



As for the relationship of a line and a plane in space, there are three possibilities: the line is included in a plane, the line and the plane will intersect, or the line and the plane are parallel to each other. When the line and the plane intersect, it is necessary to specify that the line is perpendicular to every line that passes through the point of intersection between the line and the plane in order for them to be considered perpendicular to each other, or the line is not slanted in any direction with respect to the plane. If $l \perp m, l \perp n$, then $l \perp P$. However, if you remember that, "a plane is defined by two intersecting lines," if a line is perpendicular to 2 other perpendicular lines at their point of intersection, then the first line is perpendicular to the plane defined by the 2 perpendicular lines. In other words, to check whether or not a line is perpendicular to a plane, one needs to check whether or not the line is perpendicular to two intersecting lines. When two planes in space are considered, they may either intersect with each other or they may not. A special case of intersecting planes is the perpendicularity, and when the two planes do not intersect, they are parallel to each other. When two planes intersect with each other, they intersect in a line. The positional relationships of lines and planes discussed above are the foundation for examining space figures, and we will not be able to analyse space figures without them. When these ideas are taught, as specified in the teacher's guide that, "through activities of observation, manipulation and experimentation," incorporate situations to examine concrete space figures where positional relationships of lines and planes are essential and avoid simply giving those relationships formally. For example, it is necessary to help students understand the positional relationships of lines and planes by actually building solids and observing them, or through activities of explaining their ideas using the solids. Through such investigations, help students recognize that two lines that are parallel to a third line are parallel to each other themselves or two lines that are perpendicular to the same third line may not necessarily be parallel to each other.

Composing space figures by moving plane figures

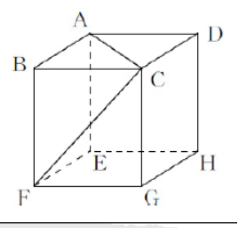
When examining space figures, if you focus on their constituent parts, they may be considered as a result of moving plane figures such as a line segment, polygon or circle. This perspective is important in developing creativity and intuition about space. The idea that a plane is composed as a result of moving a line segment will be taught. For example, the lateral surfaces of a prism may be formed as a result of translating a line segment in space. Or, the lateral surfaces of right cylinders or right cones may be formed by rotating segments around a fixed line (axis of rotation). We will also teach the idea that solid figures are formed as a result of movements of plane figures. For example, prisms and cylinders can be thought of as a result of translating their bases in space. Right prisms can be considered as the result of rotating a rectangle using one of the sides as the axis of rotation, and a right cone may be formed by the rotating of a right triangle around one of its legs as the axis of rotation. Spheres can be viewed as the result of rotating a semi-circle around its diameter. To help students understand these ideas, instruction should make connections to everyday situations such as a potter's wheel or the structure obtained by stacking cards or blocks. It is important to deepen students' understanding of space figures through activities of observation, manipulation and experimentation with space figures, such as actually rotating a right triangle around one of its legs or sorting solids that can be obtained by moving line segments or plane figures.

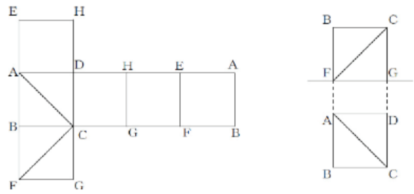
Representing and interpreting plane representations of space figures

While examining space figures, considering their components as plane figures or relating them to plane figures are important facets of understanding space figures along with considering space figures as the result of the motions of plane figures. Students will be taught about representing space figures on a plane in order to understand their characteristics. They will also learn to identify characteristics of space figures from their representations on a plane.

By drawing sketches of space figures or identifying their characteristics from sketches, students can examine space figures without having actual shapes on hand. With sketches, they can also examine the space figures considering their interior, too. Projections of space figures are studied in grades 7 mathematics for the first time. By representing a space figure by how the figure will look when seen from directly above and seen from directly in front, students will examine the characteristics of the space figure.

In this way, students will learn to develop the ways to analyse objects not from a single viewpoint but from multiple viewpoints. In some cases, when a space figure is represented as a sketch or a projection drawing, characteristics of the original space figure may not be preserved. In the sketch of a cube on the right, students might mistakenly think that the diagonals (AC and CF) are not equal by inspection. However, by examining a net or a projection drawing of the same cube, students can reason mathematically, "since the two segments are the diagonals of congruent squares, their lengths must be equal."





While learning to interpret plane representations of space figures, it is important that sketches, nets, and projection drawings are connected to each other so that students can understand space figures more realistically. Thinking about nets in order to build models of cubes or tetrahedron can lead students to examine those figures more analytically and think about the relationships between faces or edges as well as their positional relationships. In this way, students' understanding of solid figures may be deepened. Or, when calculating the surface area of cylinders or cones, how to determine the areas of curved lateral faces can be a problem. However, if they are represented as nets, those lateral faces are rectangles and sectors, respectively, and their areas can be easily calculated. In this way, thinking about nets will lead to the deepening of understanding about space figures. As discussed above, by investigating components of concrete space figures by using sketches, nets, and projection drawings, students' ability to think and represent logically is to be cultivated.

Length of an arc and area of a sector

In primary mathematics, circumference and area of circles are taught. In Grades 7, 8 and up have the students reflect on what they studied in primary and express area and circumference. In addition, students are enable to calculate the length of an arc and the area of the sector by understanding the proportional relationship between the length of an arc and the length of the corresponding arc.

Surface area and the volume of prisms, pyramids, and spheres

In primary mathematics, the volume of cubes, cuboids, prisms and cylinders are dealt with. The formula to calculate the volume of prisms, area of base x height, has been studied. In grade 7, students will deepen their understanding of how to calculate the volume of prisms and cylinders by connecting the view of prisms and cylinders as solids obtained by translating their bases. The volume of pyramids, which is discussed at this level is $\frac{1}{3}$ of the volume of a prism with the congruent base and height. The volume of a sphere is of the volume of a cylinder in which the sphere can fit snugly. To help students develop a concrete understanding of volume of pyramids and spheres, have them estimate the relationship between the volume of the pyramid or the sphere and the volume of a prism or a cylinder. Then, through demonstration or experimenting with models, verify the estimation. The surface area of prisms and pyramids should be taught through activities such as drawing nets of the given solid figures. It is also important that students understand the values of nets. As for the surface area of spheres, help students develop a concrete understanding by using models or by measuring the results of experiments.
As the measurement of solid figures is taught, have students think about measurements of what parts of the solids are needed or what types of drawings may be helpful. By treating them more holistically, we can help students further deepen their understanding of space figures. It should be noted that we only discuss those space figures with bases whose area formulas have already been learned, such as triangles and circles.

Strand: Patterns and Algebra

1. Algebraic expression using letters

Algebraic expressions help students understand the benefits of using letters, representing quantitative relationships and rules using algebraic expressions with letters; interpreting the meaning of algebraic expressions; and, calculating with algebraic expressions with letters. In teaching these ideas, give full consideration to the levels of understanding of the materials in senior primary school mathematics and carefully discuss the generality afforded by letters so that students can gradually develop an understanding of the use of letters without too much confusion. For example, incorporate activities like representing quantitative relationships and rules using algebraic expressions with numbers, words, and symbols such as $\square$ $4x$, interpreting algebraic expressions, and evaluating algebraic expressions by substituting numbers in words and symbols.

(a) Meaning and purpose for the use of letters

Algebraic expressions with letters are needed to represent quantitative relationships and rules concisely, clearly, and generally. For example, if we are to state the commutative law of addition in words, we will say, "the sum of two numbers will remain the same when the addends are reversed." If we use specific numbers, we can say it concisely, for example, "2 + 3 = 3 + 2", but we cannot express that the commutative law holds true in general. However, by using letters a and b, we can express the commutative law concisely, clearly, and generally,
by saying **"$a + b = b + a$."**

Furthermore, by using algebraic expressions with letters, we can examine quantitative relationships abstractly, by considering them as relationships of numbers, without being bound by the specific contexts. For example, $s = ab$ may be representing the relationships such as "area of a rectangle = length × width", **"cost= unit price× number of units," or "distance = speed × time",** and in every situation, we can transform the equation into a s/b to examine the quantitative relationships.

Another benefit of algebraic expressions with letters is that we can communicate our thought processes to others accurately using algebraic expressions with letters. For example, if we arrange match sticks as shown in Figure 1, the number of match sticks you need to make n squares may be represented as **$4n$ - (n-1) or $2n$ + (n+1)** as shown in Figures 2 and 3, respectively.

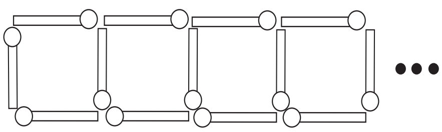


Figure 1

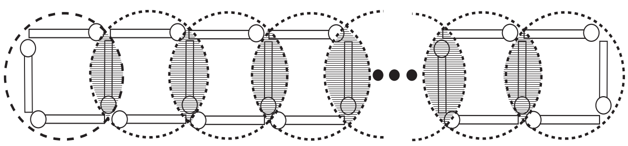


Figure 2

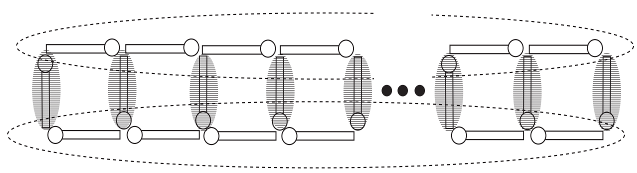


Figure 3

(b) To know about how to express multiplication and division of algebraic expressions with letters

Students are to learn that, when expressing quantitative relationships and rules using algebraic expressions with letters, the multiplication symbol, "×", is usually omitted between two letters or between a letter and a number, and division is represented by the fraction form instead of using the division symbol, " ÷", unless it is specifically needed. For example, the following are typical notations:

$$a \times b = ab, \qquad a \div b = \frac{a}{b} \qquad a \times a = a^2$$

Through these conventions, algebraic expressions can be written more concisely and their manipulation becomes more efficient. It is important to note that the expressions such as ab, a/b, $a + b$, and $a-b$, represent not only the operations to be performed but also the results of the operations. It is necessary that teachers pay close attention, especially at the beginning of the study of algebraic expressions with letters, because students often feel strange leaving operation symbols in expressions, for example, $3a + 2$ or $5x - 5$.

(c) Addition and subtraction of linear expressions

As for calculations with algebraic expressions with letters, addition and subtraction of linear expressions will be taught. The goal is to master simple calculations necessary in solving linear equations with one variable.
Therefore, the focus is on combining like terms involving linear expressions with one letter such as $(2x - 3) + (x+1)$ or $2(3x + 4)-3(x - 5)$.

To develop an understanding of the ways to calculate with algebraic expressions with letters, it is important to make connections to the world of numbers, for *example,* calculations with letters also involve terms that calculations with numbers do, and properties of operations remain valid. Moreover, enable students to use calculations with specific numbers or daily situations to understand calculations of algebraic expressions. For *example,* $a - (b + c) = a - b - c$ may be related to $5-(3 + 2) = 5-3-2$, or to the calculation of the amount of change with a-yen when a b-yen item and a c-yen item are purchased.

(d) Writing and interpreting algebraic expressions with letters
Algebraic expressions with letters are a powerful form of representation. It is important for students to experience their benefits by writing algebraic expressions with letters to represent quantitative relationships and rules and by interpreting those expressions. It is also important that students develop the disposition to utilize algebraic expressions actively.

In order to write and interpret algebraic expressions, quantities represented by letters and their relationships must be understood. For example, if the admission fee for an adult at an art museum is a-yen and the admission for a child is b-kina, the total cost for an adult and two children can be represented as $a + 2b$. In the same situation,the expression, $a - b$, can be interpreted as the difference of the admission fees for an adult and a child.

To represent with algebraic expressions with letters, we must decide what operations to perform with quantities that are represented by letters. Grasping the relationships among quantities will be easier if we consider specific numbers instead of just thinking with letters.
As for algebra expressions that represent quantative relationship, the size relationships of quantities will be represented in equations or inequalities. For example, in the previous art museum example, the relationship,"the total cost of admission for one adult and two children were 1000 kina", can be represented by, $a + 2b = 1000$.

Alternately, we can express the relationship in, $2b = 1000 - a$, or, $a = 1000 - 2b$. The equal sign here is used to represent the equality of quantities, not as a symbol to indicate an operation and its result. That is, the equation, $a + 2b = 1000$, means that "$a + 2b$ and 1000 are equal (because they are both representing the total cost of admissions, therefore, they are balanced)," not "when you perform the calculation $a + 2b$, the answer is 1000". Being able to interpret algebraic expressions this way is very closely related to the study of linear equation. In addition, if we say, "we received change of 1000 kina when we paid the admission for one adult and two children," it can be interpreted that "the total amount we paid was less than 1000 kina". Therefore, we can represent the relationship as $a + 2b < 1000$. Here the aim is to help students write and interpret quantitative relationships represented in algebraic expressions with inequality symbols and to deepen their understanding of algebraic expressions with letters.

A letter may stand for different values, and in order to help students develop that understanding, it is helpful to have them evaluate the algebraic expressions with letters by substituting different numbers in the expressions. This is also important to understand the meaning of the solutions of equations. When evaluating algebraic expressions, make sure that students can correctly substitute negative values as well. In addition, avoid making the evaluation of algebraic expressions just computation practice by connecting the expressions to concrete situations.

2. Linear Equations with one unknown

In Grades 7 and 8 mathematics, based on the study of algebraic expressions with letters in primary , the necessity and the meaning of equations and the meaning of their solutions and methods to solve linear equations with one variable based on the equality relationships will be taught. Students are also expected to understand the benefits of algebraic manipulations through the study of these ideas.

(a) The necessity and the meaning of equations and the meaning of their solutions

Equations are algebraic expressions with variables (unknowns) that represent the conditions for the equality relationships, and they are needed to determine the value that will satisfy the conditions accurately. The solutions of equations are those values that satisfy the conditions. For example, the equation, $x + 3 = 5$, is a mathematical representation of, "the sum of x and 3 equals 5," but it can also be thought of as the condition for the value of x that will satisfy the equality relationship. Whether or not the condition is satisfied depends on the value the variable x in the equation will take. If we consider the domain of x to be the set of integers, we can substitute ___ ;-3;-2;-1; 0; 1; 2; 3; ___ into x. We will then find out that when, and only when, the value of x is 2, the equality relationship is satisfied. Therefore, 2 is the solution of this equation. This way of determining the solution may be important in understanding the meaning of the solution of an equation, but it is not an efficient procedure. Equations can be solved by procedural manipulation based on the properties of equality relationships; therefore, they are useful in obtaining solutions in concrete situations efficiently.

(b) Properties of equality relationships

To solve linear equations with one variable, we transform the given equations using the properties of equality relationships into the form, $x = \_$. The four properties of equality that are used in the process are as follows:

1. If $a = b$, then $a + c = b + c$
2. If $a = b$, then $a - c = b - c$
3. If $a = b$, then $ac = bc$
4. If $a = b$ and c ≠ 0, then $a/b=b/c$

Properties 1 and 2 may be considered the same from a unified perspective of addition and subtraction using positive and negative numbers. Similarly, 3 and 4 may be considered the same, as well.

It is important that students understand the properties of equality relationships with a concrete image, perhaps developed through activities that use a pan balance, and to be able to use them in solving equations.

In studying how to solve equations using the properties of equality relationships, students should understand not only the merit of being able to solve equations through manipulation but also the fact that the properties of equality relationships justify the transformation of equations. It is particularly important that teachers help students to understand and to be able to explain the transposition of terms, which is a useful technique in solving equations, based on properties 1 and 2 above. It is also possible to extend students' abilities to perceive and think mathematically based on what have been validated previously through this type of instruction.

(c) Solving linear equations with one variable

We teach how to solve linear equations with one variable, $ax + b = cx + d$, by transforming it into the form, $x = \propto$, by using the properties of equality relationships. That is, by using properties 1 and 2 , we generate the idea of transposing terms, and by transposing terms, we transform the given equations into the form Ax=B(A≠0). Then, by using properties 3 and 4, we change the coefficient of x to 1 to find the solution. Students are expected to understand that equations can be solved by transforming the given equation step by step through a series of equivalent equations into the form, $x = \propto$ By examining the process, help students summarize the process to solve equations generally and become proficient at solving equations. It is necessary to note that transforming equations in the solution process is different from the transformation of algebraic expressions during the calculation, such as addition and subtraction of numbers or linear expressions, in the following way: Calculation of numbers and algebraic expressions means that the given expression is being transformed into a more simplified expression. Transformations of equations during the solution process not only change the given equation into more simplified equations but also equations with the same solutions. Help students understand that transforming the given equation into a series of equivalent equations is different from the transformation of algebraic expressions studied before this point. As for solving linear equations with one variable, enable students to solve equations necessary to solve concrete problems

so they can actually use what they learn.

$5x + 3 - (2x - 6)$	$5x + 3 = 2x - 6$
$= 5x + 3 - 2x + 6$	$5x - 2x = -6 - 3$
$= 5x + -2x + 3 + 6$	$3x = -9$
$= 3x = +9$	$x = -3$

(d) Use of linear equations with one variable

To solve problems by using equations, the following series of activities are performed:

1. Identify the quantity to be determined and represent it with a letter

2. Identify the quantity that can be represented in two different ways based on the quantitative relationships in the problem situation, and represent the relationship in algebraic expressions with letters and numbers
3. Create an equation by connecting the two expressions with the equal sign, and solve the equation
4. Examine the solution in the context of the original problem to determine the answer

Step 2 closely relates to the study of writing and interpreting quantitative relationships using algebraic expressions with letters. In step 4, the solution of the equation is examined in the context of the original problem to determine if it is appropriate as the answer to the problem. This means to re-examine the conditions that could not be incorporated into the equation. It is important to develop the disposition to reflect on the results obtained through mathematical manipulation in terms of their appropriateness through a study like this one. In our daily life, we often encounter problems involving ratios. As possible types of situations to use linear equations, we can think of those situations in which simple proportions must be solved.

For *example,* consider this problem: two types of solutions will be mixed in the ratio of 3 : 5 in terms of their weights. If we use 150 g of solution B, how many grams of solution A are needed? If we consider that x g of solution A is used, we can establish a proportion, $\frac{3}{5} = \frac{x}{150}$. This can be considered as a linear equation with one variable, and the solution of the equation is $x = 90$.

In other words, we need to mix 90g of solution A. In this manner, in those situations where ratio is used to determine quantities, we can solve the problem by solving the equations obtained as proportions.

2. Functions

The study of direct and inverse proportional relationships is the foundation of exploration of quantitative relationships in our daily life. In this study, cultivate students' abilities to identify, represent, and examine functional relationships through observation of concrete phenomena, and avoid approaching these topics just formally and generally. It is also important to re-conceptualize direct and inverse proportional relationships, which students studied in primary school, as functions based on the expansion of the range of numbers.

Meaning of functional relationships

A functional relationship exists between two quantities when the value of one quantity is determined, the value of the other quantity will also be fixed to one and only one value. The aim here is to help students deepen their understanding of functional relationships by focusing on the way quantities change and correspond, and capture the relationships using expressions, "…….. and …….. are in a functional relationship," or, "…….. is a function of ……...."

The instruction in grade 7 will centre around the study of direct and inverse proportional relationships, as they are examples of functions. In the early stage of the study of functions, some students might mistakenly think that only direct and inverse proportional relationships are functions. Therefore, care should be taken so that students can experience the expansion of the concept of functions and cultivate their abilities to identify, represent and examine functional relationships.

Representing quantitative relationships in algebraic expressions, variables and constants must be distinguished. It is necessary to clearly define which quantity will be considered as x and which will be y. When quantitative relationships are represented in algebraic expressions, it can be seen that when the value of one variable is fixed, the value of the other variable is also fixed. Moreover, tables and graphs can be easily constructed based on algebraic expressions. Care should be taken to help students realize that there are functional relationships that cannot be represented in an algebraic expression.

To graph quantitative relationships in graphs, points whose coordinates represent the values of corresponding quantities must be plotted. Help students understand that they can determine from graphs that when the value of variable x is fixed, the value of variable y is also fixed. When tables, algebraic expressions, and graphs are used, it is important that connections are made among them, instead of treating them in parallel or independently, so that students can understand them more holistically.

For example, while examining a quantitative relationship represented in a table, by representing the same relationship as an algebraic expression or a graph, students can deepen their understanding of the quantitative relationship by seeing that other correspondences not shown on the table can be gathered from the alternative representations. Also, while examining a quantitative relationship represented in an algebraic expression, if one represents the relationship in a table or graph, the way quantities change and correspond can be more concretely captured. In this way, Meaning of direct and inverse proportional relationships.

In primary mathematics, simple cases of direct proportional relationships are discussed in grade 5, and, in grade 6, students are expected to understand direct proportional relationships based on the study in grade 5. They also learn to investigate characteristics of direct proportional relationships by examining their tables and graphs. To help students deepen their understanding of direct proportional relationships, students are expected to know about inverse proportional relationships. Students have seen the following three meaning of direct proportional relationships in primary mathematics:

When one quantity increases 2, 3, ... times as much, or decreases $\frac{1}{2}$, $\frac{1}{3}$, the other quantity also increases 2, 3, ... times as much, or decreases $\frac{1}{2}$, $\frac{1}{3}$,respectively. When one quantity becomes m times as much, the other quantity also becomes m times as much.

The ratio of the two quantities (ratio) is constant.

Students have also seen the following three meaning of an inverse proportional relationship:

Given two quantities, when one quantity increases 2, 3, ... times as much, or decreases $\frac{1}{2}$, $\frac{1}{3}$, the other quantity becomes $\frac{1}{2}$, $\frac{1}{3}$, times as much or 2, 3, ... times as much, respectively.

When one quantity become m times as much, the other quantity becomes $\frac{1}{m}$ times as much. The product of the two quantities remains constant. The range of numbers for the quantities is limited to non-negative numbers.

From grade 7 mathematics, building upon the study in primary level, the range of numbers in direct and inverse proportional relationships including negative numbers and represent them in algebraic expressions with letters. For direct proportional relationships, using a as the proportion constant, they can be represented generally as $y = ax$ or $\frac{x}{y} = a$. Inverse proportional relationships may be represented generally as $y = \frac{a}{x}$ or $xy = a$, where a is the proportion constant.

Meaning of coordinates

In Grades 5 and 6 students learn to represent the position of objects in a plane or in space, which leads to the meaning of coordinates. In addition, students learn to represent the changes of quantities using broken line graphs. However, the approach taken there was to connect the top of bars in bar graphs, not based on the idea of coordinates, that is, to represent the position in a plane using a pair of numbers.

A position on a plane may be represented by a pair of numbers which correspond to two intersecting number lines as axes, generally. This is the concept of coordinates on a plane. In Grade 7 and 8 mathematics, students are expected to understand the meaning of coordinates as a way to uniquely represent a position on a plane by using two perpendicular number lines intersecting at the origin, 0. By using coordinates, it is possible to represent graphs as a set of points.

To explain concrete phenomena using the ideas of direct and inverse proportional relationships It is important that students can explain concrete phenomena using the ideas of direct and inverse proportional relationships. There are many phenomena related to direct and indirect proportional relationships in our daily life as well as other subject matters, particularly in science. Moreover, it is possible to study direct and inverse proportional relationships using mathematics contents that have already been discussed, such as the length of a side and the area. If we can capture the relationship between two quantities as a direct or inverse proportional relationship, we can capture many characteristics of the way those quantities change and correspond. In addition, it is possible to explain those characteristics more easily by using tables, algebraic expressions and graphs. For example, consider the question, ”what would happen to the circumference when the radius becomes 2, 3, ...times as long?”, which is about a direct proportional relationship between the radius, r, and the circumference,

We can respond to this question more easily if we use the meaning of algebraic expression, $l=2\pi r$, without having to calculate with specific numbers. Furthermore, when we consider the question, “if the radius becomes 2, 3, ... times as long, would the area also becomes 2, 3, ... times as much?”, we can detect from the algebraic

expression $S = \pi r^2$, where S is the area of the circle, the radius and the area are not in a direct proportional relationship. With respect to inverse proportional relationships, some students might mistakenly think that, “since B decreases as A increases, B is inversely proportional to A.” It is important that care should be taken so students will not develop this misunderstanding by examining the characteristics of tables, algebraic expressions and graphs of inverse proportional relationships.

Finally, when dealing with concrete situations, it is important to pay attention the range of values the variables can assume. For example, when considering the relationship between the length and the area using a direct or inverse proportional relationship, negative values will not make sense for either length nor area. Enable students to pay attention to the range of numbers for the variables as they explain concrete phenomena.

Strand: Statistics and Probability

Making use of Data

In Grades 5 and 6, students learn about bar graphs, broken line graphs, circle graphs, and percentage bar graphs. Students also learn to examine and represent data statistically through activities such as representing frequency distribution in tables or graphs and investigating the average and spread of data. In grade 5, the average of measurements, and in grade 6, the idea of examining and representing statistically are taught.

In secondary mathematics, students in Grade 7 are expected to understand the importance of efficient and some purposeful ways to collect and organize data as well as some reasonable ways of processing the data. In addition, they are expected to understand ideas such as histograms and representative values for a set of data, and to identify trends in data through using those representations in explaining the data.

Need for and meaning of histograms

Frequently, we have to make decisions based on data in our daily life. Data collected for different purposes may be qualitative, such as gender distribution in a population survey, or quantitative, such as changes in the temperature at noon in the previous month. In either case, in order to make appropriate decisions, we must process the data purposefully and use the results to interpret the trends in the data.

Frequency distribution tables and histograms are methods for such statistical processing. By using histograms, the spread of data can be captured. By dividing the data into a number of classes and counting the number of data in each class, it is easier to capture such features as the shape of the data, the range of distribution of the data, the location of the peak, and symmetry.

16,	12,	27,	18,	18,	23,	22,	24,	15,	13
26,	12,	24,	24,	15,	10,	18,	15,	18,	18
18,	18,	15,	16,	21,	11,	12,	20,	26,	27
16,	20,	25,	21,	18,	18,	23,	16,	18,	24
16,	18,	14,	18,	14,	14,	18,	15,	14,	18
23,	23,	23,	14,	14,	21,	21,	27,	25,	23
20,	22,	27,	18,	18,	14,	18,	18,	27,	24
15,	25,	15,	24,	23,	21,	25,	25,	15,	16
24,	11,	25,	23,	13,	13,	20,	15,	20,	26
18,	20,	25,	22,	23,	23,	21,	22,	16,	22

Figure 1

When using histograms to interpret trends in data, it is necessary to pay careful attention to the width of each class of data. For *example,* Figure 1 shows the records of handball tosses by 100 grade 1 students at Grade 7.
Figures 2 and 3 are the histograms for this data set using 3m and 2m as the width of classes, respectively.

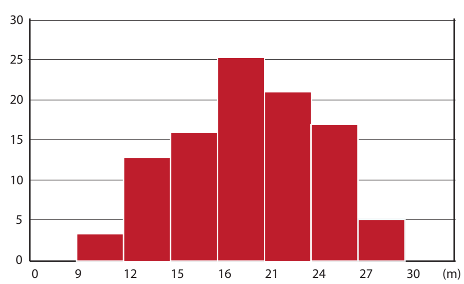


Figure 2

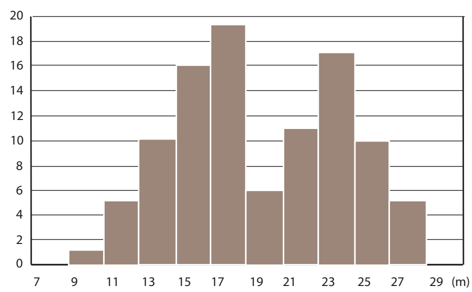


Figure 3

In the histogram in Figure 2, the distribution of the data appears to include one peak, while the histograms in Figure 3 show two peaks. Therefore, the answer to the question, ”are there many students who threw the handball between 19 m and 20 m?”, may be different depending on which histogram students use.
Thus, depending on the width of classes, histograms may show different trends for the same set of data. Therefore, it is necessary to make several histograms with different widths of classes and compare them when histograms are used to identify trends in data for specific purposes.

Making histograms by hand can deepen students’ understanding of the meaning of histograms. However, for the study such as the one described above, it is important to provide sufficient time for thinking by, perhaps, using computers.

Need for and meaning of representative values

When identifying trends in data, in addition to frequency tables and histograms, representative values are often used.
A unique feature of representative values is expressing the characteristics of the data distribution in one numerical value from a particular perspective. Mean, median, and mode are most frequently used. By representing a data set in a single value, characteristics of the set can be concisely expressed and the comparison with another set of data becomes easier. On the other hand, it should be kept in mind that some information such as the shape of data distribution may be lost. Mean can be determined from a data set that has yet to be organized in a frequency distribution table. On the other hand, there are situations where means are not the appropriate representative values.

Need for and meaning of relative frequency

When comparing two data sets with different number of data, we cannot simply compare the frequency of each class directly. In those situations, we can compare the relative frequency of each class because a relative frequency will show the ratio of the frequency in each class to the whole data set.
A relative frequency is a value showing the ratio of a part (frequency in a class) to the whole (the total number of data), and we can consider it as a frequency of the class. In teaching this idea, it should be kept in mind that this is the foundation for probability discussed in Grade 8. By using relative frequencies for a single data set, we can more easily determine the ratio of each class as well as the ratio of combinations of classes (above or below a certain class).

Grasping and explaining trends in data

Enable students to grasp and explain trends in data using histograms and representative values. The purpose of histograms and representative values is not actually to create or calculate them. Rather, when they are used to identify and explain trends in data, they become meaningful. Therefore, when teaching these topics, provide students a series of activities such as the following: identify a problem from everyday situations, gather data necessary to solve the problem, make histograms and calculate representative values – perhaps with the help of computers, identify trends in the data, and explain the solution using the results of the analysis. For example, let's think about how the number of runners in a relay race between two classes will influence the results. The data from the physical education class may be used, or a new set of data can be gathered for this investigation. By making frequency distribution tables and histograms from the data set, we can predict, "which class will win if we select 10 runners from each class?" or, "what if we increased the number of runners to 20 students from each class?" based on the distribution of the data.

What is important here is not whether or not the prediction turns out to be correct, rather, making clear on what the identified trends and explanations are based. Through activities of explaining and communicating, students can know that even from the same data set several interpretations are possible. By listening to each other's explanations and their bases, students may also deepen their understanding of the ideas being used. In our daily life, to capture trends of a data set, representative values alone are often used because of their simplicity. However, it is necessary that students can grasp trends of data with a clear understanding that representative values may not capture some information.

Use of tools such as computers

While dealing with a large number of data, or data that have large numbers or numbers with fractional parts, use tools such as computers to efficiently process the data, and the emphasis should be placed on interpreting the results of statistical processing to identify trends. However, consideration should be given that making histograms or calculating representative values by hand may be important while teaching the need for and the meaning of histograms and representative values. It is necessary to think about how to use those tools effectively such as having a computer available to each student, and using a computer as a tool to display during the whole class discussion. In addition, when gathering data using various information networks, the data are no longer primary data. Therefore, care should be taken to check for the reliability of the data by, for example, identifying who gathered the data using what technique.

Errors and approximations

In primary mathematics, students learned about approximate numbers and how to use them purposefully. They have also learned about the decimal numeration system. Here, errors, approximation, and the idea of expressing the numbers in the form of $a \times 10^n$ will be handled.
Every measurement includes an error. For example, suppose you measure a student's height using a tool whose smallest interval is mm. If the student's height measures 157.4 cm, it means his true height is greater than equal to 157.35 cm but less than 157.45 cm.

In other words, if the height of the student is x cm, then $157.35 \leqq x < 157.45$.
Help students understand that measurements always involve errors, perhaps by representing the measurements on a number line, and 157.4 cm is being used as the approximation. Students are to understand experientially the meaning of approximations and errors.

Mathematics Grade 7

Guided Lessons

Lesson No: 1 to 84

LO1 : What kinds of numbers are there?

Positive and Negative Numbers

Lesson Objective: : To define positive and negative numbers and explain their uses in daily life. **(7.1.1.1)**
Materials: Calendar, world and weather map, thermometer, pictures showing positive and negative numbers such as number line, win-lose scores of sports teams.

ASK-MT and Assessment	
Attitudes/Values	Become interested in finding out about numbers with (-) signs and enjoy identifying the use of negative and positive numbers in everyday situations
Skills	To be able to read and explain numbers less than 0 on a number line and on thermometers.
Knowledge	Positive and negative numbers and their use in daily life
Mathematical Thinking	Think about the meaning and representation of positive and negative numbers and how it is used in daily life
Assessment	Use the ASK-MT above to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1. **Express the following temperatures using negative (-) sign.**

(a) The temperature 2°C below 0°C. (b) The temperature 3.5°C below 0°C.

Example.2. **What can we find out about positive numbers, negative numbers and zero?**

Express the following with a positive or negative sign

(a). The number 9 less than 0. (b). The number 11greater than 0.

(c).The number 5.5 greater than 0. (d). The number $\frac{2}{3}$ less than 0.

Example.3. **Natural numbers**

Which of the following are natural numbers? Which are integers? 0.3, -5, -6, 4, - 0.7, 1/7 , 0, (-1)/3, +1

Key Ideas

Temperature below 0°C is expressed using numbers less than zero like these.
Numbers that are less than zero, like -3, - 3.5, and $-\frac{1}{2}$, are called **negative numbers.**
Number that are greater than 0 like 4, 0.5 and $\frac{3}{4}$, are called **positive numbers.**
The numbers zero is neither positive nor negative.
Negative numbers are expressed by putting a - in front of them, and **positive numbers** are sometimes shown with a +.

For example if the number 2 were represented as +2, it would be read as "positive two". Also, when + and – are used this way, they are known as a "positive sign" and "negative sign"
Positive integers greater than zero such as 1, 2, 3 and so on are called **natural numbers** (counting numbers).

$$\overbrace{\underbrace{\ldots\ldots,\ -3,\ -2,\ -1,}_{\text{Negative intergers}}\ 0,\ \underbrace{1,\ 2,\ 3,\ \ldots\ldots}_{\text{Positive integers}}}^{\text{integers}}$$

LO2 : The number line

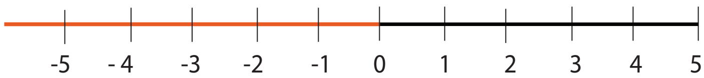
Lesson Objective: To use positive and negative numbers to represent on a number line **(7.1.1.1/3)**
Materials: Number line charts

ASK-MT and Assessment	
Attitudes/Values	Appreciate the meaning and use of positive and negative numbers in daily life.
Skills	Be able to represent negative and positive numbers on a number line.
Knowledge	The meaning of positive and negative numbers and how to represent it on number line.
Mathematical Thinking	Be able to think of how to represent positive and negative number on a number line.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

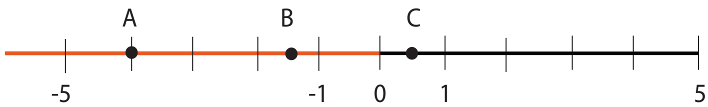
Teaching and Learning Activities

Introductory

How should we express integers on the number line?
Express the point +2 on the number line. How can you express the point -2?
On the number line, numbers greater than 0 are shown on the right of 0. If we extend the number line to the left of 0, we can express numbers less than 0 as well.

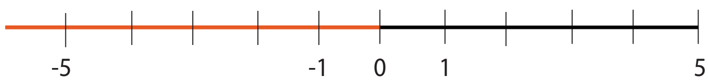


Example.1 Read and say the numbers represented by points A, B and C on the number line below.



Example.2 Plot the following numbers on the number line below.

$-3, \frac{7}{2}, +4.5, -25$



Exercises

(1) Express the following numbers with a positive or negative sign.

(a) The number 18 greater than 0.

(b) The number 36 less than 0.

(c)The number $\frac{1}{3}$ greater than 0.

(2) Which of the following are negative numbers?

$-3.2, \quad 0, \quad \frac{2}{3} \quad -10, \quad -\frac{5}{6}, \quad 0.2, \quad -1, \quad +9, \quad -0.1$

LO3 : Expressing quantity with positive and negative numbers

Positive and Negative Numbers

Lesson Objective: To identify and express the quantities used in opposite properties. **(7.1.1.1/3)**
Materials: blackboard

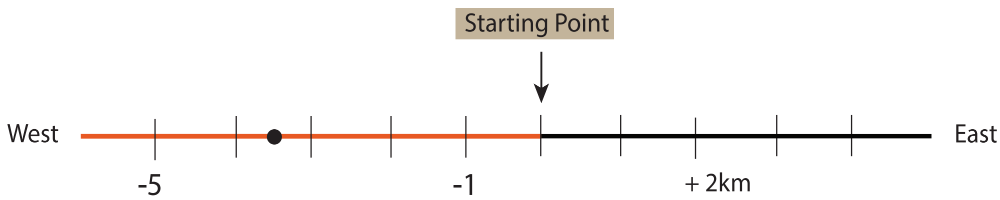
ASK-MT and Assessment	
Attitudes/Values	Appreciate different quantities that express positive and negative numbers.
Skills	Discuss and explain the quantities that have opposite properties and represent with positive or negative sign.
Knowledge	Quantities that have opposite properties.
Mathematical Thinking	Think about how to express quantities using positive and negative numbers.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Like income and expense, the height of a mountain and the depth of the ocean are quantities that have opposite properties. We can express quantities such above and below using positive and negative numbers.

Example.1 **Income and expenses**
If 5000 kina is expressed as + 5000 Kina, 4000 kina of expenses can be expressed as - 4000 Kina.

Example.2 **East and west**
If a point 2km to the east from the starting point is expressed as +2 km, *a* Point 3.5 km to the west from the starting point can be written *a* - 3.5km.



Example.3: **Setting a target as a standard value**

Rubi makes a goal to score 10 points during each basketball game. The difference from the goal is expressed as follows;

+6 points, if scores 16 points, -3 points, if he scores 7 points

Key Ideas

Quantities with opposite properties are described with two different phrases, like “over” and under. Using negative numbers allows us to describe all the quantities with the same phrase.
For example, if we use the negative number -3 to express the phrase 3 under, we can say -3 over instead.
Setting a target as 3 under.......................... -3 a standard value

Exercises

Use the words in [ ] to express the following.

(1) 4 under [over] (2) 6cm short [long] (3) 3kg light [heavy] (4) 10kina short [over]

L04 : The number line

Lesson Objective: To use positive and negative numbers to represent on a number line **(7.1.1.1/3)**
Materials: Blackboard

ASK-MT and Assessment	
Attitudes/Values	Appreciate the meaning and use of positive and negative numbers in daily life.
Skills	Be able to represent negative and positive numbers on a number line.
Knowledge	The meaning of positive and negative numbers and how to represent it on number line.
Mathematical Thinking	Be able to think of how to represent positive and negative number on a number line.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

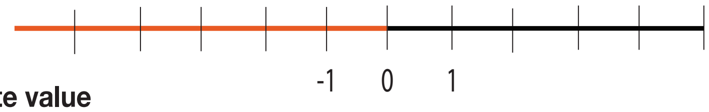
Teaching and Learning Activities

Introductory

What can you find out?
Show the following numbers on the number line below

+3, -3, -4, +4, -1.5, + 1.5

What can you say about a positive and negative number that have the same numerals?



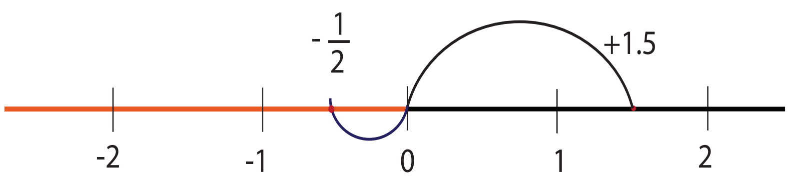
Example 1: **Absolute value**

The absolute value of +3 is 3

The absolute value of -4 is 4.

The absolute value of +1.5 is 1.5.

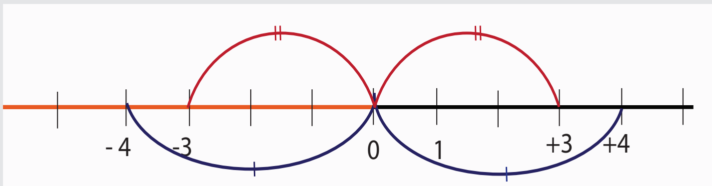
The absolute value of $-\frac{1}{2}$ is $\frac{1}{2}$.



Key Ideas

Switching the + or – of a number (for *example,* changing +3 into -3 or -4 into + 4) is called **changing the sign.** A digit with two opposite sign are on opposite sides of the number line and have an equal distance away from zero.

The absolute value of a number is its distance from 0 on the number line.
The absolute value of 0 is 0.



Exercises

Read and explain the absolute values of the following numbers.

(1) -5 (2) + 8 (3) – 3.5 (4) 1/4

L05 : Size of numbers

Positive and negative numbers

Lesson Objective: To express relative size of positive and negative numbers using inequality signs. **(7.1.1.3)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Appreciate learning about relative size of positive and negative numbers.
Skills	Express relative size of positive and negative numbers using inequality signs.
Knowledge	Relative size of positive and negative numbers.
Mathematical Thinking	Identify ways of expressing the relative size of numbers.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example1. **Size of a number**

All numbers expressed on a number line are in order of size.

The numbers that are further to the right are greater.

Key Ideas

Size of numbers
Following facts about the size of numbers.
Positive numbers are greater than negative numbers. Positive numbers are greater than zero and become greater as their absolute value increases. Negative numbers are less than zero and be come less as their absolute value increases. The relative size of numbers can be expressed using inequality signs. Inequality signs are: > greater than, < less than;

For example,
-5 < 3 means that -5 is less than 3
-2 > -3 means that -2 is greater than -3

Exercises

1. Fill in the missing inequality sign to express the relative size of numbers below.
   (a) 4☐5 (b) -3 ☐ -7 (c) - 1.6 ☐ -0.6 (d) $-\frac{3}{8} \square -\frac{5}{8}$

LO6 : Using a number line

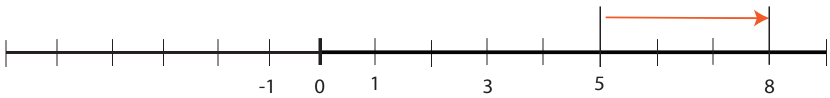
Lesson Objective: To compare the size of a number using a number line **(7.1.1./3)**
Materials: Number line chart

ASK-MT and Assessment	
Attitudes/Values	Appreciate and value the usefulness of number line when comparing the size of numbers
Skills	Compare and explain the size of numbers and their position on a number line
Knowledge	The size of numbers and their position on the number line, words used to compare such as greater and less than
Mathematical Thinking	Reason comparing the size of numbers on a number line
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

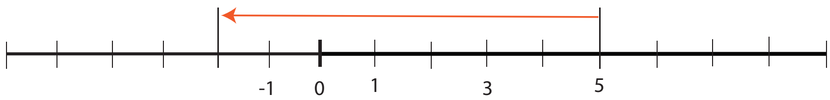
Example.1: **A number greater than or less than a given number.**

If you want to know what number is 3 greater than 5, you can look for the 3 units to the right of 5 on a number line. The number line below shows that the number is 8.



(1) The number 3 is greater than 5

In the same way, to find the number that is 7 less than 5, you can look for the 7 units to the left of 5 on the number line, as shown below.



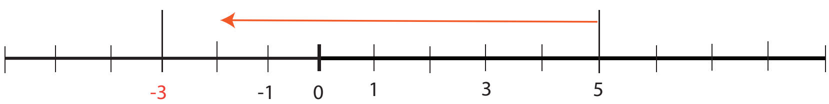
(2) The number 7 is greater than 5

The size of numbers and their positions on a number line are used to figure out a number greater than or less than a given number.

Example.2: **Another number by a certain negative amount.**

the number that is -8 greater than 5 is 8 less than 5. When you use a number line to find the answer, you look for the 8 units to the left of 5. The figure below shows that the number is -3.

-8 greater
↓
8 less



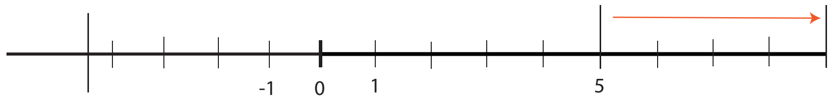
(3) The number -8 is greater than 5

You can change the words in an expression that uses negative numbers so that you don't have negative numbers. For example, "-3 greater" is the same as " 3 less". You can use this idea to find numbers that are greater or less than another number by a certain negative amount.

Teaching and Learning Activities

In the same way, the number -4 less than 5 is 4 greater than 5. The number line below shows the 4 units to the right of 5.

-4 less
4 greater



(4) The number -4 is greater than 5

Key Ideas

From the above, we know the following
The number that is a positive quantity △ greater than the number ❍ is the one that corresponds to △ units to the right of ❍ on the number line.
The number that is a positive quantity △ less than the number ❍ is the one that corresponds to△ units to the left of ❍ on the number line.

To find a number that is a negative quantity greater or less than a certain number, first change the expression so that it does not use a negative number. Then find the answer with the method used for positive numbers

Exercises

Use the number line below to find the following numbers.

(1) the number 3 greater than -5
(2) the number 5 greater than -3
(3) the number 6 less than 3
(4) the number 4 less than -1
(5) the number -4 greater than 1
(6) the number -3 greater than -1

L07 : Addition with positive and negative numbers (1)

Lesson Objective: To explore the idea of adding the positive and negative numbers using prior knowledge and number line. **(7.1.1.2/3)**

Materials: Number line charts

ASK-MT and Assessment	
Attitudes/Values	Appreciate the use of numbers lines to add positive and negative numbers
Skills	Use a number line to calculate positive and negative numbers
Knowledge	Ideas on calculations of Positive and negative numbers on a number line
Mathematical Thinking	Use previous knowledge to construct new ideas on how to calculate positive and negative numbers
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introductory

1 What number is the calculation trying to find?

What number is 5 +(- 6) trying to calculate?

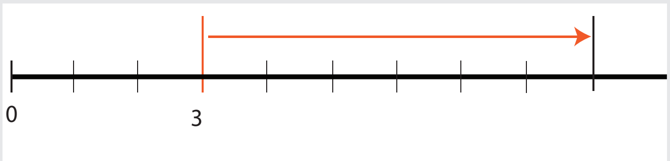
Given the above problem, Koivi reviewed what he learned in school so he could do the calculation.

Review

If six more kids come, how many will there be?

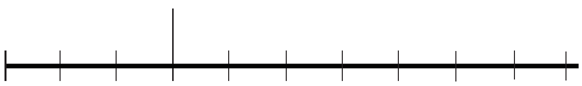
The math expresion used to solve the question on the left is 3+6.

This calculation is used to find the number that is **6 greater than 3.**
if we think about it using a number line.



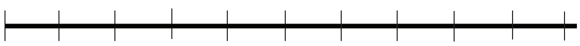
2 If we used the same idea as the one in review, what number are we trying to calculate with (-4) + 6?

- We are calculating the number that is 6 ☐ than - 4.



3 If we use the same idea as one in review, what number are we trying to calculate with 5 +(- 6)?

We are calculating the number that is ☐ greater than 5



L08 : Addition with positive and negative numbers (2)

Lesson Objective: To explore the idea of adding the positive and negative numbers using prior knowledge and number line. **(7.1.1.2/3)**

Materials: Number line chart

ASK-MT and Assessment	
Attitudes/Values	Appreciate the use of numbers lines to add positive and negative numbers
Skills	Use a number line to calculate positive and negative numbers
Knowledge	Ideas on calculations of Positive and negative numbers on a number line
Mathematical Thinking	Use previous knowledge to construct new ideas on how to calculate positive and negative numbers
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

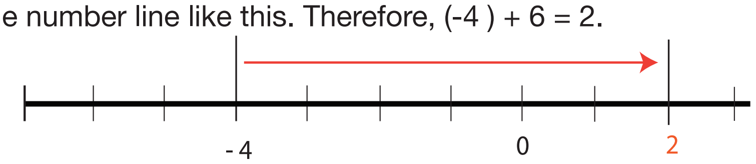
Teaching and Learning Activities

Example.1: **Adding Positive + Positive**

When we add a positive number to another positive number, we express the calculation used to find "6 greater than 3" as 3+ 6.

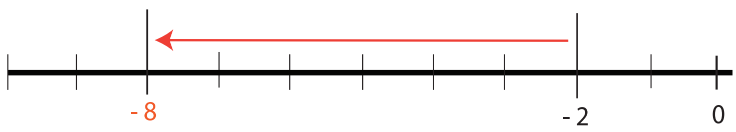
Example.2: **Negative + Positive**

Using the same concept, we can find the number that is 6 greater than (-4) + 6. We represent this calculation on the number line like this. Therefore, (-4) + 6 = 2.



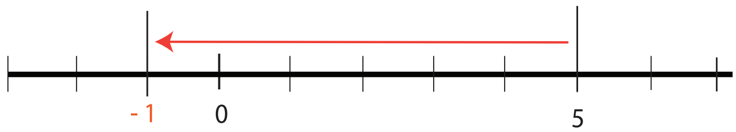
Example.3: **Positive+ Negative**

5 + (-6) is the calculation used to find the number that is -6 than 5. Therefore 5 + (-6) = -1.



Example.3: **Negative + Negative**

(-2) + (-6) is the calculation used to find the number that is -6 greater than -2. (-2) + (-6) = -8



Key Ideas

Pay attention to signs and absolute value when you are adding two numbers together. You can add the negative using the same idea like Adding positive.

LO9 : Addition with Positive and negative numbers (3)

Lesson Objective: To find the sum of two numbers with the same absolute value and opposite signs. **(7.1.12/3)**

Materials: Playing Cards

ASK-MT and Assessment	
Attitudes/Values	Appreciate the usefulness of addition of numbers in real life
Skills	To find the sum of two numbers with the same absolute value and opposite signs of whole numbers, decimals or fractions
Knowledge	The sum of two numbers with the same absolute value and opposite signs is same whether they are whole numbers, fractions or decimals
Mathematical Thinking	Analyse the sum of two opposite numbers that have the same absolute value
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Review

We can say the following about what we've studied so for

Addition with positive and negative numbers

$(+3) + (+5) = +(3+5)$

$(-3) + (-5) = -(3+5)$

(a) Sum of two numbers with same signs
Sign: Same as the two numbers.

Absolute value: Sum of the two numbers will have the sign of the larger absolute value.

$(+3) + (-5) = -(5-3)$

$(-3) + (+5) = +(5-3)$

(b) Sum of two numbers with opposite signs
Sign: Same as the signs of the number with the larger absolute value.
Absolute value: The difference when you take the smaller absolute value from the larger absolute value.

The sum of two numbers with the same absolute value and opposite signs are 0. The sum of 0 and any positive or negative number is the number itself.

Example.1: **Sum of two numbers with the same sign**

$(-12) + (-7) = -(12 + 7)$

$= -19$

Example.2: **Sum of two numbers with different signs**

(a) $(-7) + (+13)$

$= +(13-7)$

$= +6$

(b) $(+5) + (-15)$

$= -(15 - 5)$

$= -10$

Example.3: **Sums of decimals and fractions.**

(a). $(-4.7) + (+2.4)$

$= -(4.7 - 2.4)$

$= -2.3$

(b). $\left(-\frac{1}{2}\right) + \left(-\frac{1}{3}\right)$

$= -\left(\frac{1}{2} + \frac{1}{3}\right)$

$= -\frac{5}{6}$

L10: Laws of additions

Lesson Objective: To explain the meaning of Commutative and Associative laws of Addition **(7.1.1b.c)**
Materials: Number line chart

ASK-MT and Assessment	
Attitudes/Values	Appreciate the usefulness of Commutative and Associative laws of Addition
Skills	Explain the meaning of Commutative and Associative laws of Addition in calculation
Knowledge	Commutative and Associative laws of Addition
Mathematical Thinking	Examine the commutative and associative laws of addition
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Review

You already know that for any positive number, it is true that

$2+3=3+2$ $\qquad$ $(2+3)+4=2+(3+4)$

or

$a+b=b+a$ $\qquad$ $(a+b)+c=a+(b+c)$

These properties are called; the commutative law of addition and the associative law of addition. These properties hold when negative numbers are involve as well.

Key Ideas

There are four mathematical properties that involve addition. These are commutative, associative, additive identity and distributive properties. Commutative property is when two numbers are added, the sum is the same regardless of the order of addends. These properties also apply to the negative number

Exercises

(1) (4 + 3) (2) 5 + 6 (3) 7 + 3 (4) 6 + 7 (5) 5 + 2

(6) (8 + 9) + 3 (7) (4 + 6) + 8 (8) 5 + (2 + 7) (9) 8 + (9 + 2) (10) (8 + 7) + 20

L11 : Subtraction

Lesson Objective: To subtract positive and negative numbers of whole numbers, decimals and fractions. **(7.1.1.2/3)**

Materials: Playing Cards

ASK-MT and Assessment	
Attitudes/Values	Communicate ideas effectively with friends on how to subtract positive and negative numbers
Skills	Subtract positive and negative numbers
Knowledge	Subtraction of positive and negative numbers
Mathematical Thinking	Examine ways to subtract positive and negative numbers
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introductory

Explain why subtracting number (-5) – (+7) is the same as adding a positive number (-5) – (+7)

Example.1: **Subtraction**

(a) (-6) - (+10)

= (-6) + (-10)

= -16

(b) (-8) - (-3)

= (-8) + (+3)

= -5

Key Ideas

Subtraction with positive and negative numbers

When subtracting positive and negative numbers, you can add a number of the opposite sign instead of subtracting.

Exercises

Calculate.

(1) (-1.6) – (+ 0.6) (2) (+3.5) – (-2.3) (3) $(-\frac{1}{6})-(-\frac{5}{6})$ (4) $(+\frac{1}{2})-(-\frac{1}{3})$

L12: Calculation with addition and subtraction

Lesson Objective: To perform addition and subtraction with more positive and negative numbers. **(7.1.1.2/3)**
Materials: Number line chart

ASK-MT and Assessment	
Attitudes/Values	Appreciate the processes used to calculate positive and negative numbers
Skills	Set a math expression and calculate with addition and subtraction signs
Knowledge	Maths expressions of addition and subtractions and their calculations
Mathematical Thinking	Think about how to change maths expression with addition and subtraction to addition only and calculate
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Review

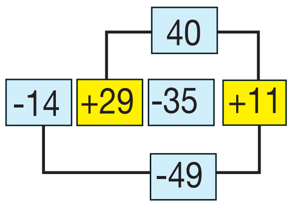
Calculate math expression (+7) - (+8) - (- 5) - (-9), with both addition and subtraction by changing to addition only.

TN: In this addition only math sentence: (+7) - (-8) - (-5) - (-9)

(+7 , -8 , -5, and +9) are called the terms of the math sentence. (+7 and +9) are positive terms, while -8 and – 5 are negative terms.

Example.1: **Calculation with both addition and subtraction**

-14 – (-29) + (-35) + 11
= -14 + 29 – 35 + 11
= 29 + 11-14 -35
= 40 - 49
= -9



Key Ideas

You can calculate math sentence with both addition and subtraction by changing them to addition only math sentences first then finding the sums of the positive and negative terms separately

Exercises

(1). 3- -4 (2) -2 + 8 -6 (3) 1-2 + 3-4 (4) -8 – 4 + (-1) – (-7) (5) -24 – (-15) + (-35) + 24

L13: Multiplying by a positive number

Lesson Objective: To explore multiplication of positive and negative numbers. **(7.1.1.2/3)**
Materials: Number line chart

ASK-MT and Assessment	
Attitudes/Values	Appreciate learning about multiplication of positive and negative numbers.
Skills	Discover and explain ways on how to multiply positive and negative numbers and their product.
Knowledge	Multiplication of positive and negative numbers.
Mathematical Thinking	Investigate ways on how to multiply positive and negative numbers.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introductory

What would happen if you expressed (-2) x 3 as addition?

If we think about a negative number multiply by a positive number in the same way;

$(-2) \times 3 = (-2) + (-2) + (-2) = -6$

-6 is equal to $-(2 \times 3)$

Example.1: **Negative number x positive number**

$(-4) \times 6 = -(4 \times 6)$
$= -24$

Key Ideas

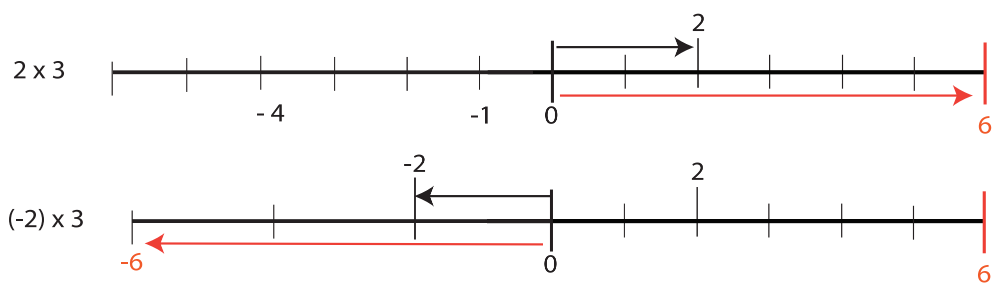
Calculate a negative number x a positive number by putting a negative sign to the product of their absolute values.

$(-2) \times 3$
$= -(2 \times 3)$

Exercises

(1) $(-3) \times 7$ (2) $(-6) \times 8$ (3) $(-12) \times 6$

(4) On a number line, multiplying a number by the positive number 3 means moving three times that number's distance from 0 in the same direction.



L14: Multiplying by a negative number

Lesson Objective: To explore ways on how to multiply by a negative number. **(7.1.1.2/3)**
Materials: Number line chart

ASK-MT and Assessment	
Attitudes/Values	Appreciate learning about multiplication of negative numbers.
Skills	Discover and explain ways on how to multiply negative numbers and their product.
Knowledge	Multiplication of negative numbers.
Mathematical Thinking	Investigate ways on how to multiply negative numbers.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

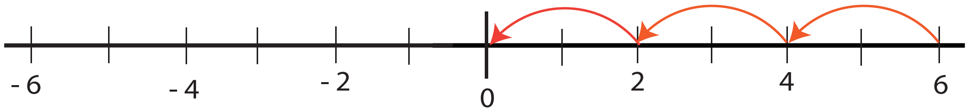
Teaching and Learning Activities

Let's look at a positive number multiplied by a negative number, like in the math sentence below.
(+2) x (-3)

$(+2) \times (+3) = +6$
$(+2) \times (+2) = +4$
$(+2) \times (+1) = +2$
$(+2) \times 0 = 0$
$(+2) \times (-1) = \boxed{}$
$(+2) \times (-2) = \boxed{}$
$(+2) \times (-3) = \boxed{}$

TN: The figure on the right shows multiplying a number by a positive number that decreases by 2 with each calculation.

When multiplier is 0, (+2) x 0 = 0.
If the multiplier decrease again by 1 in (+2) x (-1), the product should be two less than 0 or -2.



If we keep going, it should end up like this.

(+2) x (-1) = -2.......... - (2 x 1)
(+2) x (-2) = -4...........- (2 x 2)
(+2) x (-3) = -6...........- (2 x 3)

Example .1: **Positive number x negative number**

7 x (-5) = - (7 x 5)
= - 35

Example.2: **Negative number x a negative number**

(-8) x (-5) = + (8 x 5)
= 40

Key Ideas

- Calculate a positive number multiply by a negative number by putting a negative sign to the product of their absolute values.
- Calculate a negative number x a negative number by putting a positive sign to the product of their absolute values.
- You can think about multiplying a negative number by a negative in the same way as multiplying a positive number by a negative number.

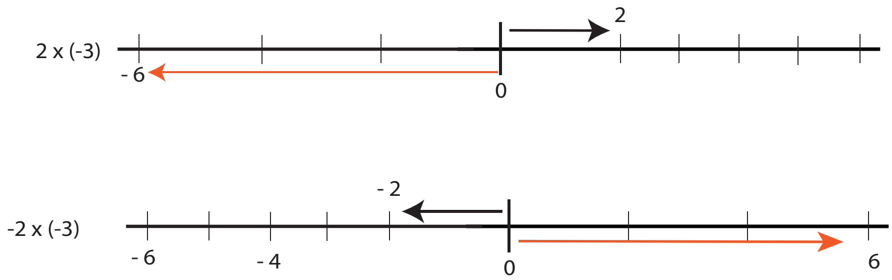
Teaching and Learning Activities

Exercises

(1) Calculate

(a). 5 x (-6) (b). 9 x (-8) (c). 10 x (- 10) (d). (-4) x (-9) (e). (- 8) x (- 7) (f). (-10) x (-10)

(2) On a number line, multiplying a number by the negative number, let's say -3 means moving three times that number's distance from 0 in the opposite direction by 2 in this case.



L15 : Dividing with positive and negative numbers

Lesson Objective: : To explore division of positive and negative numbers. **(7.1.1.2)**
Materials: Number line Chart

ASK-MT and Assessment	
Attitudes/Values	Enjoy dividing positive and negative numbers.
Skills	Investigate and perform calculation of division with positive and negative numbers.
Knowledge	Division of positive and negative numbers, their quotient and absolute values.
Mathematical Thinking	Think about how to write and express a positive or negative sign on the quotient of their absolute values.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introductory

When you calculate a positive number divide by a positive number, like 6 ÷ 2, you are looking for the missing number in □ x 2 = 6. You can think about dividing with negative numbers in the same way, where (-6) ÷ 2 means finding the missing number in the □ x 2 = - 6

What will the missing number be in the □ below?

(a) □ x 2 = -6 (b) □ x (-2) = 6 (c) □ x (-2) = 6

TN: From the above, we know that:

(-6) ÷ 2 = -3
6 ÷ (-2) = - 3
(-6) ÷ (- 2) = 3

Example.1: **Dividing with positive and negative numbers**

(a) (-12) ÷ 6 = - (12 ÷ 6) = -2 (b) (-28) ÷ (-4) =+ (28 ÷ 4) = 7 (c). 9 ÷ (-12) = - (9 ÷12) =-3/4

Key Ideas

Negative number ÷ positive number … Put a negative sign on the quotient
Positive number ÷ negative number of their absolute value
Negative number ÷ negative number …… Put a positive sign on the quotient of their absolute value.

Exercises

(1) (-56) ÷ (-7) (2) 15 ÷ (-21) (3) (-45) ÷ (-6)

L16: Multiplying and dividing by both positive and negative numbers

Lesson Objective: To explain the product or quotient of two decimal numbers with the same sign and absolute value. **(7.1.1.1/2)**

Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Appreciate multiplying and dividing decimals with positive and negative signs
Skills	Find the quotient of two decimal numbers with the same or different signs
Knowledge	Product of two decimal numbers with the same or different sign and absolute value
Mathematical Thinking	Think of ways on how to multiply and divide decimal numbers with positive and negative signs.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1: **Multiplying and dividing with decimals**

(a) (- 4.3) x (-0. 2)

= + (4.3 x 0.2)

= 0.86

(b) 3.2 ÷ (-4)

= - (3.2 ÷ 4)

= - 0.8

TN: The product of 0 and a positive number or 0 and a negative number is always 0. The quotient of 0 divided by a positive or negative is also 0. However, no number can be divided by 0 because the quotient will be undefined.

Multiplication and division with positive and negative numbers is the same whether they are whole number, decimal or fractions.

Key Ideas

Product or quotient of two numbers.

Product or quotient of two numbers with the same sign { Sign: Positive; Absolute value: Product or quotient of the two numbers

Product or quotient of two numbers with opposite signs { Sign: Negative; Absolute value: Product or quotient of the two numbers

Exercises

Calculate

(1) 9 x (-7) (2). (-5) x 4 (3). (-15) x 10 (4) 4 x (- 0.1)

(5) (-0.3) x (- 0.2) (6). (-0.7) x 10

L17: Dividing with positive and negative numbers

Lesson Objective: To multiply and divide with two or more fractions.
Materials:

ASK-MT and Assessment	
Attitudes/Values	Enjoy multiplying and dividing with fractions.
Skills	Multiplying and dividing with fractions.
Knowledge	Multiplication and division of positive and negative numbers, and reciprocal of positive and negative numbers.
Mathematical Thinking	Think about ways to multiply and divide positive and negative numbers, and reciprocal of positive and negative numbers.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

TN: Multiplying positive and negative numbers is done in the same way whether you are calculating with whole numbers or fractions.

Example.1: **Multiplying with fractions**

(a) $\left(-\frac{5}{6}\right) \times -\frac{4}{3}$

$= +\left(\frac{5}{6}\right) \times \frac{4}{3}$

$= -\frac{20}{18} = -\frac{10}{9}$

(b) $\left(-\frac{1}{3}\right) \times -\frac{5}{8}$

$+\left(\frac{1}{3}\right) \times \frac{5}{8}$

$= -\frac{5}{24}$

Example.2: **Finding a reciprocal of a negative number.**

$\left(-\frac{3}{4}\right) \times \left(-\frac{4}{3}\right) = 1$, so the reciprocal of $\left(-\frac{3}{4}\right)$ is $\left(-\frac{4}{3}\right)$

$(-4) \times \left(-\frac{1}{4}\right) = 1$ so the reciprocal of -4 is $\left(-\frac{1}{4}\right)$

You can also change division into multiplication with negative numbers, like this.

$5 \div \left(-\frac{3}{4}\right) = -\left(5 \div -\frac{3}{4}\right) = -\left(5 \times \frac{3}{4}\right) = 5 \times \left(-\frac{3}{4}\right)$

Example.3: **Dividing with fractions**

(a) $\left(-\frac{2}{3}\right) \div \left(-\frac{2}{5}\right)$

$= \left(-\frac{2}{3}\right) \div \left(-\frac{2}{5}\right)$

$= -\frac{10}{6}$

$= -\frac{5}{3}$

(b) $\left(-\frac{3}{5}\right) \div (-10)$

$= \left(-\frac{3}{5}\right) \times \left(-\frac{1}{10}\right)$

$= \frac{3}{50}$

Teaching and Learning Activities

Key Ideas

- A number is called the reciprocal of another number if the product of the two numbers is 1. It is the same for negative numbers.

Changing division to multiplication

- When dividing by a positive number or a negative number, you must multiply by the reciprocal of that number.

Exercises

1. What is the reciprocal of the following numbers?

(a) $-\frac{2}{5}$ (b) $-\frac{1}{6}$ (c) -3

2. Change division to multiplication and calculate.

(a). $\frac{5}{4} \div (-15)$ (b) $(-2/3) \div 1/6$ (c). $(-3/8) \div (-9/16)$

L18: Laws of multiplication

Lesson Objective : To apply commutative and associative laws of multiplication to solve problems.**(7.1.1.3)**
Materials:

ASK-MT and Assessment	
Attitudes/Values	Appreciate application of commutative and associative laws
Skills	Calculate and compare results of math expression which involve multiplication
Knowledge	Application of the commutative and associative laws of multiplications
Mathematical Thinking	Think about how to calculate and compare results of math expression based on commutative and associative laws
Assessment	Use the ASK-MT to assess the students learning progress during the lesson

Teaching and Learning Activities

Example.1 **Communicative and Associative law of multiplication**

When multiplying any positive number, it is true that;

2 x 3 = 3 x 2 (2 x 3) x 4 = 2 x (3 x 4)
Or
$a \times b = b \times a$ $(a \times b) \times c = a \times (b \times c)$

These properties are called the **commutative law of multiplication** and the **associative law of multiplication.** These properties hold when negative numbers are involved as well.

Since the math expression only involves multiplication, we can use the multiplication laws to change the order and calculate.

(-4) x 9 (-25) = 9 x (-4) x (-25) → Commutative law
= 9 x 100 → Associative law
= 900

Exercises

1. Calculate and compare the results of 3 x {(-4) x (-5) } and 3 x [(-4) x (-5)].

2. Calculate.

(a) 25 x 11 x (-2) (b) (-2) x 12 x (-15)

L19: Calculation with both multiplication and division

Lesson Objective: To multiply by three or more numbers and determine the sign for the result of calculation. **(7.1.1.3)**

Materials:

ASK-MT and Assessment	
Attitudes/Values	Enjoy calculation and determining the sign of the result
Skills	Identify the sign of the results from multiplying and dividing
Knowledge	Correct sign for the result of a calculation
Mathematical Thinking	Evaluate multiplying three or more numbers and the sign of the result
Assessment	Use the ASK-MT to assess the students learning progress during the lesson

Teaching and Learning Activities

Example 1: **Multiplying three or more numbers**

(a). (- 2) x 5 x 7 x (- 3)
= + (2 x 5 x 7 x 3)
= 210

(b). $\frac{3}{4} \times \left(-\frac{2}{5}\right) \times \frac{5}{3}$

$$= -\frac{3}{4} \times \frac{2}{5} \times \frac{5}{3}$$
$$= -\frac{1}{2}$$

TN: You can calculate math sentence with both multiplication and division by changing them to multiplication negative (-) only math sentences first and then determine the sign of the result.

Example.2: **Multiplying 3 or more numbers involving fractions**

$$(-27) \text{ x } (-\tfrac{2}{3}) \div (-9) = (-27) \text{ x } (-\tfrac{2}{3}) \text{ x } (-\tfrac{1}{9})$$
$$= -(27) \text{ x } (-\tfrac{2}{3}) \text{ x } (-\tfrac{1}{9})$$
$$= -2$$

Key Ideas

The sign of the result in a calculation that only involves multiplication is as follows;
If the number of negative signs is; $\begin{cases} \text{Even: +} \\ \text{Odd: -} \end{cases}$

Exercises

Calculate.

(1) (-4) x (-12) x (-5) (2). $\left(-\frac{3}{5}\right) \times \frac{5}{6} \times (-3)$ (c) (-12) x (-5) ÷3

(3) – 24 ÷ (-3) x 4

L20: Multiplying a number by it self

Lesson Objective : To apply commutative and associative laws of multiplication to solve problems.**(7.1.1.3)**
Materials:

ASK-MT and Assessment	
Attitudes/Values	Appreciate and share ideas on how to multiply a number by itself
Skills	Express and multiply a number by itself
Knowledge	Multiplication of a number by itself
Mathematical Thinking	Think about how to express and calculate number with exponents.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Review
You can express the product of a number multiplied by itself like this;

$5 \times 5 = 5^2$, $5 \times 5 \times 5 = 5^3$
5^2 is read as "5 to the second power" and 5^3 is read as "5 to the third power".

The small 2 or 3 to the upper right in 5^2 and 5^3 indicate the number of times 5 is multiplied by itself and is called **an exponent.**

The second power is sometimes called **"squared"** and the third power is sometimes called **"cubed".**

Example.1: **Expand and compare the products of $(-2)^4$ and -2^4.**

(a) $(-2)^4 = (-2) \times (-2) \times (-2) \times (-2)$
$= 16$

(b) $-2^4 = -(2 \times 2 \times 2 \times 2)$
$= -16$

Example.2: Calculations with exponents

$(-2)^3 \div (-3)^2 = (-8) \div 9 = -\frac{8}{9}$

Exercises

Calculate

(1) 4^2 (2) 3^3 (3) $(-3)^3$ (4) -5^3

(5) – 1.52 (6) $(-4)^3 \times (-7)$ (7) $(-6^2) \div (-2)^3$

L21 : Calculation with four arithmetic calculations

Lesson Objective: To calculate mixed math expression involving addition, subtraction, multiplication and division. **(7.1.1.1/2)**

Materials:

ASK-MT and Assessment	
Attitudes/Values	Enjoy solving problems with four arithmetic calculations
Skills	Perform calculations that include the four arithmetic calculations in order (BODMAS)
Knowledge	Order of operation that include the four arithmetic calculations (BODMAS)
Mathematical Thinking	Application of BODMAS in arithmetic calculation
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Review

Applying addition, subtraction, multiplication, and division of 3 or more numbers are called the four arithmetic calculations.

Example.1: **Calculations with both addition/subtraction and multiplication and division**

(a) 3 - x (- 2) x 5 = 3- (-10)

= 3 + 10

= 13

(b).(- 6) x 7 + 75 ÷ (-5^2)

= (- 6) x 7 + 75 ÷ (- 25)

= (- 42) + (-3)

= - 45

Example.2: **Calculations with parentheses**

3 x { -4 – (19 -8) }

= 3 x { -4 – 11}

= 3 x (-15)

= - 45

Key Ideas

Order of calculations **(BODMAS)**

- When a math sentence contains both addition/ subtraction and multiplication /division, per form the multiplication and division first.
- When a math sentence includes parentheses, the items inside the parentheses are normally calculated first.

Exercises

1. Calculate.

(a). -4 -6 x (-3) (b). -3 x (-7) – 9 x (-8) (c). 5 x (-12) + 14 ÷

2. Calculate.

(a) – 5 + (13 – 7) ÷ 3 (b). 7 – { (- 2) 2 – (9 – 14) }

L22: Distributive laws

Lesson Objective : To examine the distributive law and use in calculation **(7.1.1.1/2)**
Materials:

ASK-MT and Assessment	
Attitudes/Values	Appreciate and share ideas on distributive law
Skills	Apply distributive law to calculate and compare results
Knowledge	Distributive law and its applications
Mathematical Thinking	Think of ways to apply distributive law in calculations
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1: **Distributive laws**

The following is true for any numbers a, b, and c;

$(a + b) \times c = a \times c + b \times c$
$c \times (a + b) = c \times a + c \times b$

This property is called the **distributive law.**

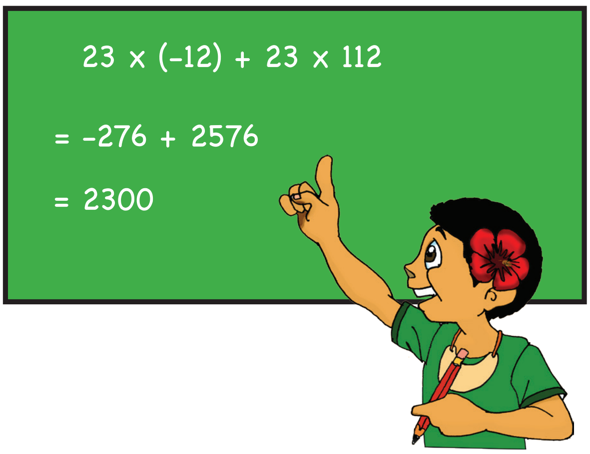
Exercises

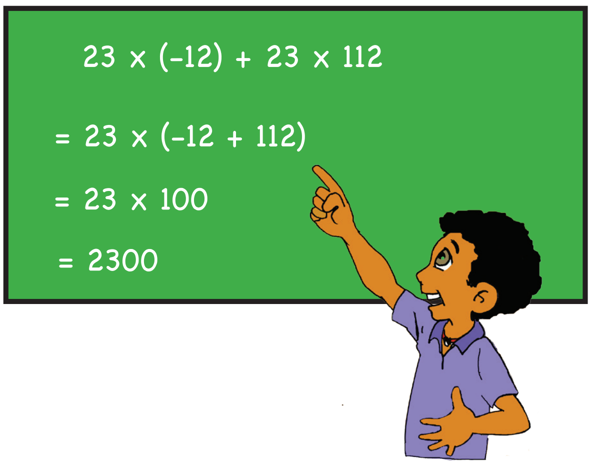
1. Calculate and compare the results of {3 + (-4)} x (-5) and 3 x (-5) + (-4) x (-5).

2. Apply the distribute law and calculate

(a) $\left(\frac{1}{3} + \frac{1}{2}\right) \times (-6)$ (b). $12 \times \left(-\frac{1}{3} + \frac{3}{2}\right)$

3. Koivi and Vaipa each calculated 23 x (-12) + 23 x 112 in a different way. Explain their Thinking.





L23: Expanding numerical range and the four Arithmetic calculations

Lesson Objective: To apply the four arithmetic calculations when the numerical range is expanded. **(7.1.1.2/3)**

Materials:

ASK-MT and Assessment	
Attitudes/Values	Enjoy learning how the four arithmetic calculations apply when numerical range is expanded
Skills	Calculate using the four-arithmetic calculations when numerical range is expanded
Knowledge	The application of the four arithmetic operations in the expansion of numerical range
Mathematical Thinking	Think of how the four arithmetic operations apply when numerical range is expanded
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Review

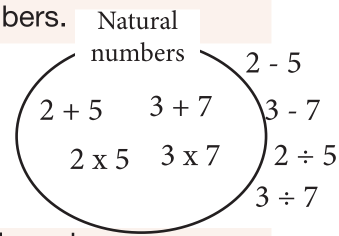
You have two cards, one with a 2 and one with a 5. When you place them in the following calculations, which calculation always produces a natural number?

(a) □ + □ (b) □ - □ (c) □ × □ (d) □ ÷ □

Example.1: **Addition and multiplication of natural**

When a and b are natural numbers, the answer to $a + b$ and $a \times b$ will always be natural a number. Based on this we can say the following;

Addition and multiplication of natural numbers always produce natural numbers.



However, the answer to $a - b$ or $a \div b$ may not be a natural number, as in

$2 - 5 - 3$ $\quad 2 \div 5 = \frac{2}{5}$

Based on this we can say the following;

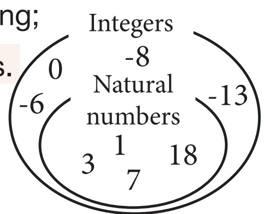
Subtraction and division of natural numbers will not always produce natural number.

Example.2 **Addition, multiplication, and subtraction with integers**

If you had a □ and □ card, which of the calculation (a) through (d) would always produce an integer?

If the range is expanded from natural numbers to integers, we can say the following;

Addition, multiplication, and subtraction can always be performed among integers.



However, division cannot always be performed among integers.

For example, the answer to $3 \div 5 = 3/5$ is not an integer.

TN: Every natural number together is called the set of natural numbers. Every natural number (positive integer), 0, and negative whole numbers are called the set of Integers. If we include decimals and fractions in the group, we have the set of Real numbers (All numbers in the diagram change to real numbers).

Teaching and Learning Activities

Key Ideas

We can say the following is based on what we've studied so far;

- Addition and multiplication can always be performed within the set of natural numbers
- Addition, multiplication, and subtraction can always be performed within the set of integers.
- All four arithmetic calculations can always be performed within the set of all numbers.

Exercises

Which of the four arithmetic calculations (addition, subtraction, multiplication, and division) can you always perform within the set of natural number, the set of integers, and the set of all numbers? Mark the calculations you can always perform in each set with a O in the table below, and the ones you cannot perform with a Δ. Exclude dividing by O.

	Addition	Subtraction	Multiplication	Division
Set of natural numbers				
Set of integers				
Set of all numbers				

Unit Checkpoint

Review on positive and negative numbers

Review on Positive and Negative numbers

Expressing positive and negative numbers

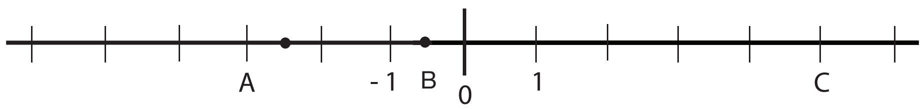
1. Express the following numbers with a positive or negative sign.

   (a) The number 8 less than 0 (b) The number 15 greater than 0

2. What numbers are marked by **A,B,** and **C** on the number line below?

   Represent the following numbers on the number lines as well

   $-5, -3.5 \frac{1}{2}$



Using positive and negative numbers to express amount

3. Use the word in [ ] to express the following.

   (a) 6 under [over] (b) -4 greater [less]

Absolute value

4. What is the absolute value of -3?

Size of positive and negative numbers

5. Use an inequality sign to express the relative size of the numbers below.

   (a) 4,-6 (b) -7,-8 (c) -0.1,0

Addition and Subtraction with positive and negative numbers

6. Calculate.

   (a) (-6) + (+4) (b) (+5) – (+9) (c) (-3) + (-7)

   (d) (+9) - (-6) (e) -2+5-8 (f) 7+(-11)-(-5)

Multiplication and Division with positive and negative numbers

7. Calculate.

   (a) 3 × (-2) (b) (-3) × (-2) (c) (-8)÷ 2

   (d) (-8)÷ (-2) (e) (-3) × (-2) × (-5) (f) 30 ÷ (-5) × (-2)

Mixed Calculation

8. Calculate.

   (a) 3^4 (b) $(-3)^2$ (c) 3^4 (d) 6 - 12 ÷ (-3) (e) 6 - 3 × (7-4)

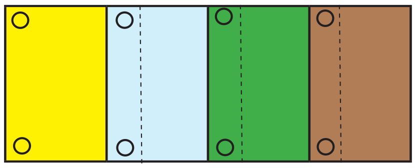
L24 : Using letters to express quantity

Lesson Objective: To use letters to represent different quantities like numbers of objects or cost. **(7.1.3.1)**
Materials: picture charts

ASK-MT and Assessment	
Attitudes/Values	Confidently share ideas on the use of Algebraic Expressions
Skills	Represent different quantities with numbers, words and symbols and interpret the expressions
Knowledge	Representation of the relationships and rules of numbers and quantities in algebra
Mathematical Thinking	Think of ways to express, simply and represent algebraic expressions
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introductory
In the given situation, the number of magnets needed as the number of pictures increases from 1 to 2 and the 3 can be expressed like this:



Different pictorial objects can be used to expressed one idea in different ways.

When there is 1 picture 2 x 1 x 2 (magnets)
When there are 2 pictures 2 x 2 x 2 (magnets)
When there are 3 pictures 2 x 3 x 2 (magnets)

Number of pictures	Number of magnets
1	2 x 1 + 2
2	2 x 2 + 2
3	2 x 3 + 2
4	
5	
6	
⋮	⋮

What expressions can be used to indicate the number of magnets needed to put up 4, 5, and 6 pictures? Complete the table on the right.

Example.1: **Expressing quantities with two different letters**

To find the total cost of a 12 kina notebooks and b 3 kina ballpoint pens, a notebooks when 1 notebook is 12 kina is 12 x a kina and b ballpoint pens when 1 pen is 3 kina is 3 x b kina

Therefore, if we put them together, we get the expression

12 x a + 3 x b kina

Exercises

Write an expression to indicate the following quantities.

(1) The total amount of kina when there are x 10 kina coins and y 1 kina coin.

(2) The number of people that can sit in an airplane with a rows of 2 seats and b rows of 3 seats.

L25: How to write algebraic expressions

Lesson Objective : To explore and apply the rules for writing algebraic expressions. **(7.1.3.1/2)**
Materials:

ASK-MT and Assessment	
Attitudes/Values	Share ideas on the use of algebraic expressions
Skills	Represent different quantities using pronumerals and apply the rules for writing algebraic expressions.
Knowledge	Ways of representing the relationships and rules of numbers and quantities in algebra
Mathematical Thinking	Think of ways to simply and represent algebraic expressions
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1: **How to express products**

(a) $a \times b = ab$ (b) $a \times 4 = 4a$ (c) $a \times a = a^2$ (d) $(a + b) \times 2 = 2\ (a+b)$

TN: $b \times a$ would normally be , but we write it in regular alphabetic order as ab.
For 1 x a, we would normally take out the X symbol and write 1a, but we write it as just a.
(-1) x a is written as – a

Example.2: **How to express quotients**

a) $a \div 5 = \frac{a}{5}$ b) $(a + b) \div 5 = \frac{a+b}{5}$

Important $\div$ 5 is the same operation as $\times\frac{1}{5}$, so you can write $\frac{1}{5}a$ *and* $\frac{a+b}{5}$ *as* $\frac{1}{5}(a + b)$

Example.3 **Expressions that do not use × or ÷ symbols**

$$6{\times}a + b \div 3 = 6a + \frac{b}{3}$$

Key Ideas

Use this rule to indicate products and quotients in an expression that uses letters
How to write algebraic expression (products)

1. Take out the x multiplication symbol
2. When showing the product of a letter and a number, write the number in front of the letter.
3. When showing the product of a letter and itself, use exponents

How to write algebraic expressions (quotients)

4. Do not use the ÷ symbol. Write the expression in fraction form.
When showing the product of a letter and a number, write the number in front of the letter.
When showing the product of a letter and itself, use exponents.

Teaching and Learning Activities

1. Write the following expressions using the rules for writing algebraic expressions.

   (a) $50 \times n$ (b) $x \times 8$ (c) $y \times (-1) \times x$ (d) $c \times c \times c$ (e) $3 \times a \times a \times b$

2. Rewrite the following expressions using the x symbol.

   (a) $7ab$ (b) $2\ xy^2$

3. Write the following expressions in fraction form.

   a) $x \div 2$ b) $3 \div y$

4. Rewrite the following expressions using the ÷ symbol

   (a) $\frac{a}{3}$ (b) $\frac{8}{t}$

L26: Algebraic Expressions and quantity

Lesson Objective: To use expressions to indicate various quantities using the rules for algebraic expressions. **(7.1.3 2 /4)**

Materials:

ASK-MT and Assessment	
Attitudes/Values	Appreciate and share their ideas on the use of Algebraic Expressions
Skills	Represent different quantities using pronumerals and apply the rules for writing algebraic expressions
Knowledge	Relationships and rules of numbers and quantities in algebra
Mathematical Thinking	Think of ways to simply and represent algebraic expressions
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1: **Total cost and change**

You buy 6 pieces of cake with 50 kina. Each piece of cake costs x kina. You can express your change as

50 kina – total cost

The total cost is

$x \times 6 = 6x$ (kina)

Therefore, you can express your change like this

$50 - 6x$ (kina)

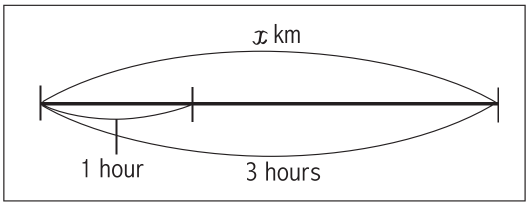
Example.2: **Speed, Time and travel distance**

A school trip takes students on an x km long hiking course that takes 3 hours to walk. The students' walking speed can be calculated using

Travel distance ÷ Time

so it can be expressed like this

$x \div 3 = \frac{x}{3}$ (km/h)



Example.3 : **Relative amount**

The area of a park is a m^2, and a pond covers 7% of it. The relative amount 7% can be expressed as the fraction $\frac{7}{100}$, so the area of the pond can be expressed like this;

$$a \times \frac{7}{100} = \frac{7}{100}a(m^2)$$

Important 7% is expressed as the decimal 0.07, so we can also express it as $a \times 0.07 = 0.07a(m^2)$

Teaching and Learning Activities

Example.4: **The Meaning of expressions**

The entrance fee for an art museum is a kina for adults and b kina for children.
The expression;

$$2a + 3b \text{ (kina)}$$

shows the total entrance fee for 2 adults and 3 children

Exercises

1. Write an expression to indicate the following quantities

   (a). The money left over when 4 people each chip in a kina to buy something that costs 500 kina.

   (b). The total cost when you buy $3x$ kina pineapples and $5y$ kina oranges.

2. Write and expression to indicate the following quantities.

   (a) The travel distance you can run in x hours at a speed of 4 km per hour.

   (b) The time it takes to walk to a town x km away at a speed of 2 km per hour.

3. Write an expression to indicate the following quantities.

   (a) 47% of a piece of land with an area of a m^2

   (b) The total cost when purchasing an item regularly priced at a kina at 3/10 off.

4. What do the following expressions indicate?

   (a). $3a$ kina (b). $a + 2b$ (kina) (c). $a - b$ (kina)

L27: The value of expressions (1)

Lesson Objective : To Find the value of the pronumerals by substitution method. **(7.1.3 2./4)**
Materials:

ASK-MT and Assessment	
Attitudes/Values	Collaborate with others and share their ideas on algebraic expressions
Skills	Represent different quantities using pronumerals, apply the rules and write algebraic expressions
Knowledge	Be able to understand how to represent the relationships and rules of numbers and quantities in algebra
Mathematical Thinking	Be able to think of ways to simply and represent algebraic expressions
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introductory

Replacing a number for a letter in an expression is called Substitution. A number that is substituted for a letter is called the **Pronumeral**, and the result of calculating with the substitution is called the **value of expression.**

Let's find values in a variety of expressions.

Example 1: **Value of 6 – 4x**

(a) When $x = 2$

$6 - 4x = 6 - 4\times2$
$= 6 - 8$
$= -2$

(b) When $x = 2$

$6 - 4x = 6 - 4\times2$
$= 6 - 8$
$= -2$

Example 2: **Value of – x**

When $x = -3$,

$-x = (-1)\times x$
$= (-1)\times(-3)$
$= 3$

Example 3: **Value** $\frac{6}{x}$

When $x = -2$,

$\frac{6}{x} = 6 \div x$
$= 6 \div (-2)$
$= -3$

Exercises

1. Find the value of 12 - 2x when the value of x is as follows.

(a) $x = 7$ (b) $x = -8$

2. Find the value of -x - 2 when the value of x is as flows.

(a) $x = 3$ (b) $x = -5$

3. Find the value of the following expressions when $x = -3$

L28: The value of expressions (2)

Lesson Objective: To substitute numbers for two or more letters in expressions to find their values. **(7.1.3.2/4)**

Materials:

ASK-MT and Assessment	
Attitudes/Values	Share their ideas on substituting numbers for two or more letter in expressions to find their value
Skills	Substitute numbers for two or more letter in expressions to find their value
Knowledge	Substitution of numbers for two or more letter in expressions to find their value
Mathematical Thinking	Think of ways to substitute numbers for two or more letter in expressions to find their value
Assessment	use the ASK-MT to assess the students learning progress during the lesson

Teaching and Learning Activities

Example 1: **Value of a^2,value of-a^2**

When $a = -3$,

$$a^2 = (-3)^2$$
$$= (-3)\times(-3)$$
$$= 9$$

When $a = -3$,

$$-a^2 = -(-3)^2$$
$$= -\{(-3)\times(-3)\}$$
$$= -9$$

Example 2. Value of $\mathbf{3x + 2y}$

When $x = 5$ and $y = 4$,

$$3x + 2y = 3\times5 + 2\times4$$
$$= 15 + 8$$
$$= 23$$

Example 3: Value of $\mathbf{-5x - 6y}$

When $x = 3$ and $y = -2$

$$-5x - 6y = (-5)\times3 - 6\times(-2)$$
$$= -15 + 12$$
$$= -3$$

Key Ideas

When there are 2 or more letters, you can still find the value of the expression in the same way.

Exercises

1. Find the value of a^2 when the value of a is as follows.

   (a) $a = 6$ (b) $a = -2$

2. Find the value of -x^2 when the value of x is as follows.

   (a) $x = \frac{1}{2}$ (b) $x = -1$

3. Find the value of the following expressions when $x = -2$, and $y = 6$

   (a) $2x + y$ (b) $4x - 3y$ (d) $\frac{3}{2}x + y$

L29: Terms and coefficients in algebraic expressions

Calculating algebraic expressions

Lesson Objective : To define terms and coefficients in algebraic expression. **(7.1.3.1 /3)**
Materials:

ASK-MT and Assessment	
Attitudes/Values	Show interest in learning about terms and coefficients in algebraic expressions.
Skills	Define and represent terms and coefficients in an expression.
Knowledge	Terms and coefficients in an algebraic expression.
Mathematical Thinking	Think of about how to state terms of expression and the coefficient of terms that contains letters.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Review

The expression $3x + 1$ represents the sum of $3x$ and 1. The items being joined by the + addition sign, $3x$ and 1 are called **Terms** of the expression $3x+1$. In the expression $3x + 1$, the term $3x$ is in the form of a product of a number and a letter-expressed another way, $3 \times x$. In situations like this, the 3 is called the **coefficient** of x.

$\underline{3x} + \underline{1}$
Term Term

Example 1. **Terms and Coefficients (1)**

$x - 4y + 2$, so the terms are x,$-4y$ and 2.

The coefficient of x is 1, and the coefficient of y is – 4.

$x = 1 \times x$
coefficient

Example 2. **Terms and Coefficients** (2)

The terms of $\frac{a}{3} - b$ are $\frac{a}{3}$ and $-b$.

$\frac{a}{3} = \frac{1}{3}a$, so the coefficient of a is $\frac{1}{3}$

$-b = (-1) \times b$, so the coefficient of b is – 1.

Key Ideas

Terms like $5x$ and $-4y$ that contain only one letter are called terms of the first degree.

Expressions that contain only terms of the first degree or that indicate the sum of terms of the first degree and numbers are called the expression of the first degree.

Expressions of the first degree
$5x$, $-4y$
$4x + 3$
$2x - 3y + 1$

Exercises

State the terms of the following expressions. State the coefficients of terms that contain a letter.

(1) $9 - 2x$ (2) $\frac{x}{4} - 3y$ (3) $a - b + 8$

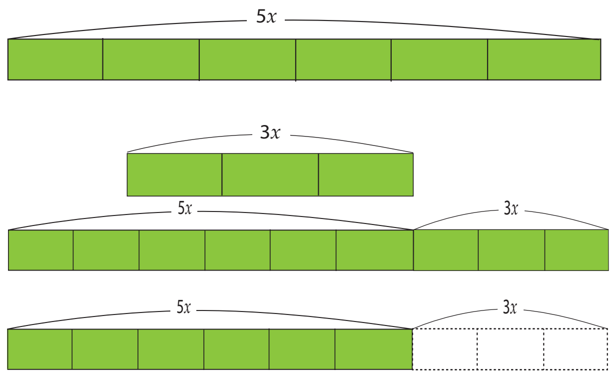
L30 : Simplifying Expression

Lesson Objective: To apply the laws of calculation and simplify expression. **(7.1.3.4)**
Materials:

ASK-MT and Assessment	
Attitudes/Values	Share their ideas on how simplify expressions.
Skills	Apply the laws of calculation to simplify expression.
Knowledge	Application of the laws of calculation to simplify expression.
Mathematical Thinking	Think of ways on how to simply expressions.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Let's look at simplifying expressions. Kila bought 5 clear folders and Karin bought 3 clear folders. Each folder costs x kina. Use an expression to indicate the total cost of the folders that the two students bought. Also use and expression to indicate the difference in the cost for the two students.



In $5x + 3x$ and $5x - 3x$,
$5x$ is 5 times x and $3x$ is 3 times x.

Therefore,
$5x + 3x = (5 + 3)x$
$= 8x$
$5x - 3x = (5-2)x$
$= 2x$

Each expression follows the laws of calculation.

$$mx + nx = (m+n)\,x$$

When $m = 5$, $n = 3$ and $m = 5$, $n = -3$.

We can use these calculation rules to simply the expression.

Example.1: **Combining terms with the same letter portion**

(a) $-3x + 2x$
$= (-3 + 2)x$
$= -\,x$

(b) $7x - x$
$= (7 - 1)\,x$
$= 6x$

Example. 3: **Simplify by removing the parentheses**

(a). $3x + (5x + 2)$
$= 3x + 5x - 2$
$= 8x - 2$

(b) $-3x + 2x$
$= (-3 + 2)x$
$= -\,x$

Teaching and Learning Activities

Example. 3: **Simplify by removing the parentheses**

(a). $3x + (5x + 2)$
$= 3x + 5x - 2$
$= 8x - 2$

When there is a + in front of the parenthesis, you can simply take terms out of the parentheses and express their sum.

(b). $3x - (5x + 2)$
$= 3x - 5x + 2$
$= -2x + 2$

When there is a – in front of the parentheses, you switch the sign of each term in the parentheses and then express their sum.

Key Ideas

- You can simplify an expression like $8x + 4 - 6x + 1$ by combining the terms with the same letter portion and the terms that are just numbers.
- Simplify by removing the parentheses. You can remove the parenthesis in expressions like these.

$a + (b + c) = a + b + c$ $\qquad$ $a - (b + c) = a - b - c$

Exercises

1. Simplify the following expressions.

(a) $6x - 2x$ (b) $x - 8x$ (c) $3/5\ x + 1/5x$

2. Simplify the following expressions

(a) $6x + 4 + 3x$ (b). $-5x + 7 + 4x$ (c). $2x - 8 - 4x + 7$

3. Remove the parentheses and simplify the following expression.

(a) $2x + (5 - x)$ (b). $6y - 3 + (-4y - 3)$ (c). $4x - (x + 5)$

L31: Adding Expressions and Subtracting Expressions

Lesson Objective : To add and subtract two algebraic expression. **(7.1.3.3/4)**
Materials:

ASK-MT and Assessment	
Attitudes/Values	Share their ideas on the use of Algebraic expressions.
Skills	Represent different quantities using pronumerals and apply the rules for writing algebraic expressions.
Knowledge	The relationships and rules of numbers and quantities in algebra.
Mathematical Thinking	Think of ways to simply and represent algebraic expressions.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Examples.1 : **Adding expressions**

Add $7x + 6$ to $3x - 4$
$(3x - 4) + (7x + 6)$
$= 3x - 4 + 7x + 6$
$= 10x + 2$

Examples.2 **: Subtracting expressions**

Subtract $7x + 6$ from $3x - 4$
$(3x - 4) - (7x + 6)$
$= 3x - 4 - 7x - 6$
$= -4x - 10$

Key Ideas

You can add or subtract two expressions by enclosing each expression in parentheses and linking them with + or – signs. Then remove the parentheses and simplify.

Adding two expressions
( ) + ( )
Subtracting two expressions
( ) - ( )

Exercises

1. Add the following pairs of expressions. Then subtract the expression on the right of each pair from the expression on the left.

(a) $5x + 9$,$6x - 1$ (b) $4x - 2$,$x - 2$ (c) $-3y + 4$,$y - 8$ (d) $7x - 5$,$-7x + 6$

2. Calculate.

(a) $3a - (5a - 1)$ (b) $2x + (3x - 4)$ (c) $-2a + 7 - (6a - 7)$ (d) $3x - 9 - (2x + 1)$

L32: Multiplying and dividing expressions with one term

Lesson Objective: To calculate an expression that involve multiplying and dividing an algebraic expression by a number. **(7.1.3.2/4)**

Materials:

ASK-MT and Assessment	
Attitudes/Values	Share their ideas on how multiplying and dividing an algebraic expression.
Skills	Represent different quantities using pronumerals and apply the rules for writing algebraic expressions.
Knowledge	Be able to understand how to represent the relationships and rules of numbers and quantities in algebra.
Mathematical Thinking	Think of ways to simply and represent algebraic expressions.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example 1. **Algebraic expression x number**

(a) $2x \times 5$

$= 2 \times x \times 5 = 2 \times 5 \times x$

$= 10x$

(b) $6x \times (-3)$

$= 6 \times x \times (-3)$

$= 6 \times (-3) \times x$

$= -18x$

Example 2. **Algebraic expression ÷ number**

(a) $12x \div 3 = \dfrac{12x}{3}$

$= \dfrac{12x \text{ x } x}{3}$

$= 4x$

$\boxed{a \div b = \dfrac{b}{a}}$

(b) $4x \div (-\frac{2}{5}) = 4x \text{ X } (-\frac{2}{5})$

$= 4 \text{ X } (-\frac{2}{5}) \text{ X } x$

$= 10x$

$\boxed{a \div \dfrac{n}{b} = a \text{ X } \dfrac{m}{n}}$

Key Ideas

- Let's about calculations that involve multiplying as algebraic expression by a number or dividing and algebraic expression by a number.
- You can change the multiplication order so that you're multiplication numbers with number.
- You can take an expression with 2 or more terms and multiply or divide it by a number using calculations.

$m(a + b) = ma + mb$ $\qquad$ $\dfrac{(a+b)}{m} = \dfrac{a}{m} + \dfrac{b}{m}$

Exercises

Calculate.

(a) $3x + 2$ (b) $4x \times (-7)$ (c) $-x + 9$ (d) $18x \div 6$ (e) $10x \div (-5)$

(f) $-12x \div (-4)$

L33: Multiplying and dividing expressions with two or more terms

Lesson Objective : To calculate expressions with two or more terms or in a fraction form by a number. **(7.1.3.2/4)**

Materials:

ASK-MT and Assessment	
Attitudes/Values	Share their ideas on how to calculate expressions with two or more terms.
Skills	Calculate expressions with two or more terms in it by a number.
Knowledge	Algebraic expression multiplying and dividing by a numbers.
Mathematical Thinking	Think of about ways on how to calculate expressions with two or more terms in it by a number.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example 1. **Multiplying an expression with 2 or more terms by a number**

(a) $3\ (4x+5) = 3 \times 4x + 3 \times 4$
$=12x + 15$

$\boxed{3\ (4x + 30)}$

(b) $(2x - 4) \times (-5) = 2x \times (-5) + (-4) \times (-5)$
$= -10x + 20$

$\boxed{(2x - 4)\ \text{x}(-5)}$

(c) $(9x - 6) = \quad (-5) = \ \times 9\ (-5) + (-4) \times (-5)$
$= -10x + 20$

Example 2: **Dividing an expression with 2 or more terms by a number**

(a) $(15x + 30) \div 5 = \frac{15x}{5} + \frac{30}{5} = 3x + 6$

(b) $(18x - 21) \div \frac{2}{3} = (18x - 21)\ \text{x}\ \frac{2}{3}$
$= 12x - 14$

$$\boxed{\begin{aligned}&(15x + 30) \div 5\\ &= \frac{15x + 30}{5}\end{aligned}}$$

Example 3: **Multiplying an expression in fraction form by a number**

$\frac{5x + 3}{2}\ \text{x}\ 6 = (5x +)\ \text{x}\ 3$
$= 15x + 9$

$$\boxed{\frac{(5x + 3)\ \text{x}\ \cancel{6}^{3}}{\cancel{2}_{1}}}$$

Example.4: **Calculating Expressions with parentheses.**

$3(2x + 1) - 4(x - 7)$
$= 6x + 3 - 4x + 28$
$= 2x + 31$

Exercises

1. Calculate.

(a) $(4x + 8) \div 2$ (b) $(6x - 15) \div (-3)$ (c) $(-\frac{3}{2}x + 4) \div 4$

2. Calculate.

(a) $\frac{2x + 3}{4}\ \text{x}\ 8$ (b) $15\ \text{x}\ \frac{3x - 10}{5}$ (c) $\frac{-3x - 5}{8}\ \text{x}\ (-6)$

3. Calculate.

(a) $8(x - 2) + 4(2x + 6)$ (b) $6(a + 5) + 3(a - 10)$ (c) $5(x - 3) - (x + 1)$ (d) $\frac{1}{2}(2x-4)-3(x+1)$

L34: Expressions showing equal relationships

Lesson Objective: To understand the expressions showing relationships of equality expressions. **(7.1.3 .2/3/4)**

Materials:

ASK-MT and Assessment	
Attitudes/Values	Share their ideas on how to represent the relationship of Expressions and rules of numbers and quantities.
Skills	Represent different quantities using pronumerals and apply the rules for writing algebraic expressions.
Knowledge	Be able to understand how to represent the relationships and rules of numbers and quantities in algebra.
Mathematical Thinking	Evaluate multiplying three or more numbers and the sign of the result.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example 1: **Equalities showing equal quantities** 1

A boy is a cm tall. He is 4 cm taller than his younger brother, who is b cm tall. In this case, we express the relationship between the two quantities like this.

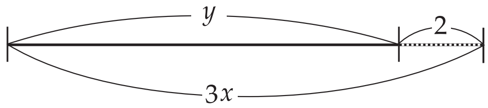
$$a = b + 4$$

Because the difference between the older and younger brother's height is 4 cm, we can express the relationship between the two quantities like this as well.

$$a - b = 4$$

Example 2: **Equalities showing equal quantities** 2

You have y pencils. You want to distribute them to x people so that each person gets 3 pencils, but you are 2 pencils short. In this case,



The number of pencils you have now: y pencils.
The number of pencils you need to distribute them evenly: $3x$ pencils.

y Pencils is 2 pencils less than $3x$ pencils, so the relationship between these two quantities can be expressed like this.

$$y = 3x - 2$$

Key Ideas

An expression like this that uses the equal sign = to indicate and equivalent relationship between two quantities is called **equality.** The expression to the left of the equal sign is called the **left side** of and equality, and the expression to the right is called the **right side** of the equality. Together they are referred to as both sides of the equality.

Exercises

Express the relationship between the quantities below with an equality.

(1) The total cost of $3x$ Kina tennis balls is y Kina .
(2) You get b Kina in change when you use K1000 to pay for a ticket that costs a Kina.

L35: Expressions showing relative size

Lesson Objective : To understand the expression showing relationship of inequality expressions.**(7.1.3.1/2/4)**
Materials:

ASK-MT and Assessment	
Attitudes/Values	Share their ideas on the use of algebraic expressions.
Skills	Represent different quantities using pronumerals and apply the rules for writing algebraic expressions.
Knowledge	Be able to understand how to represent the relationships and rules of numbers and quantities in algebra.
Mathematical Thinking	Think of ways to simply and represent algebraic expressions.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example 1: **Expressing relationships using ≥ and ≤**
A box 2 kg have several 3kg objects inside. You want to keep the total weight to 20 kg or less. If the number of objects is represented as x objects, we can express the relationship this way.

$$3x + 2 \leq 20$$

Example. 2: **The meaning of expressions showing relationship**
The entrance fee at an aquarium is a kina for adults and b kina for children. The inequality

$$2a + 3b \leq 8000$$

Indicates that the total cost for 2 adults and 3 children is 8000 kina or less.

Key Ideas

An expression like this that indicates the relative size of two quantities is called an **inequality.** The left side of an inequality and the expression to the right is called the **right side** of the inequality. Together they referred to as both sides on the inequality.

Inequality

$$\underbrace{\underbrace{8a}_{\text{Left side}} < \underbrace{2b + 3000}_{\text{Right side}}}_{\text{Both sides}}$$

In addition to > and <, ≥ and ≤ are also inequality signs. For any numbers a and b, the phrase "a is at least b" means that either $a>b$ or $a = b$

When we use the symbols ≥ and ≤ to express this as $a \geq b$ or $b \leq a$

Exercises

1. Express the relationship between the quantities below with an inequality.

   (a) 4 people each chip in x kina for a total of at least 1000 kina.
   (b) A girl has a kina and her younger sister has b kina. Together they are able to buy something that costs 1200 kina

2. What do the following expressions from example.2 indicate?

   (a) $2a + b = 5000$ (b) $a - b = 700$ (c) $a + 2b > 3500$ (d) $3a \leq 7b$

Unit Checkpoint

Review on Positive and Negative numbers

Review on Positive and Negative numbers

1. Rewrite the following expressions according to the rules for writing algebraic expressions.

   (a) $25 \times a$ (b) $-x \times y \times x$ (c) $x \div 3$ (d) $(m+n) \div 2$

2. Write the following expressions using × and ÷ symbols.

   (a) $8a + 3b$ (b) $4(x+y) - \frac{z}{5}$

How to write algebraic expression

3. Write an expression to indicate the following quantities.
   (a) The total cost of 5 juice cartons priced at x kina each.
   (b)The cost of one pencil when 12 pencils cost x kina.

Using expressions to indicate quantity

4. Find the value of the following expressions when x =-3
   (a) $5x + 2$ (b) $4-7x$

Absolute value

5. Calculate.
   (a) $9x - 2$ (b) $-8x + 3x$ (c) $5x + 7 + 3x$
   (d) $-2a - 3 - 8a$ (e) $7a + 4 + 3a - 5$ (f) $9y - 8 - 4y + 7$

Addition and subtraction with algebraic expression

6. Calculate.
   (a) $2x \times (-2)$ (b) $-9x \div \frac{3}{2}$
   (c) $-2(4x-3)$ (d) $(-12x + 8) \div (-2)$

Multiplication and division with algebraic expression and numbers

7. Express the relationship between the quantities below with an equality or inequality.

   (a) The sum is 12 when 6 is added to a number x
   (b) There are 3 pencils left over when a pencils are divided among b people so that Each person gets 5 pencils.

Expression showing relationship

L36: Equations and their solutions

Equations

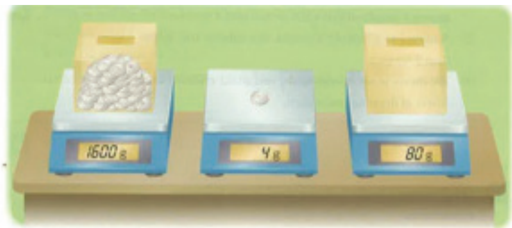
Lesson Objective : To understand the equations and how to find their solutions **(7.1.3.1/2)**
Materials:

ASK-MT and Assessment	
Attitudes/Values	Be able to share ideas on how to find the value of a letter in equality
Skills	Be able to find the value of a letter in equality
Knowledge	Understand equations and their solutions
Mathematical Thinking	Be able to think about how to find the value of a letter in equality.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

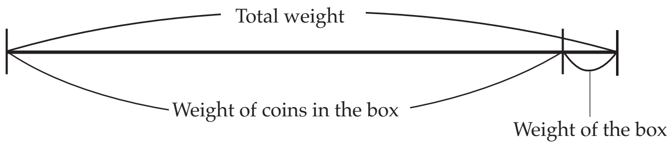
Teaching and Learning Activities

Introductory

The students at Wardstrip primary school held a fundraising to collect 50 toea coins for people who were affected by earth quake at Hela Province. Aaron wanted to find out how many coins were in the coin box without opening it so he decided to weigh the box. Find the number of coins in the box using this weight relationship



(a) Think about using a diagram
Diagram expressing weight relationships



The weight of the coin in the box is, 2500g - $\square$ = $\square$ (g)

Since one coin weighs 5g, the number of coins in the box is

$\square \div \square = \square$ (coins)

(b) Let x be the number of coins and write an equality expressing weight relationships
When the number of coins in the box is x coins, the weight relationships are;

(weight of x coins) + (weight of box) = (total weight).
Therefore we can create the equality.

$\square$

Teaching and Learning Activities

Example.1 **How to find solutions to equation**

Find out whether 4 is the solution to the equation; $2x - 3 = x + 1$.
if we substitute 4 for x,
Left side $2 \times 4 - 3 = 5$ and right side $4 + 1 = 5$
The left and right sides of the equation are equal, so 4 is the solution to this equation.

Key Ideas

- The equality that contains letters like these are called **equations**
- The value that works for the letter in an equation is called the solution to the equation.
- Finding the solution is called solving the equation.

Exercises

State which of the following equation proves 3 as the solution.

(a) $x - 8 = 5$ (b) $4x - 7 = 5$ (c) $x + 2 = 3x - 4$

L37 : Properties of equalities

Lesson Objective: To apply the properties of equalities to make equations and their solutions.**(7.1.3.2/3)**
Materials:

ASK-MT and Assessment	
Attitudes/Values	Be able to enjoy using the ideas of properties of equalities to make equations to their solutions
Skills	Be able to make equations using the properties of equalities
Knowledge	Be able to understand the Properties of equalities
Mathematical Thinking	Be able to extend known ideas to think about ways to make equations using the properties of equalities
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introductory
Let's think about ways to solve an equation using properties of equalities
You put an envelope, 3g of weights, and 10g of weights on a scale as shown on the figure and scale balances exactly.
Find the weight of an envelope?

Discuss
If we let the weight of the envelope be xg and we know that the weight of the two sides are equal, we can make the following equation.

$$x + 3 = 10$$

If we subtract 3 from both sides of the scale of the equation,
the remaining value will be also be equal.
Since

$$x + 3 = 10 -3$$

then

$$x = 7$$

meaning that the weight of the envelope is 7g.

Key Ideas

The properties of equalities

① An equality holds true if the same number is added on both sides

If $A = B$, then $A + C = B + C$

② An equality holds true, if the same number is subtracted on both sides

If $A = B$, then $A - C = B - C$

③ An equality holds true, if both sides are multiplied by the same number

If $A = B$, then $A \times C = B \times C$

④ An equality holds true, if both sides are divide by the same number

If $A = B$, then $A \div C = B \div C$

Important Property ④ does not apply when C is 0

L38: Solving equations of addition and subtraction

Lesson Objective : To solve equations using properties of equalities of addition and subtraction. **(7.1.3.2/3)**
Materials:

ASK-MT and Assessment	
Attitudes/Values	Be able to enjoy using the ideas of properties of equalities to add and subtract
Skills	Be able to apply the properties of equalities # 1 and 2 to add and subtract equations
Knowledge	Be able to understand the properties of equalities for adding and subtracting to solve equations
Mathematical Thinking	Be able to think about ways of solving equations using properties of equalities
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1 **Adding the same number to both sides**

$$x - 5 = -1$$
$$x - 5 + 5 = -1 + 5$$
$$x = 4$$

In *example*.1 $x = 4$ Indicates that 4 is the solution to the equation. In other words, it means the equation is solved

Example.2: **Subtracting the same number from both sides**

$$x + 13 = 8$$
$$x - 13 + 13 = 8 - 13$$
$$x = -5$$

Exercises

1. Use the properties of equalities to solve the following equations.

   (a) $x - 9 = 3$ (b) $x - 8 = -10$

2. Use the properties of equalities to solve the following equations.

   (a) $x + 7 = 15$ (b) $x + 1.2 = 0$

L39: Solving equations of multiplication and division

Lesson Objective: To solve the equations using properties of equalities of multiplication and division. **(7.1.3.2/3)**

Materials:

ASK-MT and Assessment	
Attitudes/Values	Be able to enjoy using the ideas of properties of equalities to multiply and divide
Skills	Be able to apply the properties of equalities # 3 and 3 to multiply and divide equations
Knowledge	Be able to understand the properties of equalities for adding and subtracting to solve equations
Mathematical Thinking	Be able to think about ways of solving equations using properties of equalities
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1: **Multiplying both sides by the same number**

$$\frac{x}{4} = -3$$
$$\frac{x}{4} \times 4 = (-3) \times 4$$
$$x = -12$$

Example.2: **Dividing both sides by the same number**

$$-7x = 14$$
$$-7x \div (-7) = 14 \div (-7)$$
$$x = -2$$

Exercises

1. Use the properties of equalities to solve the following equations.

(a) $\frac{x}{7} = 3$ (b) $\frac{x}{4} = -5$ (c) $-\frac{1}{6}x = 2$

2. Use the properties of equalities to solve the following equations.

(a) $5x = 45$ (b) $-8x = 48$ (c) $12x = 3$

L40: How to solve equations

Lesson Objective : To think about the best way to solve equations using properties of equalities. **(7.1.3.2/3)**
Materials:

ASK-MT and Assessment	
Attitudes/Values	Be able to share ideas on best ways to solve equations
Skills	Be able to use transpose approach to solve equations
Knowledge	Be able to understand how the form of an expression changes when you use the properties of equalities to solve equations
Mathematical Thinking	Be able to think about of ways to solve the equations using transpose approach
Assessment	Use the ASK-MT to assess the students learning progress during the lesson

Teaching and Learning Activities

Example.1: **Transposing terms to solve equations** ①

$$3x + 20 = 5,$$

Transpose 20 from the left side to the right side

$$3x = 5 - 20,$$
$$3x = -15$$
$$x = -5$$

$3x + 20 = 5$
Tranpose
$3x = 5 - 20$

Example.2: **Transposing terms with letters to solve equations** ②

$$8x = 5x - 21$$

Transpose 5x from left side to right side

$$8x - 5x = 5x - 5x - 21$$
$$3x = - 21$$
$$3x = -21$$
$$x = -7$$

$8x = 5x - 21$
Tranpose
$8x - 5x = - 20$

Example.3: **Transposing both numerical terms and letter terms to solve equations** ③

$$7x - 2 = 6 + 3x$$

Transpose –2 and $3x$

$$7x - 3x = 6 + 2$$
$$4x = 8$$
$$x = 2$$

$7x - 2 = 6 + 3x$
Tranpose Tranpose
$7x - 3x = 6 + 2$

Key Ideas

- You can move a term from one side of an equality to the other side if you change its sign. This is called transposing the term.
- When you transpose terms to solve an equation, put all the letter terms on one side and all of the numerical terms on the other

Exercises

Solve the following equations

(a) $5x + 8 = 23$ (b) $6x - 5 = 17$ (c) $10x = 6x - 8$

(d) $3x = 5x - 14$ (e) $9x + 2 = 4x + 17$ (f) $5x - 8 = -17 - 4x$

L41 : : Various types of equations

Lesson Objective: To solve equations with parentheses and that contain fractions and understanding linear equations. **(7.1.3.1/2/3)**

Materials:

ASK-MT and Assessment	
Attitudes/Values	Be able to share ideas on solving equations with parentheses and that contain fractions
Skills	Be able to solve equations with parentheses and that contain fractions
Knowledge	Be able to understand linear equation and how to solve equations with fractions, parentheses
Mathematical Thinking	Be able to think about ways to solve the linear equations with fractions, parentheses
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1: **Solving equations with parentheses**

$$7(x-5) = 9x+1$$
$$7x - 35 = 9x + 1$$
$$7x - 9x = 1 + 35$$
$$-2x = 36$$
$$x = -18$$

Example.2: **Solving equations containing fractions**

$$\frac{(x+1)}{2} = \frac{1}{5}x + 2$$
$$\frac{(x+1)}{2} \times 10 = (\frac{1}{5}x+2) \times 10$$
$$(x+1) \times 5 = 2x + 20$$
$$5x + 5 = 2x+20$$
$$3x = 15$$
$$x = 5$$

Key Ideas

Steps for solving linear equations

1. Remove the parenthesis and cancel the denominators, if necessary
2. Put all letter terms on one side and numerical terms on the other
3. Transform the equation into $ax = b$
4. Divide both sides of the equation by α, the coefficient of x

$$3(x-2) = x+2 \quad ❶$$
$$3x-6 = x+2 \quad ❷$$
$$3x-x = 2+6 \quad ❸$$
$$2x = 8 \quad ❹$$
$$x = 4$$

When transposing terms in the equations, you have learned that the equation resulted in the form $ax = b$, Equations like these are called linear equations.

Exercises

Solve the following equations.

(a) $4x+1= 3(x+2)$ (b) $2(x-4) = 9x+2$ (a) $\frac{(x-1)}{3} = \frac{1}{2}x + 4$ (b) $\frac{3}{4}x - 7 = 2x + \frac{1}{2}$

L42 Ratios and properties of proportional expressions

Lesson Objective : To understand equivalent ratios as proportional expression and properties of proportional expressions. **(7.1.3.3/4)**

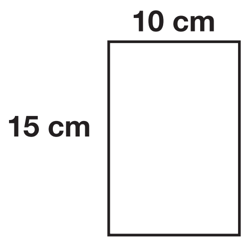
Materials:

ASK-MT and Assessment	
Attitudes/Values	Be able to share ideas on calculating value of x using proportional expressions
Skills	Be able to explore properties of proportional expressions, find the ratio value and calculate the expressions
Knowledge	Be able to understanding of proportional expressions and calculations of the solutions
Mathematical Thinking	Be able to think about the ratio values and how to solve the proportional expressions
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Review

You have a rectangle with length 15cm and width of 10cm Write a ratio of the length and width of the rectangle. How many times the width is to its length?



Example.1: **Solving proportional expressions using the property of proportional expressions**

(a) $x : 6 = 7 : 3$

$3x = 42$

$x = 14$

(b) $5 : x = 2 : 3$

$2x = 15$

$x = \dfrac{15}{2}$

Key Ideas

In general a and b in the ratio: $a : b$ is called the "terms" of ratio and the value a/b which represents a divided by the second term b is called a **ratio value.**

This can be expressed as $\dfrac{a}{b} : \dfrac{c}{d}$, the ratios are called proportional expression

In general, finding the value of a letter contained in a proportional expression is called **solving a proportional expression.**

Property of proportional expression

The product of outer terms of proportional expression is equal to the product of inner terms.

If $a : b = c : d$, then $ad = bc$

Exercises

Solve the following proportional expressions

(a) $x : 8 = 3 : 2$ (b) $3 : 4 = x : 5$

(c) $x : (14\text{-}x) = 3 : 4$ (d) $x : 0.3 = 100 : 0.2$ (e) $x : 8 = 3 : 2$

L43 : Calculation with both multiplication and division

Using equations

Lesson Objective: Be able to find the problem situations and their relationships, set the equations and solve the equations **(7.4.4.2)**

Materials:

ASK-MT and Assessment	
Attitudes/Values	Willingness to pay attention and participates, accepts responsibilities, organise and take interest in the lesson activities. Enjoy the activities in the lesson
Skills	Find quantities of problem situations and their relationships, set the equations and solve the equations
Knowledge	Understand how to solve equations in everyday problems
Mathematical Thinking	Think about steps for setting up equations and how to solve them
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1 **Problems using total cost**

The total cost of 6 mini pizza and one 2kina can drinks is 4 times the total cost of the mini pizza, and one 5 kina 2L juice.

Approach: the relationship between the total costs is as follows.
(The cost of 6 mini pizza and 1 can drink = the cost of 1 mini pizza and 1 juice) × 4
Use this relationship, set up an equation where x kina is the price of 1 cake that you are trying to find.

Solution
If we let x be the price of 1 mini pizza

$$6x + 2 = 4\,(x + 5)$$

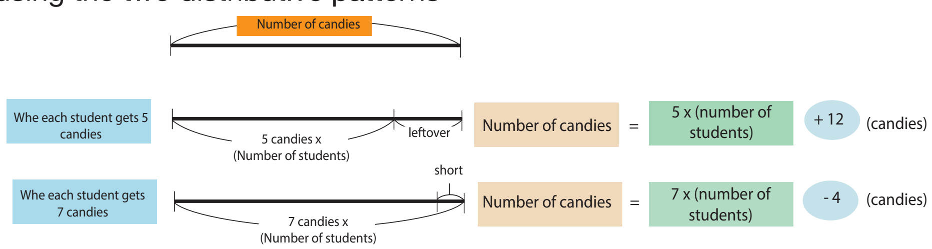
Solving this gives us

$$6x + 2 = 4x + 20$$
$$6x - 4x = 20 - 2$$
$$2x = 18$$
$$x = 9$$

Example.2 **Problems of too much or not enough**

Candy is distributed to several students so that they all have the same amount. When each student gets 5 candies so that, there are 12 left over and when each child gets 7 candies, there are 4 left over. How many students are there?

Approach: The relationship between the number of candies and the number of students can be determined by using the two distributive patterns



Teaching and Learning Activities

Solution:

If we let x be the number of students

$$5x + 12 = 7x - 4$$

Solving this gives us

$$-2x = -16$$
$$x = 8$$

<u>The number of students is 8.</u>

Example.3: **Speed, time and distance**

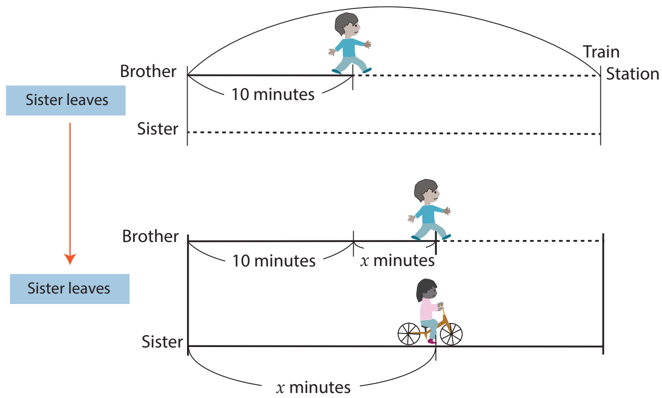
A boy left home to walk to the main road which is 2 km to catch a PMV to Port Moresby. After 10 minutes, her sister realized that he had left something and rode on the bicycle to catch up with her brother. If the boy walks at 80m per minute and sister rides her bicycle at 240m per minute, how many minutes after she leaves the house to catch up with her brother?

Approach: Let x be the number of minutes it will take for the sister to catch up with her brother. The travel distance and the time it will take for the sister to catch up with her brother should be shown as below;

Solution: If we let x be the number of minutes it will take for the sister to catch up with her brother after she leaves the house

Then, $240x = 80(10 + x)$

$$3x = 10 + x$$
$$2x = 10$$
$$x = 5$$



Key Ideas

Steps for using equations to solve problems

1. Focus on the quantities in the problem and find their relationships
2. Set up an equation, using unknown quantities where appropriate.
3. Solve equation

Important: find out where the solution to the equation actually solves the problems

Exercises

1. Rose has K78 and Miriam has K63, They both wanted to buy the same book. When the change was given, Rose had 2 times the amount that Miriam had. How much was the price of each book? Use the same approach in the last lesson and set the equation to solve the equation for this problem

2. You want to buy some post cards to send to your friends. With the money that you have you can buy 15 post cards. You are now left with K2. If you buy 20 post cards you will be short by K4. How much is 1 post card?

Unit Checkpoint

Review on Equations

Review on Equations

Equations and their solution

1. State which of the following equations have the solution of 2.

(a) $5x - 4 = 8$ (b) $10 - 3x = 8x - 12$

Properties of equalities

2. Fill in □.

Which properties of equalities are being used in (1) and (2)?

$$3x - 7 = 8$$
$$3x - 7 + \square = 8 + \square$$
$$3x - 7 = 8 + \square$$
$$3x = \square$$
$$x = \square$$

Properties of equalities and how to solve equations

3. Solve the following equations.

(a) $x - 5 = 8$ (b) $x + 13 = 4$ (c) $3x = -12$

(d) $\frac{1}{3}x = \frac{1}{2}$ (e) $5x = x - 4$ (f) $3x + 5 = x + 11$

Ratios and proportional expressions

4. Solve the proportional equations $x - 4 = 6 - 3$

Using equations

5. When you buy 5 pencils and one 50 toea eraser with K5, you get 50^t change.
Find the price of 1 pencil.

Using proportional equations

6. Find the total cost x kina when buying 3000g of sugar that cost K3.50 per 500g

L44: Understanding functions

Functions

Lesson Objective: To understand the meaning of functions by investigating the relationship between pairs of quantities which change together. **(7.4.1.1)**

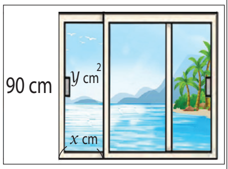
Materials: Examples of functional relationships.

ASK-MT and Assessment	
Attitudes/Values	Be able to appreciate the relationship of two quantities that determine functional relationships
Skills	Be able to investigate the relationship between pairs of quantities that change together
Knowledge	To be able to understand the meaning of functional relationships
Mathematical Thinking	To be able to determine functions using sets of quantities that vary simultaneously, focusing on their changes and correspondences
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1 **The area of the open portion of the window.**

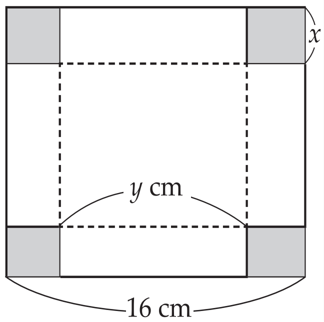
A window has a height of 90cm. The area of the open portion changes according to the distance the window is moved. If we know that distance, there can only be one area.



If we let the distance the window moved be (x)cm and the area of the open portion be (y)cm², then (x) and (y) change together and can take on many different values. The relationship between (x) and (y) is $y = 90x$

In these situations when (y) is a function of (x), there may be ways to indicate the relationship using an expression.

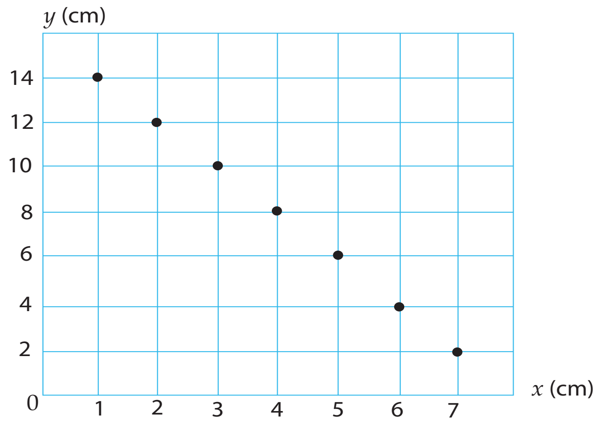
Example.2: **Finding out the nature of functions using tables and graphs.**



To make the boxes from the previous lesson we cut out squares with (x) cm sides from the corners of a square with 16cm sides. Here, (y) is a function (x) when the length of the side forming the base of the box is (y)cm.
If we use a table or graph to express the way that the corresponding value of (y) changes as the value of (x) changes we get the following:

x (cm)	1	2	3	4	5	6	7
y (cm)	14	12	10	8	6	4	2

As the value of (x) increases, the value of (y) decreases.



Teaching and Learning Activities

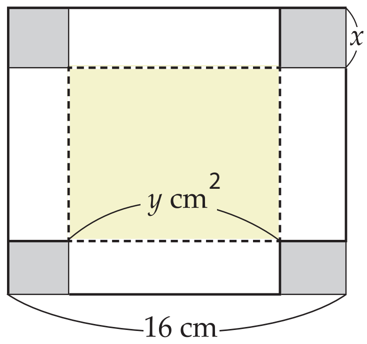
Key Ideas

Letters like (x) and (y) which can take on a variety of values, are called variables.

When you have two variables (x) and (y) that change together, once you determine the value of (x), there is a unique corresponding value of (y). In this case, (y) is said to be a function of (x).

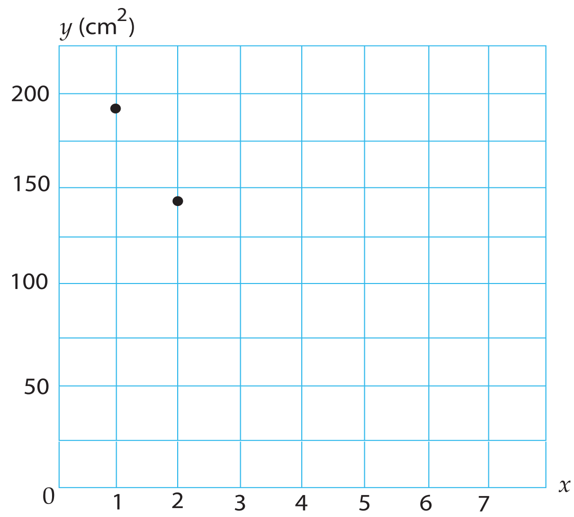
Exercises

To make the boxes, let the area of the base of the box be (y)cm^2 when we cut out squares with (x)cm sides from the corners.



Express the way (x) and (y) change in this situation using the table and graph below.

x (cm)	1	2	3	4	5	6	7
y (cm^2)	196	144					



L45: Quantities which change together

Lesson Objective : To explore meaning domain and its relationship between pairs of quantities which change together. **(7.4.1.1)**

Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Appreciate the relationship of two quantities that determine functional relationships.
Skills	Investigate the relationship between pairs of quantities that change together to identify the domain.
Knowledge	The meaning of domain in functional relationships.
Mathematical Thinking	Think of how to determine the domain of functions using sets of quantities that vary simultaneously.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1. **How to express the relationship.**

When the value of (x) must be at least -2 the relationship is expressed as, $x \geq -2$

Similarly, (x) must be less than 5 is expressed as, $x < 5$

Key Ideas

When x is used to express the relationship of a function is called a domain.

Exercises

Use inequality signs to express the domain of the variable (x), when its value must be at least 3 and less than 10.

L46: Proportional expressions

Functions

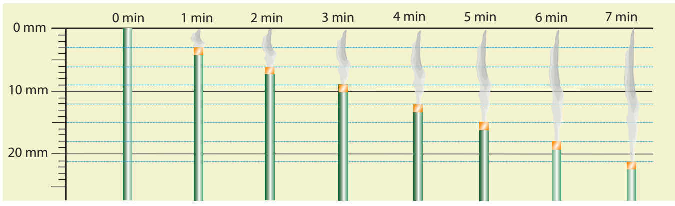
Lesson Objective: To express proportional relationships using tables. **(7.4.1.2)**
Materials: Blank table of results, candle stick

ASK-MT and Assessment	
Attitudes/Values	Share ideas on the relationship of a burning incense stick and appreciate the concept of proportion in their daily lives.
Skills	Identify the changes in quantities of length to time as x and y variables.
Knowledge	The concept of proportional relationships.
Mathematical Thinking	Examine two quantities in concrete situations that vary simultaneously focusing on their changes and correspondences.
Assessment	use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introductory

Try an experiment to find out the relationship between the time since an incense stick is lit and the length of the portion that burns away.



Summarize the relationship between (x) and (y) in the table below if (x) minutes is the amount of time since the incense was lit and (y) mm is the length of the portion that was has burned away.
(Complete the table)

x	0	1	2	3	4	5	6	7
y								

Expected results

The relationship between the time that had passed since the incense was lit (x minutes) and the length of the portion that had burned (y mm) can be expressed in the following table.

x	0	1	2	3	4	5	6	7
y	0	3	6	9	12	15	18	21

If we focus on the corresponding numbers in the upper and lower part of the table, we see that the value of (y) is 3 times the value of (x).
We can use this information to express the relationship between (x)) and (y) with the following expression;

$$y = 3x$$

TN: In the expression $y = 3x$, the value of the variable x can be 1, 2, 3, and so on. The proportional relationship $y = ax$ can also be expressed as the function $y = ax$, a is the constant of proportion and can stand for any number.

Teaching and Learning Activities

Example.1 **When variables take a negative value**

You fill an aquarium with water at a rate of 5 L per minute. Starting from a certain time, the volume of water will increase by y L after x minutes.

Therefore,

$y = 5x$

When x = -3 in this expression, the value of y is

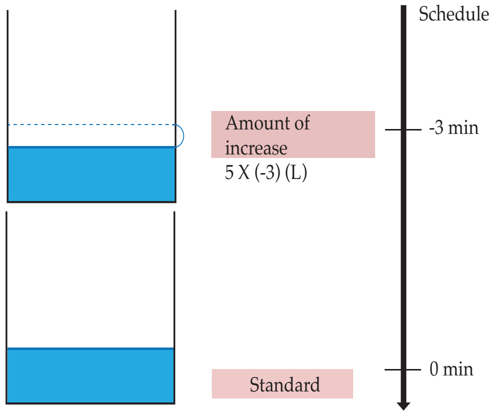
$y = 5 \times (-3) = -15$

This means that

after -3 minutes, the water will increase -15 L.

In other words, it indicates that

3 minutes ago, the water was 15 L less.



x	···	- 4	- 3	- 2	- 1	0	1	···
y	···	- 20	- 15	- 10	- 15	0	5	···

Example.2 **Setting up proportional expressions**

y is proportional to x, and y =16, when x = 8. Write an expression indicating (x) and (y).

Approach: Because (y) is proportional to (x), we can express it as $y = ax$

Solution:

If we let the constant of proposition be a, $y = ax$

y =16 when x, = 8, so

16 = a x 8

a = 2

Therefore, $y = 2x$

Key Ideas

- A fixed number in an expression, like the 3 in $y = 3x$, is called a **constant**
- Variables that change together like x and y can sometimes have a negative value. Even in this case, y is still said to be proportional to x if the relationship $y = ax$ still holds true.

We can say the following about the proportional relationship $y = ax$

(a) As the value of x is multiplied 2 times, 3 times, 4 time, and so on the value of y is also multiplied 2 times, 3 times, 4 time, and so on

(b) the quotient of corresponding x and y values ($\frac{y}{x}$) is fixed and equal to the constant of proportion a. In other words, the relationship between x and y can also be expressed as $\frac{y}{x} = a$

x	1	2	3	4	5
y	3	6	9	12	15

Exercises

1. For each of the items below, check to see if y is proportional to x. Then state the constant of proportion in each.
   (a) The total cost y kina when buying x 50 kina stamps
   (b) The area y cm² of a triangle with a base of 8 cm and height of x cm.
2. Find the values of y that correspond to the values of x when y = -2x and complete the table below.
3. Write an expression indicating the relationship between x and y below.
   (a) y is proportional to x, and y = 32 when x = 8
   (b) y is proportional to x, and y = 40 when x = -4

L47 : Coordinates

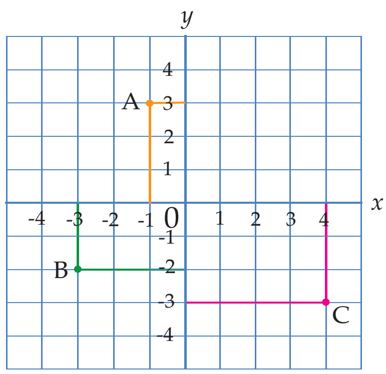
Lesson Objective : To locate and express the position of points on a plane. **(7.4.1.4)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Appreciate usefulness of the relationship of two quantities that determine functional relationships in their daily lives
Skills	Identify and correctly plot coordinates on graphs
Knowledge	Points on a graph using the x and y axis
Mathematical Thinking	Think of ways to determine the position of x and y coordinates on graphs from the point of origin (0)
Assessment	Use the ASK-MT to assess the students learning progress during the lesson

Teaching and Learning Activities

Example.1 **Coordinates of points**

The coordinates of point A are (-1,3).
The coordinates of point B are (-3,-2).
The coordinates of point C are (4,-3) and the coordinates of the origin are (0,0).



Key Ideas

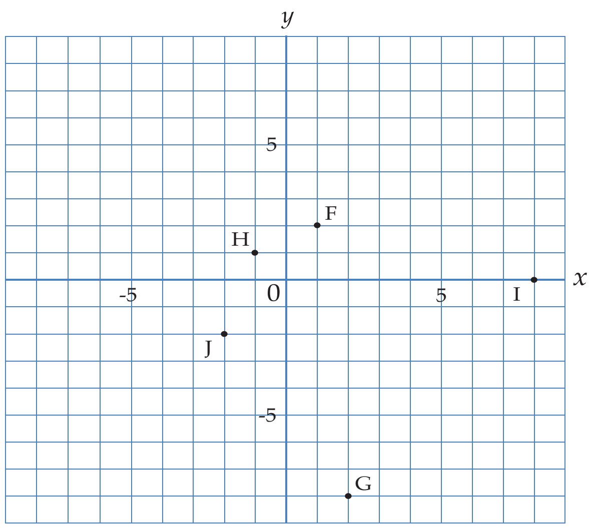
We us e the following terms to describe a situation like this;

Horizontal number line:	**x axis**
Vertical number line:	**y axis**
Both number line together:	**coordinate axes**
Intersect point 0 of coordinate axes:	**origin**

Exercises

1. Plot the points represented by the following coordinates on the right.
   (a) (0,-3) (b) (-6,4) (c) (4,6)
   (d) (-8,-6) (e) (3,-1)

2. Write the coordinates of points F,G,H,I and J from the figure on the right.
   F(___, ___), G(___, ___),
   H(___, ___), I(___, ___), J(___, ___)



L48: Graphing proportion

Lesson Objective: To identify and draw and proportional relationship $y = ax$ on a graph. **(7.4.1.2/3)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Appreciate the relationship of two quantities that determine functional relationships.
Skills	Use pairs of x and y values in table to plot points on a graph.
Knowledge	Graphs of direct proportion on a number plane.
Mathematical Thinking	Determine the position of x and y coordinates on graphs from the point of origin (0).
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

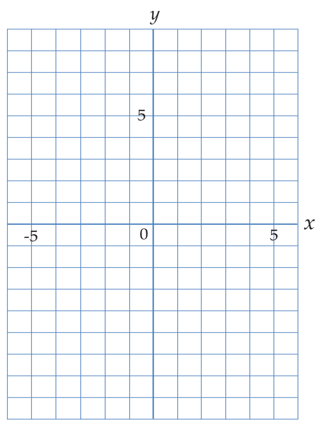
Teaching and Learning Activities

Introductory

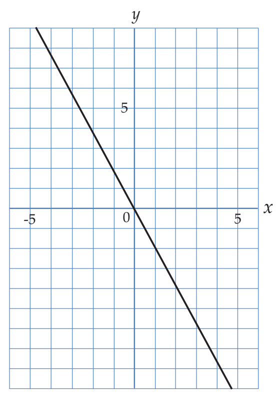
Let's look at the graph of the proportional relationship $y = ax$ when the constant of portion a is a negative number.
The table below shows the corresponding x and y values in y = -2x

x	…	- 4	- 3	- 2	- 1	0	1	2	3	4 …
y	…	8	6	4	2	0	-2	-4	-6	-8 …

Use the pairs of x and y values in the table above as coordinates and plot the points on the graph below



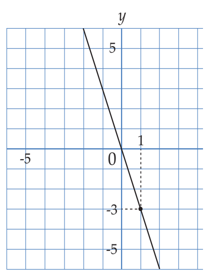
Use the pairs of x and y values in the table above as coordinates and plot the points on the graph below



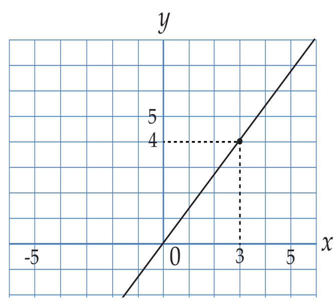
TN: The graph of the proportional relationship $y = ax$ is a straight line that passes through the origin

Example.1 **Graphing proportion**

(1) The graph of y = -3 passes through the origin and the (1,-3)



(2) The graph of y = 4/3 x passes through the origin and the (3,-4)

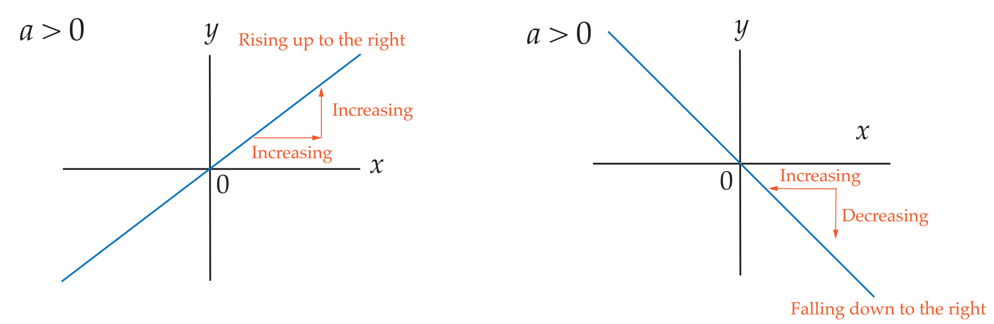


Teaching and Learning Activities

Key Ideas

Graphing proportion

The graph of the proportional relationship $y = ax$ is a straight line that passes through the origin. It varies according to the value of a as shown below.



Exercises

Draw the graphs of (a) to (d)

(a) $y = 4x$ (b) $y = -3x$ (c) $y = \frac{2}{3}x$ (d) $y = -\frac{3}{4}x$

L49: Graphing with domains

Lesson Objective : To be able to express the proportional relationship $y = ax$ on a graph when there is a domain. **(7.4.1.2/3)**

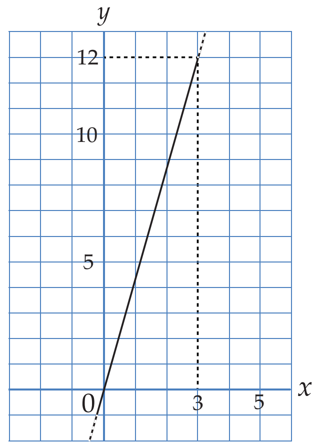
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Be able to appreciate the relationship of two quantities that determine proportional relationships with a domain.
Skills	Be able to examine the graph for proportion when there is a domain.
Knowledge	To be able to understand proportional relationship of $y = ax$ when there is a domain.
Mathematical Thinking	To be able to determine the position of x and y coordinates on graphs and express the domain.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1 **Graphing Domain**

You walk from the station to the park that is 12km away at a speed of 4km per hour. Write an expression indicating the relationship between your walking time (x) hours and the distance (y) km that you cover during that time.
Then draw the graph.



Solution
If we write the expression for the relationship between x and y as

$$y = 4x$$

Since it will take 3 hours to get to the park,
The domain of x is expressed as

$$(0 \leq x \leq 3)$$

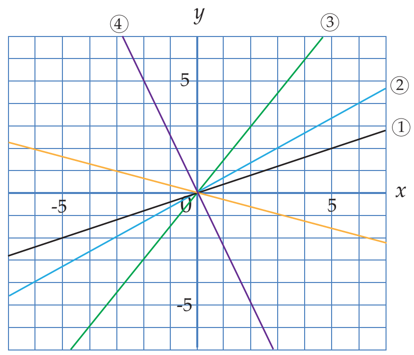
The graph is the solid portion of the line in the figure

Exercises

1. You pour water into an 18L container at a rate of 2L per minute. Write an expression and draw a graph to indicate the relationship between the time the water runs (x minutes) and the amount of water filled during that time (yL).

2. Which of the lines on the right represent the graphs of (1) to (40) below?

(a) $y = \frac{3}{2}x$ (b). $y = -4x$ (c). $y = \frac{2}{3}x$ (d). $y = -\frac{1}{3}x$



L50: Inversely proportional expressions

Inverse Proportion

Lesson Objective: To be able to identify inversely proportional relationships and indicate them in expressions. **(7.4.1.2/3)**

Materials: Grid paper and table of results.

ASK-MT and Assessment	
Attitudes/Values	Be able to appreciate the relationship of two quantities that determine inverse proportional relationships
Skills	Be able to use pairs of x and y values in table to express inversely proportional relationships
Knowledge	To be able to understand the meaning of a relationship that is inversely proportional
Mathematical Thinking	To be able to determine inversely proportional relationships between two variables (x) and (y)
Assessment	use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introductory

The table below shows the relationship between width (x cm) and length (y cm) in the rectangles from the previous activity.

x	1	2	3	4	5	6
y	6	3	2	1.5	1.2	1

If we focus on corresponding numbers in the upper and lower part of the table, we see that

$$y = 6 \div x.$$

We can therefore express the relationship between x and y with the following expression

$$= \frac{6}{x}\ .$$

When y is a function of x and the relationship between them can be expressed as

$$y = \frac{a}{x},\quad a \text{ is a constant}$$

The constant of proportion in $y = \frac{6}{x}$ is 6.

The inversely proportional relationship $y = \frac{a}{x}$ is also the function $y = \frac{a}{x}$

Example.1 **When variables take a negative value.**

If we make a table indicating the values of y in $y = \frac{12}{x}$ when x takes on different positive and negative values, we get the following.

x	⋯ -6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6 ⋯
y	⋯ -2	-2.4	-3	-4	-6	-12	x	12	6	4	3	2.4	2 ⋯

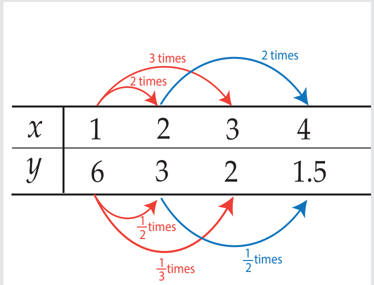
Important: In an inversely proportional relationship $y = \frac{a}{x}$, there is no value for y when the value of x is 0

Teaching and Learning Activities

Key Ideas

- When y is a function of x and the relationship between them can be expressed as $\frac{a}{x}$, a is a constant, it is said that y is inversely proportional to x, and that the constant a is the constant of proportion
- In the inversely proportional relationship = $\frac{a}{x}$, there is no value for y when the value of x is 0.
- In an inversely proportional relationship $y = \frac{a}{x}$, it is possible for the constant of proportion a to be a negative number.
- We can say the following about inversely proportional relationship $y = \frac{a}{x}$.

  (a) As the value of x is multiplied 2 times, 3 times, 4 times and so on, the value y is multiplied $\frac{1}{2}$ times, $\frac{1}{3}$ times, $\frac{1}{4}$ times, and so on.



  (b) The product xy of corresponding x and y values is fixed and equal to the constant of proportion a.

The relationship between x and y can also be expressed as $xy = a$

Exercises

1. It takes y hours to travel a distance of 15km at x km per hour. Check to see if y is inversely proportional to x.

2. Find the values of y that correspond to the values of x when y =-6/x and complete the table below.

x	...	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	...							x						

L51: Graphing inverse proportion

Lesson Objective : To be able to express inversely proportional relationships $y = \frac{a}{x}$ on a graph. **(7.4.1.2/3)**
Materials: Grid paper and table of results.

ASK-MT and Assessment	
Attitudes/Values	Be able to appreciate the relationship of two quantities that determine inverse proportional relationships on graphs
Skills	Be able to use pairs of x and y values in table to express inversely proportional relationships on a graph
Knowledge	To be able to read and understand the relationship that is inversely proportional on a graph.
Mathematical Thinking	To be able to determine inversely proportional relationships between two variables (x) and (y) on a graph.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

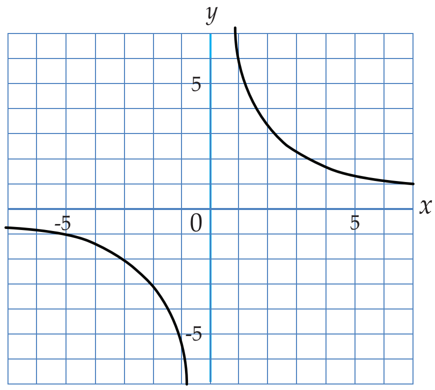
Teaching and Learning Activities

Example.1: $y = \frac{6}{x}$ **when x is a negative value.**

Look at the graph for $y = \frac{6}{x}$ when x is a negative value.

x	••• -6	-5	- 4	- 3	-2	-1	0	1	2	3	4	5	6••
y	••• -1	-1.2	-1.5	-2	-3	-6	x	6	3	2	1.5	1.2	1 •••

If we take all the corresponding values in $y = \frac{6}{x}$ except $x = 0$, the resulting points all fall along the two curves in the figure below. These two curves are the graph of the inversely proportional relationship of $= \frac{6}{x}$.

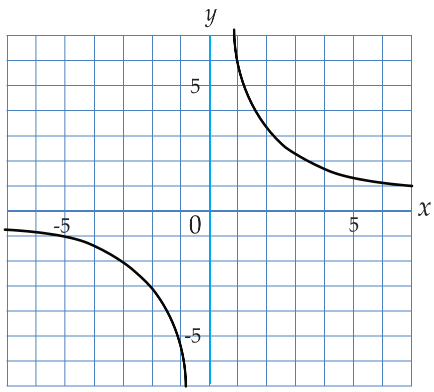


Example.2: **y=a/x when the constant of proportion a is a negative number**

Now let's look at the graph of the inversely proportional relationship $y = \frac{a}{x}$ when the constant of proportion a is a negative number. *Eg.* $y = -\frac{6}{x}$

x	••• -6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6 •••
y	••• 1	1.2	1.5	2	3	6	x	-6	-3	-2	-1.5	-1.2	1 •••

relationship $y = -\frac{6}{x}$ is shown below.

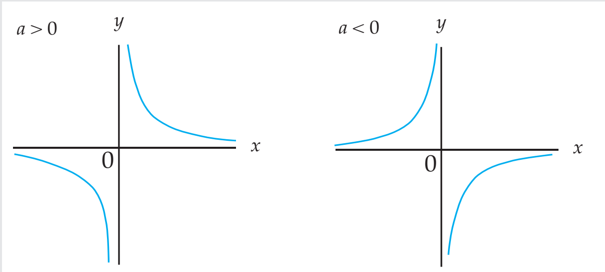


Teaching and Learning Activities

Key Ideas

Graphing inverse proportion

The graph of the inversely proportional relationship $y = a/x$ is two curves. Curves like these are called hyperbolas. It varies according to the value of a as shown below



Exercises

1. Draw the graph of $y = 12/x$.
2. Draw the graph of $y = -12/x$

L52: Applying proportion

Applying proportion and inverse proportion

Lesson Objective: : To be able to apply proportion $y = \frac{a}{x}$ to solve problems. **(7.4.1.2)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Be able to appreciate the importance of proportional relationships to solve problems.
Skills	Be able to solve problems using proportional relationships.
Knowledge	To be able to read and understand proportional relationships when solving problems.
Mathematical Thinking	To be able to relate proportional relationships $y = ax$ in situations that involve solving problems.
Assessment	use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example. **Using proportional to solve real life problems**

In this situation, let x be the number of handouts and (y grams) be their weight.
Since y is proportional to x we can express this as $y = ax$.
Since we know that $y = 80$ when $x = 25$, $a = \frac{16}{5}$
and the relationship between the number of handouts and weight is $y = \frac{16}{5}x$

$80 = a \times 25$

$a = \frac{80}{25}$

$= \frac{16}{5}$

To find the weight of the 195 handouts needed for grade 1, substitute 195 for x in this expression.
This gives $y = \frac{16}{5} \times 195 = 624g$

Exercises

1. In the situation above, how many grams needs to be measured to separate the number of handouts needed for Grades 2 and 3?

2. How many handouts weigh 960g?

L53: Applying inverse proportion

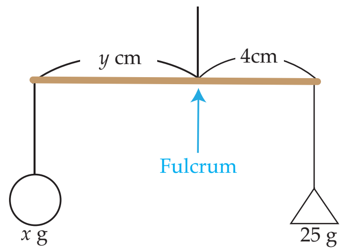
Lesson Objective : To be able to apply inversely proportional relationships $y=\frac{a}{x}$ to solve problems. **(7.4.1.2)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Be able to appreciate the importance of inversely proportional relationships to solve problems
Skills	Be able to solve problems using inversely proportional relationships
Knowledge	To be able to read and understand inversely proportional relationships when solving problems
Mathematical Thinking	To be able to relate inversely proportional relationships $y=\frac{a}{x}$ in situations that involves problem solving.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

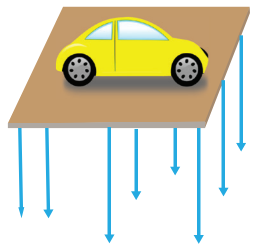
Teaching and Learning Activities

Review

Decorations that move in the breeze are called mobiles. A mobile is constructed so that each rod is balanced on the left and right sides.
For every rod, (weight of decoration) × (distance from fulcrum)
According to the principal of leverage, no matter what the length of the poles are, the weight of the decoration × distance from the fulcrum is equal.



For example, when the left and right sides are balanced like in the figure,
The expression becomes $x \times y = 25 \times 4$ or relationships $y=\frac{100}{x}$ indicating
That y has an inversely proportional relationship to x.

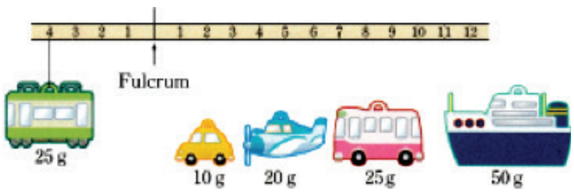


Example.1 **Area and pressure**
When you apply force to a surface, the amount of force experienced by 1m² of that surface is called **pressure.** For example, the amount of Gravitational force on a car weighing 1200kg is about 12000N (newtons)

If the car were supported by a surface with an area of S m², the Pressure on the surface P N/m² would be P=12000/S

Exercises

1. To make the mobile below, where does each of the weights need to be hung so that the left and right sides of the fulcrum are balanced?



2. Find the surface pressure if a 1200kg car is supported on a surface with an area of 10m². What about 20m²?

Unit Checkpoint

Review on Direct and Inverse Proportions

Review on Direct and Inverse Proportions

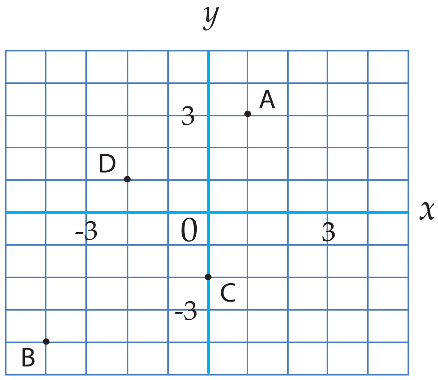
1. In which of the following is y a function of x?

In which is y proportional to x? Inversely proportional?

(a) The base (x cm) and height (y cm) of a parallelogram with an area of 10 cm^2
(b) The amount of precipitation (y mm) when the temperature is x °C
(c) A 30-L container is full in y minutes when water is poured in at a rate of x L per minute.

2. Write an expression indicating the relationship between x and y below.
   (a) y is proportional to x and $y = -4$ when $x = 2$
   (b) y is inversely proportional to x and $y = 8$ when $x = -6$

3. State the coordinates of points A, B, C, and D in the figure on the right.



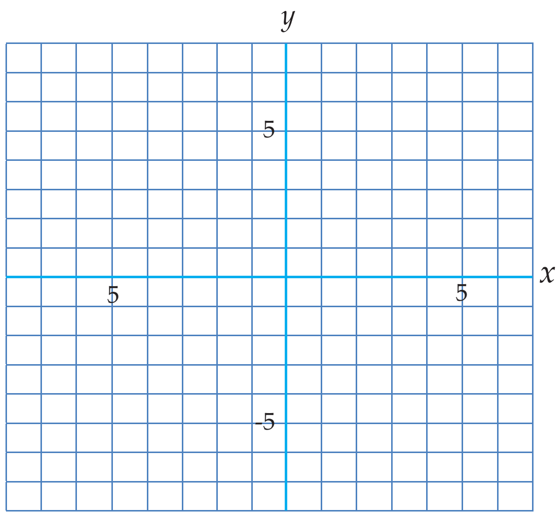
4. Draw a graph for (1) through (4) below.

(a) $y = -4x$

(b) $y = \frac{1}{2}x$

(c) $y = \frac{8}{x}$

(d) $y = -\frac{8}{x}$



Functions

Proportional expressions

Inversely proportional expressions

Coordinates

Graphing proportion

Graphing inverse proportion

L54: Lines and angles

Rectilinear figures and transformation

Lesson Objective : To explain lines and segments and refer to portions of lines that have two endpoints. **(7.2.1.1)**

Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Be able to appreciate lines and segments
Skills	Be able to recognize lines and segments
Knowledge	Be able to understand lines and segments
Mathematical Thinking	Be able to think about lines and segments
Assessment	Use the ASK-MT to assess the students learning progress during the lesson

Teaching and Learning Activities

Introductory

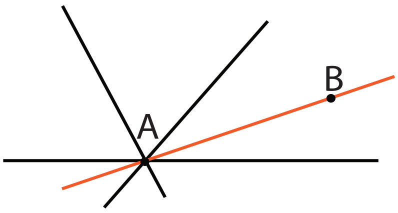
We have been calling all straight lines ”lines”, but from now on, we will use “line” to mean straight lines that extend out infinitely in both directions. We will use the term segment to refer to portions of lines that have two endpoints.

Line AB

Segment AB

A long straight road

Many different lines can pass through a single point, but there is only one line that can pass through a two points.



Key Ideas

- Segment AB is the shortest route between the two points A and B. We say that the length of segment AB is the distance between the two points A and B.
- We can also express the length of segment AB by writing AB.

Exercises

Ikai’s house is along segment AB in the map on the right. Dalona’s house is along line BC. Where are their houses? Choose from (a) through (e).

L55: Angles formed at the intersection of two lines

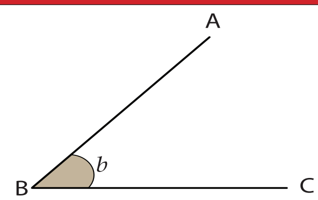
Lesson Objective: To explain how an angle is formed when two lines intersect. **(7.2.1.1)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Appreciate an angle is formed at the intersection of two line
Skills	Be able to recognize an angle is formed at the intersection of two line
Knowledge	Be able to understand how an angle is formed at the intersection of two line
Mathematical Thinking	Be able to think about how an angle is formed at the intersection of two line
Assessment	Use the ASK-MT to assess the students learning progress during the lesson

Teaching and Learning Activities

Introductory

Angles like the one in figure on the right are expressed as ∠ ABC and read angle ABC".
We can also express the size of ∠ ABC by



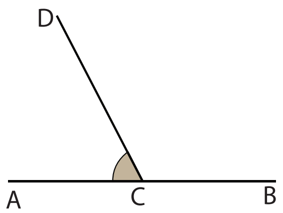
Important: ∠ ABC can also be expressed as simply by writing ∠B or ∠ b

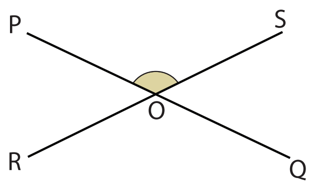
Key Ideas

- An angle is formed when two lines intersect.
- A figure enclosed by several segments is called a polygon. A triangleis the simplest type of polygon. A triangle with vertices at three points A, B and C is expressed as ∠ ABC

Exercises

1. Use symbols to indicate the angles in the figures below. Next, use protractor to measure the size of those angles.

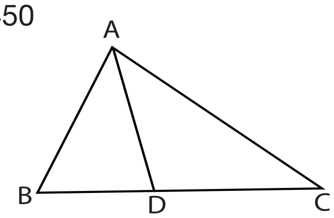




2. Use the Δ symbol to indicate all of the triangles in the figure on the right.

3. Construct triangle Δ ABC according to the information given below

   (a) AB = 5cm, BC = 6cm, CA = 4cm
   (b) BC = 6cm, ∠B = 600 , ∠C = 450



L56: Perpendicular and parallel lines

Lesson Objective : To construct perpendicular line on the left as shown in the figure and explain what relationship does the original line have to the fold line you made. **(7.2.1.1)**

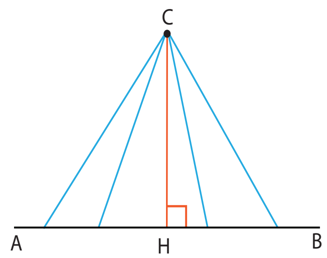
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Be able to enjoy how to fold down the line and describe what relationship does the original line have to the folded line you made.
Skills	Be able to fold down the line as shown in the figure and explain what relationship does the original line have to the fold line you made.
Knowledge	Be able to understand how to fold down the line as shown in the figure and explain what relationship does the original line have to the fold line you made.
Mathematical Thinking	Be able to think about how to fold down the line as shown in the figure and explain what relationship does the original line have to the fold line you made.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

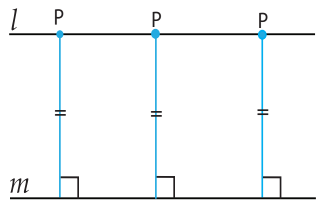
Example.1 **Perpendicular**

In the figure on the right, the line perpendicular to line AB is drawn from point C, intersecting line AB at point H. Segment CH is the shortest of the segments connecting point C with a point along line AB. We say that the length of segment CH is the distance between point C and line AB.



Example.2: **Parallel lines**

If two lines l and m are parallel, the distance between point P and line m is always the same no matter where point P is along line l.
When two lines AB and CD do not intersect, AB and CD are said to be parallel. This is expressed as
AB/CD

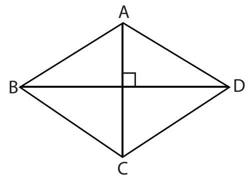


Key Ideas

- When the angle made at the intersection of two lines AB and CD is a right angle, AB and CD are said to be perpendicular. This is expressed as **AB⊥CD**
- When two lines for example, AB and CD are perpendicular, one line is said to be the line perpendicular to the other.
- When **AB ⊥ CD**, means
  AB is the line perpendicular to CD,
  CD is the line perpendicular to AB
- The fixed distance is called the distance between two parallel lines l and m

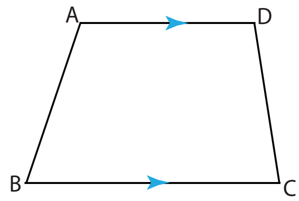
Teaching and Learning Activities

Exercises



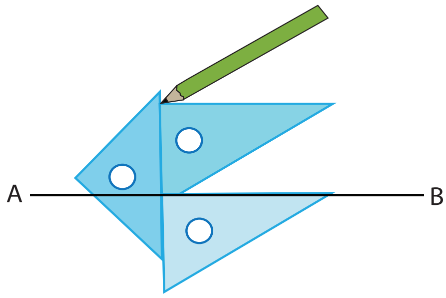
1. Use the symbol ⊥ to indicate the perpendicular line segments in the rhombus on the right.

2. Draw the lines that are perpendicular to each of the two lines l and m and that pass through point A. Next, measure the distance from point A to the l and to line m.



3. Use the symbol // to indicate the parallel segments in the trapezoid on the right.

4. Draw the line AB in your notebook. Then draw a line parallel to line AB at a distance of 2cm.
   How many lines like this can you draw?



L57 : Transformation and translation

Figure transformation

Lesson Objective: : Explore and describe figure transformations and translation and its properties. **(7.2.1.2)**

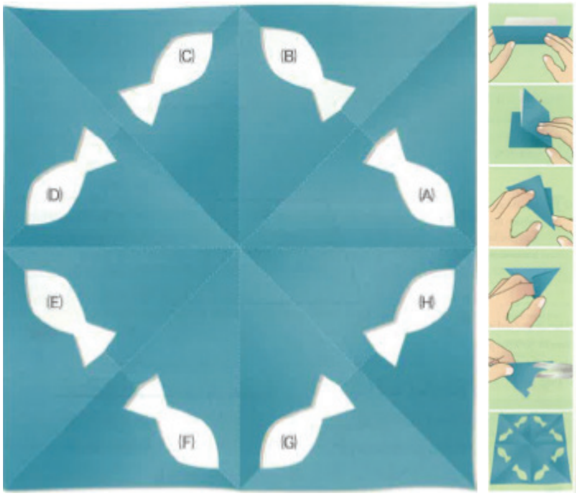
Materials: Square piece of origami paper, square grid

ASK-MT and Assessment	
Attitudes/Values	Enjoy the paper fold see significant figure change and movement of shapes.
Skills	Distinguish between transformation and translation.
Knowledge	Figure transformations and translation.
Mathematical Thinking	Think about transformation and translation of figures.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

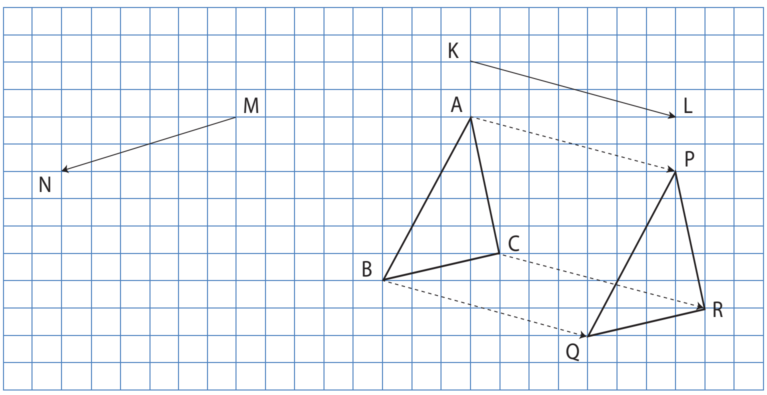
Teaching and Learning Activities

Example.1: **Figure transformation**

The figure below shows a square piece of origami paper folded as shown on the right, cut and opened up again. If you look at a shape (A) as a starting point, how have other shapes moved in relation to A?

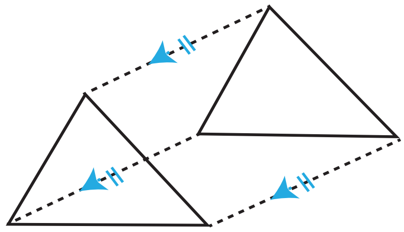


Example.2: **Translation**
In the figure below, Δ PQR is translated from Δ ABC in the direction of KL for the length of KL.



Teaching and Learning Activities

We can say the following about translation; The segments connecting pairs of corresponding points are all parallel and their lengths are equal.



Key Ideas

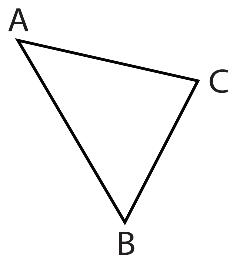
- When the figure changes but its shape and size stays the same, it is called transformation.
  The before and after points that are moved in the transformation are called corresponding points
- When a shape slides in a single direction for a certain distance along a plane, the movement,

Exercises

1. What relationship is there among the segments AP, PQ and CR that link the corresponding points in **Example.2**?

2. Construct the above shape that is translated from Δ ABC in the direction of the arrow MN for the length of MN in **Example.2**.

3. Construct the shape that is translated from Δ ABC in the figure on the right where point A moves to point P.



L58: Rotation and reflection of figures

Lesson Objective : To identify and describe the shapes within the centre of rotation as they rotate by equal lengths and equal distances and describe the properties of shape movement in the axis of reflection. **(7.2.1.2)**

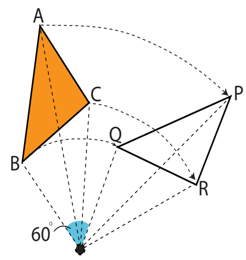
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Show interest in learning about Rotation and reflection of figures.
Skills	Explain rotation and reflection of figures their properties.
Knowledge	Rotation and reflection of shapes and their properties.
Mathematical Thinking	Think about the movement of shapes as they are rotated or reflected.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

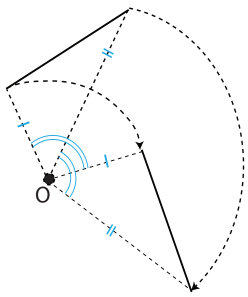
Teaching and Learning Activities

Example.1 **Rotation**

According to the figure on the right ΔPQR is rotated 60° clockwise from ΔABC around the centre of rotation 0

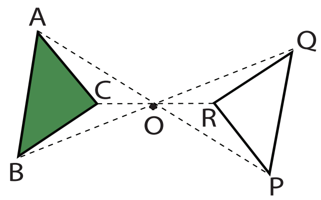


Then we can say the following about rotation of shapes in a plane;



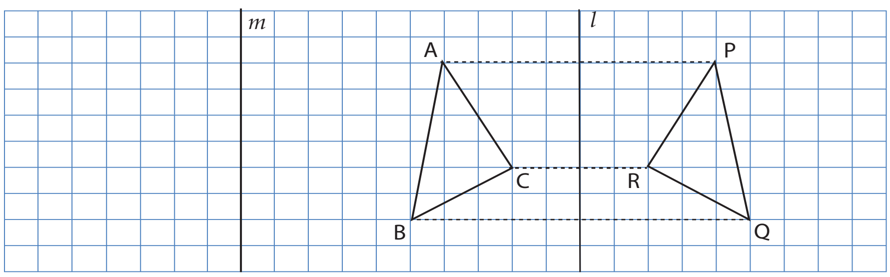
All corresponding points are at equal distance from the centre of rotation and the size of the angles made by linking each pair to the centre of rotation 0 are also equal.

In point symmetry rotation, the corresponding points and the centre of rotation 0 lie along the same line.



Example.2 **Reflection**

The figure on the right shows that shape Δ PQR is reflected from the shape ΔABC over the axis of reflection of line l



We can say the following about reflection.

(a) Reflected shapes have line symmetry with the axis of reflection.

(b) The segments linking corresponding points intersect the axis of reflection perpendicularly at a point that divides them into equal halves

L59 : Perpendicular bisectors

Lesson Objective: Explore perpendicular bisector properties and rules to construct shapes such as the Rhombus and other shapes.**(7.2.1.1/2)**

Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Enjoy constructing perpendicular bisectors of segment.
Skills	Use the tools to and perpendicular bisector properties to construct the shapes.
Knowledge	Perpendicular bisection and tools for construction.
Mathematical Thinking	Think about how to construct a perpendicular bisector and the shapes.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson

Teaching and Learning Activities

Review

A Rhombus is a figure with line symmetry and two diagonals which has an axis of symmetry.

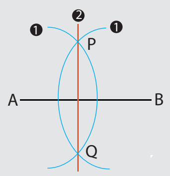
Example.1 **Perpendicular Bisector**

Within the rhombus one of the diagonals is the perpendicular bisector of the other diagonal. This means that we can construct a perpendicular bisector of segments AB, if we construct a rhombus ABPQ with one diagonal AB.

When constructing figures such tools are required;
The straight edge to draw a straight line
Compass to draw the circles and the copy the length
of the segment. You can use other ways to construct figures.

Key Ideas

Constructing perpendicular bisectors of segments



❶ Draw two circles with the two segments A and B at their centres.

❷ Label as P and Q the two points where the circles intersect and draw lines PQ connecting them.

Exercises

Construct Δ ABC in your notebook and then construct the following figures.

(a) The perpendicular bisector of side BC.

(b) The mid-point of side AB.

L60 : Angle bisectors

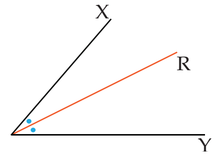
Lesson Objective : Construct an angle bisector and describe its properties **(7.2.1.1)**
Materials: protractor,triangular rulers

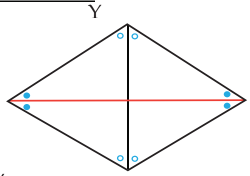
ASK-MT and Assessment	
Attitudes/Values	Enjoy constructing angle bisectors.
Skills	Construct an angle bisector and describe its properties.
Knowledge	Construction of an angle bisector in the lines.
Mathematical Thinking	Thinking about how to construct an angle bisector.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

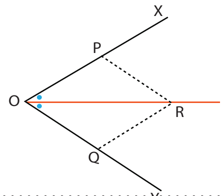
Example.1 **Angle bisectors**

In the figure on the right, line OR is the bisector of ∠XOY
In the Rhombus the diagonals bisect the angles formed at the vertices





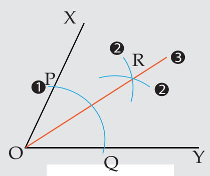
This means that we can construct the bisector of ∠XO Y if we think about the rhombus OQRP which sides are segments OP and QR along the lines OX and OY



Key Ideas

The line that divides an angle into two equal parts is called a **bisector** of that angle.

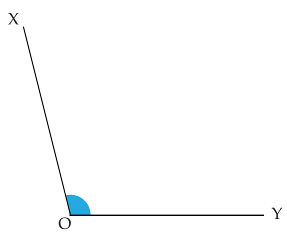
Constructing angle bisectors

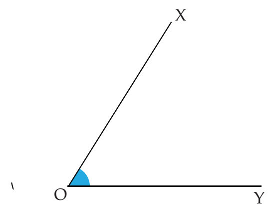


❶ Draw a circle with point O at centre, label as P and Q the points where the circle intersects lines OX and OY
❷ Draw two circles with P and Q at their centres and a radius of OP
❸ Label one of the points where they intersect R and draw line OR

Exercises

Construct the bisector of ∠XOY in the figures below





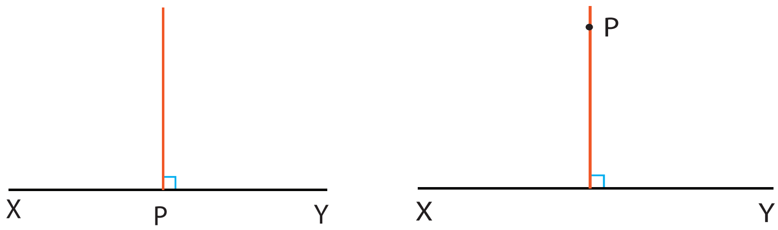
L61 : Perpendicular lines

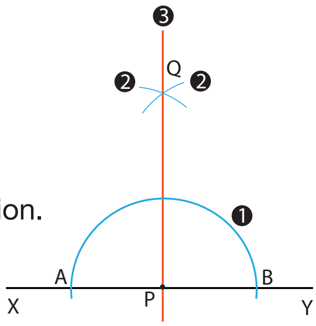
Lesson Objective: be able to construct perpendicular lines and shapes in the line segments. **(7.2.1.1)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Enjoy drawing perpendicular line.
Skills	Use perpendicular lines to construct shapes.
Knowledge	Perpendicular lines and their properties.
Mathematical Thinking	Think about how to construct shapes using perpendicular lines and their properties.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

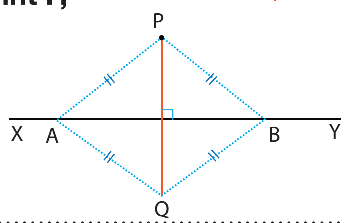
Example.1 **Line perpendicular to line XY that passes through the point P on XY**





Find the points A and B on line XY so that PA =PB
Construct a perpendicular bisector of AB and you have made the construction.
Perpendicular line bisects a 180° angle

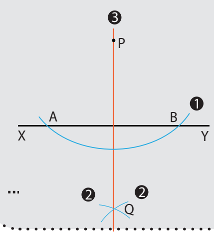
Example.2 **Line perpendicular to line XY that passes through point P, which is not on XY.**



As the figure on the right shows, if we construct rhombus PAQB so that diagonal AB is along
the line XY, then PQ ⊥XY

Key Ideas

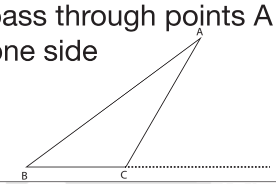
A perpendicular line bisects at 180° angle.
Constructing perpendicular lines



❶ Draw a circle with point P at the centre. Label as A and B the points where the circle intersects line XY.
❷ Draw two circles with A and B at their centres and a radius of PA
❸ Label one of the points where they intersect Q and draw line PQ

Exercises

1. In the figure on the right, draw the lines perpendicular to segment AB that pass through points A and B. Then use these lines to construct the square ABCD that has AB as one side
2. Draw the following perpendicular lines using ΔABC in the figure below.
   (a) The line perpendicular to side BC and that passes through vertex C.
   (b) The perpendicular line that extends from vertex A down to line BC.



L62: Properties of circles

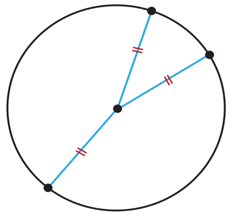
Circles and Sectors

Lesson Objective : Explore the properties of circles and sectors. **(7.2.2.3)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Appreciate and value the understanding of properties of circles and sectors.
Skills	Identify the properties of circles and sectors.
Knowledge	Properties of circles and sectors.
Mathematical Thinking	Think about ways to identify the properties of circle and sectors.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

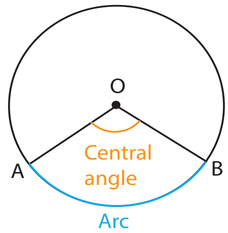
Teaching and Learning Activities

Example.1 **Circle arc and chords**



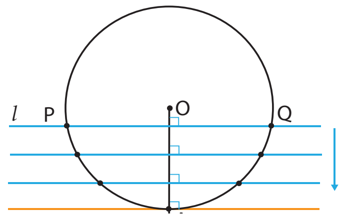
We refer to a circle with the centre point *O* as circle *O*, and the circumference around the circle is called its circumference. The lengths of the segments that link the centre point *O* with any point on the circumference are equal to, and are radii of the circle.

When we connect two points *A* and *B* on the circumference of a circle with centre O, we create $\angle AOB$ as shown in the right. In this case, we say $\angle AOB$ is the central angle of $\overset{\frown}{AB}$ and that $\overset{\frown}{AB}$ is the arc corresponding to **central angle** $\angle$ AOB

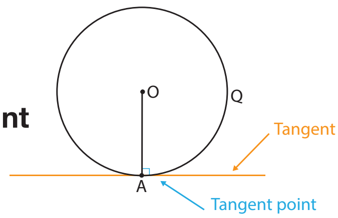


Example.2 **Circle arc and chords**

In circle *O* line l runs perpendicular to radius *OA* and intersects the circumference at *P* and Q. When then gradually move l towards A as show in the figure so that the two points of intersection get closer and closer together, finally overlapping with point A on the circumference.



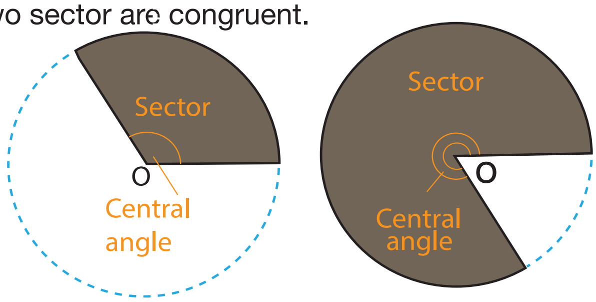
When a line intersects a circle at only one point like this, we say that the line is **tangent** to circle.
When line l is tangent to circle *O* as shown in the figure on the right, we say that line l is a **tangent** of circle *O* and that point *A* is the **tangent point**



Teaching and Learning Activities

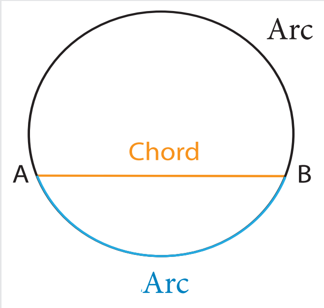
Example.2 **Sectors**

Shapes that are bound by two radii of a circle and an arc are called **sectors.**
The angle made by the radii in the sector is called the **central angle.**
The figure on the right shows two sectors whose central angles are equal.
If sector *OAB* is rotated around the centre point *O*, it will overlap sector *OCD* exactly – so the two sector are congruent.



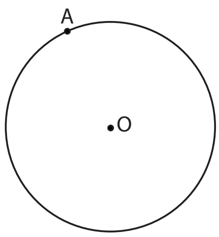
Key Ideas

The part of the circumference between any two points **A** and **B** on the circumference of a circle is **arc** *AB* and written $\overset{\frown}{AB}$
A segment that links the two endpoints of $\overset{\frown}{AB}$ is called chord *AB*
Properties of tangents to a circle
Tangents to a circle are perpendicular to the radius that passes through their tangent point.
When the radii and central angles of two sectors are equal, the length of their arcs and their areas are also equal.



Exercises

1. When chord *AB* is a diameter, what is the measure of the central angles corresponding to $\overset{\frown}{AB}$?
2. What is the relationship between diameter l and chord $\overset{\frown}{AB}$ in the figure above?
3. Draw a line tangent to circle *O* on the right where point *A* is the tangent point.



L63: Measuring circles and sectors

Lesson Objective: To find the circumference, arc length and area of circles and sector. **(7.2.2.3)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Share ideas on how to identify circumference, arc length and area of circles and sectors.
Skills	Identify circumference, arc length and area of circles and sectors.
Knowledge	Circumference, arc length and area of circles and sectors.
Mathematical Thinking	Think of ways on how to find circumference, arc length and area of circles and sectors.
Assessment	use the ASK-MT to assess the students learning progress during the lesson.

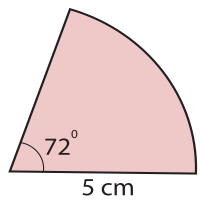
Teaching and Learning Activities

Example.1 **Circumference and area of a circle**

For a circle with a radius of 5 cm
Circumference ....$2\pi \times 5 = 10\pi$ (cm)
Area ....$\pi \times 5^2 = 25\pi$ (cm)

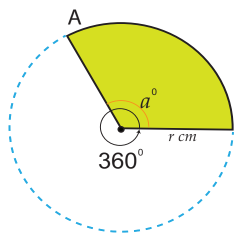
TN: Pi is the ratio of the circumference of a circle to its diameter.
More precise is

3.1415926535897932384626433832795028841971693................
but its approximate value 3.14 is often used



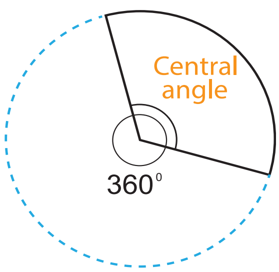
Example.2 **Sector arc length and area**

For a sector with a radius of 5 cm and a central angle of 72°
Circumference ....$2\pi \times 5 \times \frac{72}{360} = 2\pi$ (cm)
Area ....$\pi \times 5^2 \times \frac{72}{360} = 5\pi$ (cm)



For any sector with a radius of r cm and a central angle of a°, the length of the arc is $\frac{a}{360}$ of the circle
circumference $2\pi r$ cm and the area is $\frac{a}{360}$ of the circle area $2\pi r$ cm^2

In the a circle, the ratio of the arc lengths and the areas of sectors are equal to the ratio of the size of the central angles of the sectors.
If we use a propositional equation, we can also say the following about the relationship of length and are in circle and sectors.



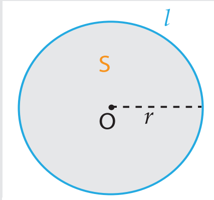
Teaching and Learning Activities

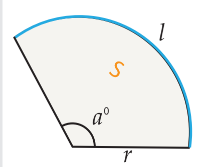
Key Ideas

Circumference and area of a circle

Given a radius r, circumference l and are S
Circumference.............$l = 2\pi r$
Area...........................$S = \pi^2$

Given a radius r, circumference l and are S
Arc length ....$l = 2\pi r \times \frac{a}{360}$
Area ....$S = \pi^2 \times \frac{a}{360}$



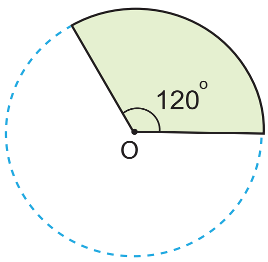


For any circle and sector with equal radii,
(length of sector arc) : (circumference of circle) = (size of central angle): 360
(area of sector) : (area of circle) = (size of central angle) : 360

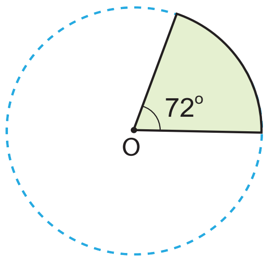
Exercises

1. Find the circumference and are of area of a circle with a diameter of 20 cm.
2. The length of the sector arc below is what portion of the circumference of a circle with the same radius? What about the area?

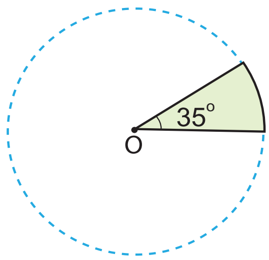
(a)



(b)



(c)



3. Find the arc length and are for the sectors below.

(a) Radius 6cm, central angle 60°

(b) Radius 4cm, central angle 225°

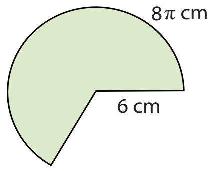
L64 : Central angle of a Sector

Lesson Objective : To find the central angle of a sector. **(7.2.2.3)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Share ideas on how to identify the central angle of a sector
Skills	Identify the central angle of a sector
Knowledge	Central angle of a sector
Mathematical Thinking	Think of ways on how to find the central angle of a sector
Assessment	Use the ASK-MT to assess the students learning progress during the lesson

Teaching and Learning Activities

Example.1 **How to find the central angle of a sector**
You have a sector with a radius of 6cm and an arc length of 8π cm.
Find the measure of the sector's central angles.



Approach: Find the circumference of a circle with a radius of 6cm.
Set up a proportional equation
where $x°$ is the central angle.

Solution: The circumference of the a circle with a radius of 6 cm is 12π,
so if we let the central angle be $x°$,

$$8\pi:12\pi = x:360$$

IF we solve this we get

$$12\pi x = 8\pi \times 360$$
$$x = 240$$

Key Ideas

Review
For any proportional equation
If $a:b = c:d$,
$ad = bc$,

Exercises

1. Find the area of the sector in *Example.1*

2. Find the size of the central angle and the area for a sector with a radius of 9 cm and an arc length of 5π cm.

Unit Checkpoint

Review on Plane figures

Review on Plane figures

Lines and angles

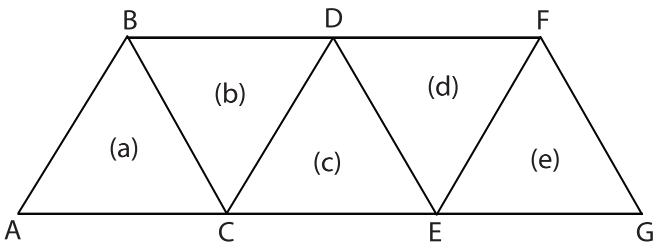
Figure transformation

1. Enter the correct word or symbol in the □ below.

(a) When two lines AB and CD intersect at a right angle, we say that AB and CD are □ and express this as AB □ CD.

(b) When two lines AB and CD do not intersect, we say that AB and CD are □ and express this as AB □ CD

2. The triangles (a) through € in the figure below are all congruent equilateral triangles. State all of the triangles that apply to (a) through (c) below.

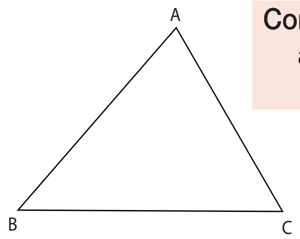


(1) The triangles(s) translated from (a)
(2) The triangle(s) rotated from (a) around the centre of rotation C
(3) The triangle(s) reflected from (a) across the axis of reflection segment BC.

Constructing figures with a straight edge and compass

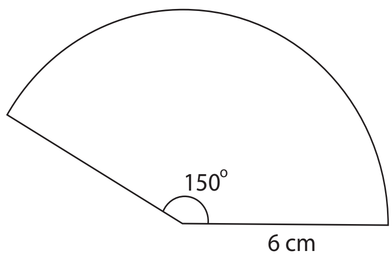
3. Construct the following figures in Δ ABC on the right.

(a). The perpendicular bisector of side AB.
(b). The bisector of side ∠ C.



4. Find the arc length and area of a sector with a radius of 6cm and a central angle of 150°.

Sector arc length and areas



L65 : Classifying Solids

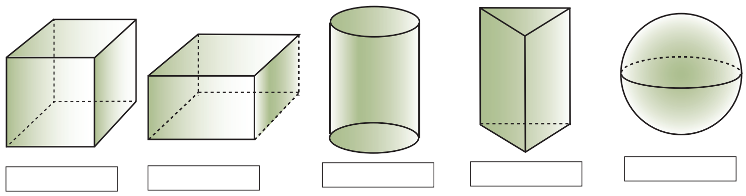
Solids and space figures

Lesson Objective : Identify the characteristics of solids by viewing them in different ways. **(7.2.2.1/3)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Be able to share their ideas on how to identify the characteristics of solids.
Skills	Be able to identify characteristics of solid through observation.
Knowledge	Be able to understand the characteristics of solid.
Mathematical Thinking	Be able to think of ways to view and identify the characteristics of the solids.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

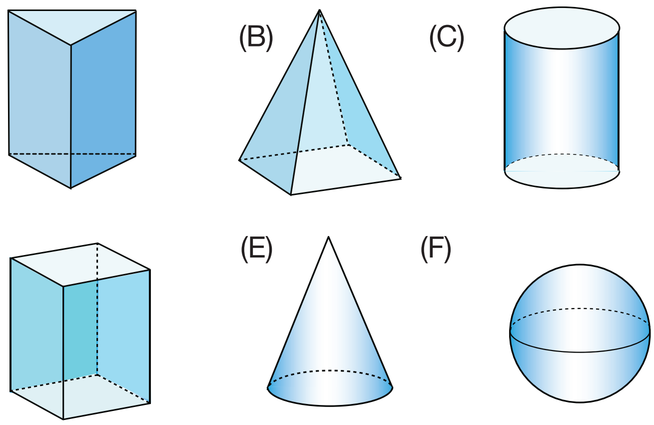
Review
Write the name of the solids below.



Example.1 **Characteristics of various solids**
Classify according to similar characteristics

(A) (B) (C)

(D) (E) (F)



Discuss

(a) What are the shared characteristics of solids (B), (E), and (G) above?

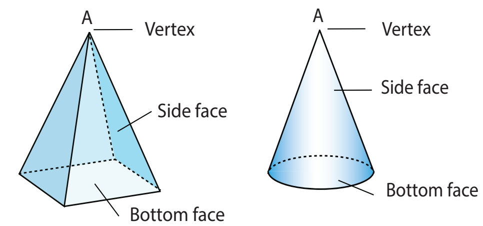
(b) In prism (A) and cylinder (C) above, which are the bottom faces and which are the side faces?

A pyramid with a triangular bottom face is called a triangular pyramid, while a pyramid with a quadrangular bottom face is called a **quadrangular pyramid.**

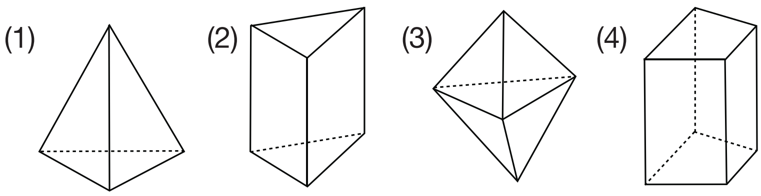
Teaching and Learning Activities

Key Ideas

- Solids (A) and (D) are prisms, solid (C) is a cylinder, and solid (F) is a sphere. Solid like (E) and (C) are called pyramids, and solids like (E) are cone.
- Pyramids and cones have bottom faces and sides. The points marked A in the figures are the vertices of the pyramid and the cone
- Solids made up of multiple of planes (flat faces) are called polyhedral (singular: polyhedron), and are typically named according to the number of faces as in tetrahedron, pentahedron, hexahedron, and so on



Exercises



1. How many faces are on each of the solids above?

2. Which polyhedron has least number of faces?.

L66 : Nets of various solids

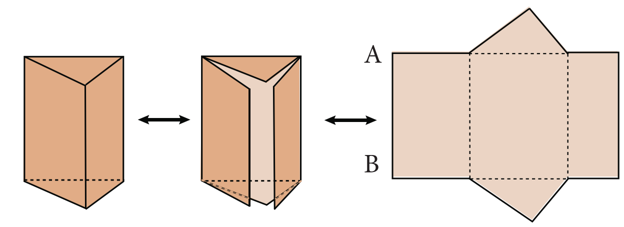
Lesson Objective: Identify different nets various solid according to the bottom faces. **(7.2.2.2)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Be able to share their ideas on identifying different nets of various solids.
Skills	Explore the different nets of various solids.
Knowledge	Nets of various solids.
Mathematical Thinking	Be able to think about the nets of various solids.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

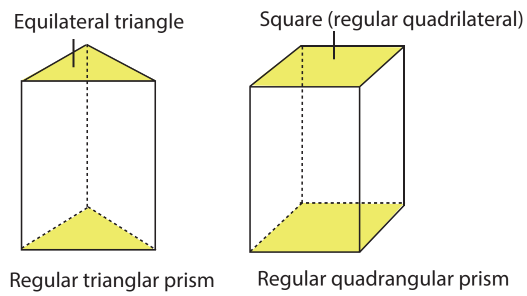
Teaching and Learning Activities

Example.1. **Prisms**

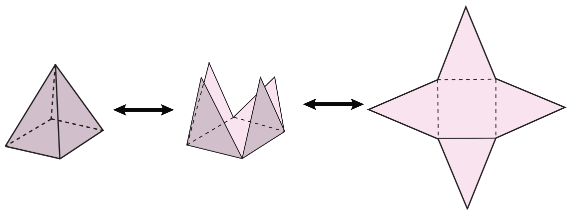
Below is a sketch and a net for a triangular prism.
Prisms have two congruent polygons as bottom faces and rectangular side faces.



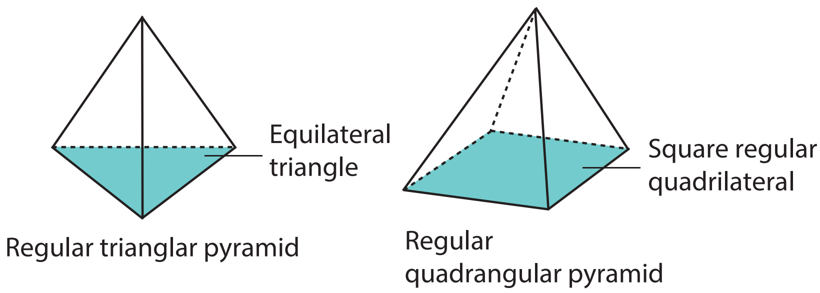
Prisms have different names depending on the shape of the bottom face. For example, a regular triangular prism has equilateral triangles for bottom faces, and a regular quadrilateral prism has squares(regular quadrilaterals) for its bottom faces.



Example.2 **Pyramid**



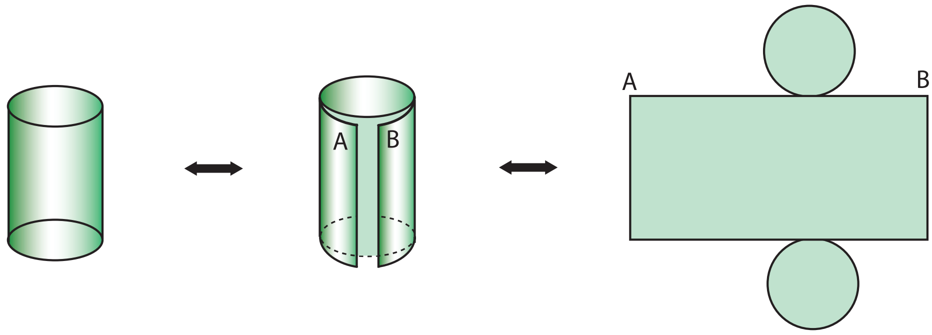
Pyramids have different names depending on the shape of the bottom face. If all side faces are congruent isosceles triangles, a regular triangular prism has an equilateral triangle for a bottom faces, a regular quadrangular pyramid has a square for a bottom face, and so on.



Teaching and Learning Activities

Example.3 **Cylinder**

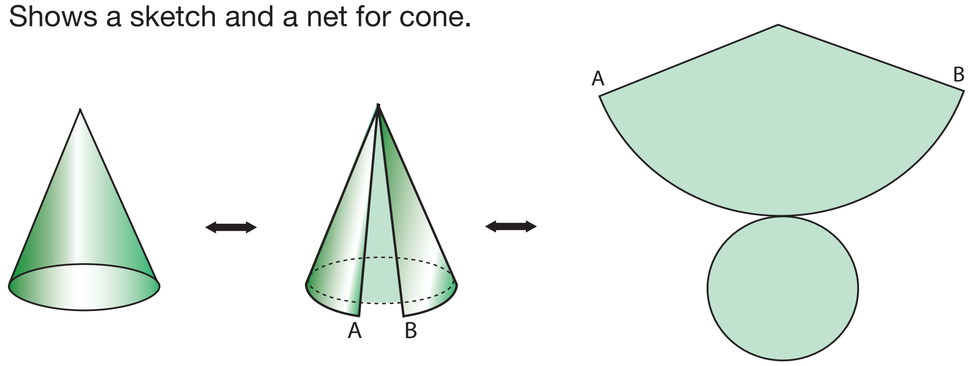
Diagram 1 Shows a sketch and a net for a cylinder.



Cylinders have two congruent circles as bottom faces, and the side face is a curved surface. In the net for a cylinder, the side face becomes a rectangle.

Example.4 **Cone**

Diagram 2 Shows a sketch and a net for cone.



Cones have one bottom face in the shape of a circle. The side face is a curve surface. In the net for a cone, the side face becomes a sector.

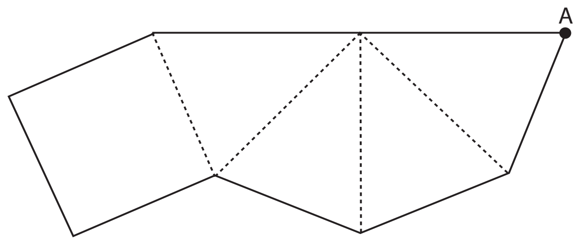
Exercises

(a) Use a “O”to mark the points that will at point A if we make a triangular prism out of the net shown above. Mark the side that will join side AB with a ~ .

(b) Use a “O” to mark the points that will join at point A if we make a quadrangular prism out of the net shown above

(c) If we make a cylinder out of the net shown above, which sides will join segment AB? Mark it with an ~.

(d) If we make a cone out of the net shown above, which part will join (AB) ? Mark it an ~.



L67 : Planes and lines in the space

Lesson Objective : Think about the positional relationship between planes and lines by looking at the faces and sides of solids **.(7.2.2.1)**

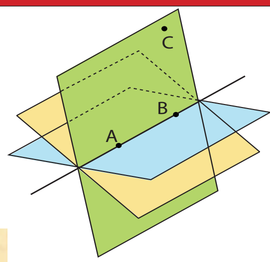
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Be able to share their ideas on how to identify the planes and parallel lines in the space.
Skills	Be able to identify the planes and parallel lines in the space.
Knowledge	Be able to understand planes and parallel lines in the space.
Mathematical Thinking	Be able to think of ways to view and identify the planes and parallel lines in the space.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

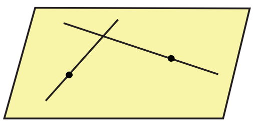
Introductory

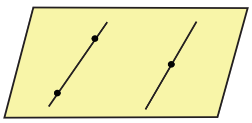
Look at the figure on the right. There are an unlimited number of planes that contains the two points A and B, but there is only one plane that passes through point C, which does not lie along line AB, Based on this, we can say that;



There is only one plane that can pass through three points that are not on the same line.

In addition, there is only one plane that includes any two intersecting or parallel lines.


Two intersecing lines


Two parallel lines

L68 : Graphing proportion

Lesson Objective: Investigate the relationship between any two lines through examining positional relationships. **(7.2.2.1/2)**

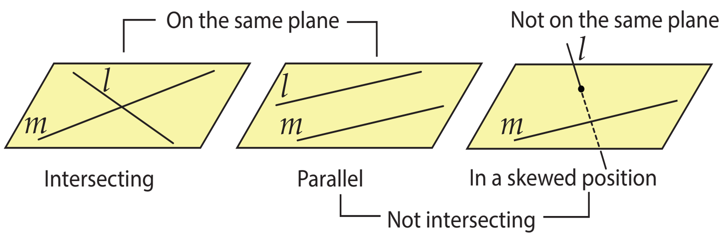
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Be able to share their ideas on how to identify relationships between any two lines in a space.
Skills	Be able to identify the relationship of any two lines by examining the positional relationship of lines.
Knowledge	Be able to understand positional relationship of any two.
Mathematical Thinking	Be able to think of ways to view any two lines in space.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Review

Use two pencils to represent two lines. Indicate various positional relationships that the two lines could have.
The three possible positional relationships that the two lines l and m can have in space are shown below.



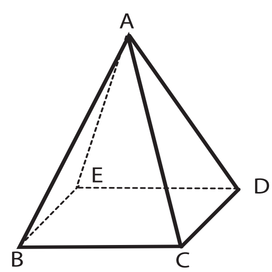
Key Ideas

If two lines in space are neither parallel nor intersect, those two lines are said to be in a skewed position

Exercises

1. If state which of the lines in the regular quadrangular pyramid on the right have the following relationship to the line BC.

   (a) Intersecting
   (b) Parallel
   (c) In a skewed position



L69 : Positional relationship between lines and planes

Lesson Objective : Identify the relationship between planes and lines. **(7.2.2.2)**
Materials: blackboard

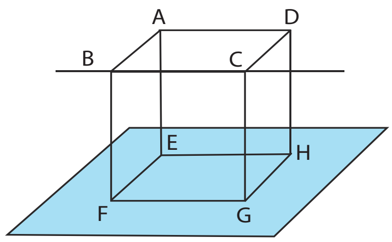
ASK-MT and Assessment	
Attitudes/Values	Share their ideas on how to identify relationship between planes and lines.
Skills	Examine and identify the relationship between planes and lines.
Knowledge	Relationship between planes and lines.
Mathematical Thinking	Think of ways to identify relationship between planes and lines.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Review

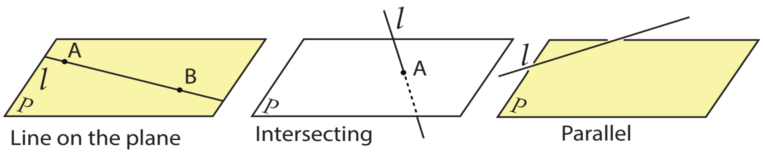
Think of the edges in the cube on the right as lines and the faces as planes. What is the positional relationship between line BC and plane EFGH?

What is the relationship between line BC and the other five planes on the figure?



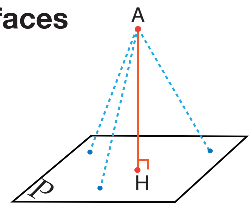
Example.1 **Possible positional relationships quadrangular faces**

The three possible positional relationships that the two lines l and m can have in space are shown below.

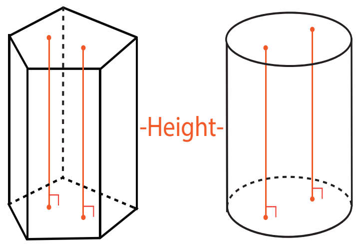


Example.2 **Possible positional relationships between prisms and cylinder faces**

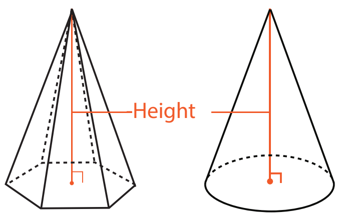
In the figure on the right, a line perpendicular to plane P is drawn from point A, and the line intersects plane P at point H. Of all the segments that connect point A to any point on plane P, segment AH is the shortest.



In prisms and cylinders, the distance between a point on a bottom face and the top face is always equal, and that distance is called the height of the prism or cylinder.



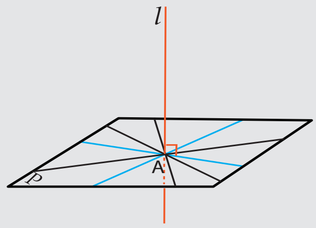
In pyramids and cones, the distance between the vertex and the bottom face is called the height of the pyramid or cone.



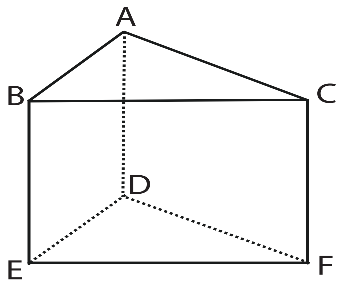
Teaching and Learning Activities

Key Ideas

- If a line l does not intersect a plane P, line l and plane P are said to be parallel
- If line l intersect plane P at point A and all lines that lie on plane P and pass through point A are perpendicular to the line l, line l and plane P are said to be perpendicular. In this case, we say that line l is perpendicular to plane P.
- The length of segment *AH* is the **distance between point A and plane P.**



Exercises



1. State which of the lines in the triangular prism on the right have the following relationship to plane ABC.

   (a) On plane ABC
   (b) Perpendicular to and intersecting plane ABC
   (c) Parallel to plane ABC

2. The figure on the right shows a triangular pyramid cut out of cube. Which length gives the height when the following faces are the bottom face of the pyramid?

   (a) When plane BCD is the bottom face?
   (b) Which plane ACD is the bottom face?

L70: Positional relationships between two planes

Lesson Objective: Identify the relationship between any two lines through examining positional relationships. **(7.2.2.1/2)**

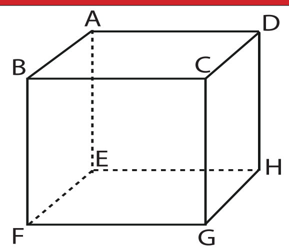
Materials: diagram chart of cube

ASK-MT and Assessment	
Attitudes/Values	Share their ideas on how to identify positional relationships between planes.
Skills	Identify positional relationships between planes.
Knowledge	Positional relationships between planes.
Mathematical Thinking	Think of ways to view positional relationships between planes.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson

Teaching and Learning Activities

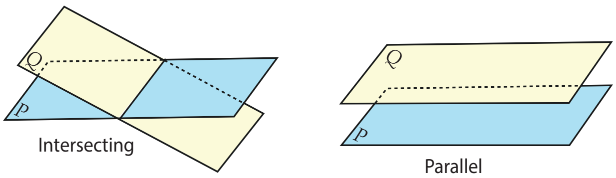
Review

Learn about the positional relationship between plane ABCD and the other planes in the cube on the right.



Example.1 **Two possible positional relationships of planes**

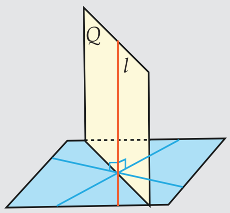
The two possible positional relationships that two planes P and Q can have are shown below.



The intersection between two planes is a line.

Key Ideas

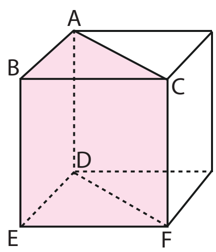
- If two planes P and Q do not intersect, they are said to be **parallel**
- Plane P and plane Q intersect in the figure on the right.
  If plane Q contains a line l that is perpendicular to plane P, plane P and Q are said to be **perpendicular.**



Exercises

The cube on the right can be cut into two triangular prisms.
State the planes that have the following relationships.

(a) The plane parallel to plane ABC
(b) The plane perpendicular to ABED



L71 : Various ways of looking at solids

Lesson Objective : Identify the solids created by the parallel movement of a face , created by revolution of a face and by moving lines. **(7.2.2.1/3)**

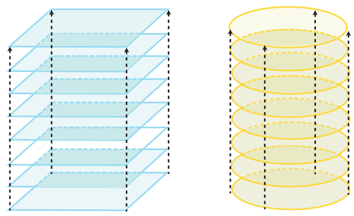
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Work collaboratively with each other to identify the solids created by the parallel movement of a face, created by revolution of a face and by moving lines.
Skills	Identify the solids created by the parallel movement of a face , created by revolution of a face and by moving lines.
Knowledge	Solids created by the parallel movement of a face , created by revolution of a face and by moving lines.
Mathematical Thinking	Think of ways to view and identify the solids created by the parallel movement of a face, created by revolution of a face and by moving lines.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

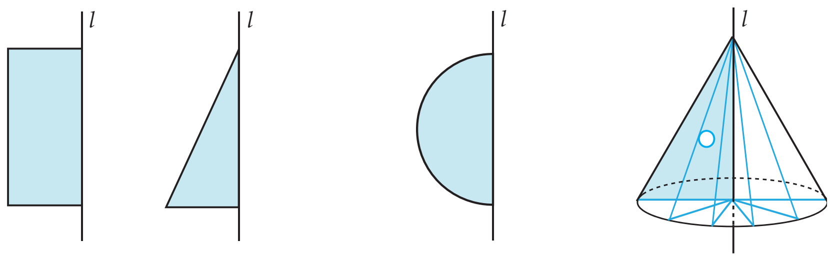
Example.1 **Solids created by the parallel movement of a face**

We can also see prisms and cylinder as solids that are created by parallel movement of polyhedron or circle for a certain distance in a direction perpendicular to that face.

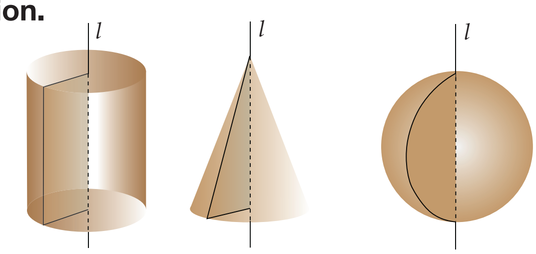


Example.2 **Solids created by the revolution of a face**

Discuss: In figure below, what solids will be created if we rotate the rectangle, right triangle and semi-circle once around line l

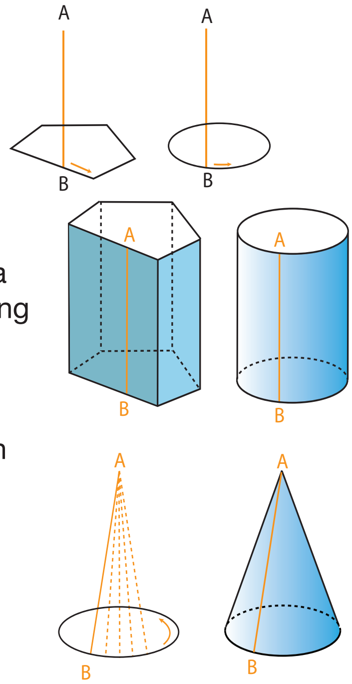


Cylinders, cones, spheres and so on can be seen as solids created by rotating a single plane figure around a line l located on that plane.Solids like these are called **solids of revolution** and line l is called their **axis of revolution.**



Teaching and Learning Activities

Example.3 **Solids created by moving lines**



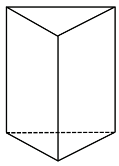
Discuss: The figure on the right shows a segment AB remaining perpendicular to polyhedron and a circle as it travels once around the perimeter of the shape. What shape will segment AB create through this movement.

We can see the side faces of prisms and cylinders as one revolution of a segment perpendicular to the polygonal or circular bottom faces travelling along the perimeter of the shapes. In this case, the segment making the revolution is called the **generatrix** of the prism or cylinder.

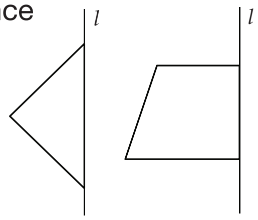
In the figure on the right, segment AB connect vertex A and a point B on the circumference of the bottom face. We can see the side face of the cone as the movement of segment AB as point B makes one revolution around the bottom face. In this case, segment AB is called the generatrix of the cone.

Exercises

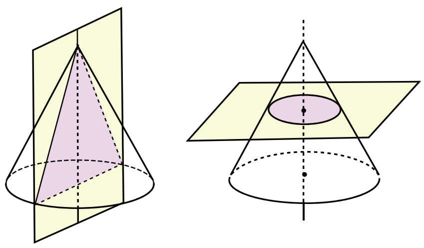
1. What shape can be moved to create a triangular prism? How is the shape moved?



2. Draw a sketch of the solid revolution that is formed when the figures on the right are rotated once around the axis of revolution line l.



3. If we pass a plane that includes the axis of revolution a cone, what shape is the cut out that we create? If we pass a plane perpendicular to the axis of revolution through a cone, what shape is the cut out?



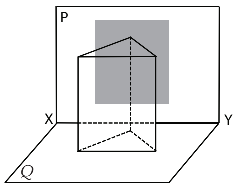
L72 : Projection of solids

Lesson Objective: Explore and understand the projection view of various solid. **(7.2.2.2.)**
Materials: Drawn pictures of solids

ASK-MT and Assessment	
Attitudes/Values	Be able to share their ideas on how to identify the solids created by revolution of faces and moving lines.
Skills	Be able to identify the solids created by revolution of faces and moving lines.
Knowledge	Be able to understand the solids created by revolution of faces and moving lines.
Mathematical Thinking	Be able to think of ways to view and identify the solids created by revolution of faces and moving lines.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introductory



Two perpendicular planes P and Q intersect at line XY. A triangular prism is positioned as shown in the figure on the right.
What is the shape of the shadow created by parallel beams of light that are perpendicular to plane P?
What is the shape when the parallel beam of light strikes plane Q at a perpendicular angle?

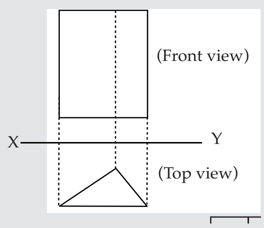
Example.1 **Projection view of solids**

There is a way to represent a solid by combining its shape when we look at it directly from the front and from the top.

The projection view for the triangle prism as an example is shown on the right.
When drawing projections, sides that can actually be seen are represented as a line_____, while sides that cannot be seen are represented as dotted lines .

Key Ideas

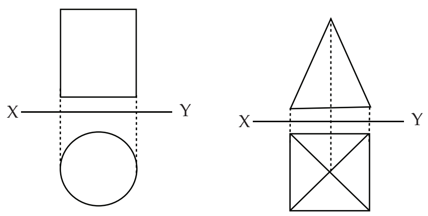
- If two planes *P* and *Q* do not intersect, they are said to be **parallel**
- Plane *P* and plane *Q* intersect in the figure on the right.
  If plane *Q* contains a line l that is perpendicular to plane *P*,
  plane *P* and *Q* are said to be **perpendicular.**



Exercises

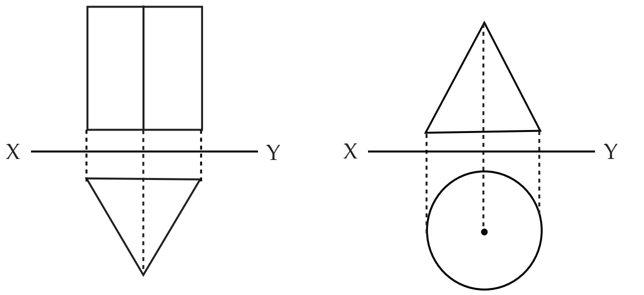
1. Which solids are represented in the projections below, a cuboid, triangular pyramid, quadrangular pyramid, cylinder, cones, or sphere?

(1

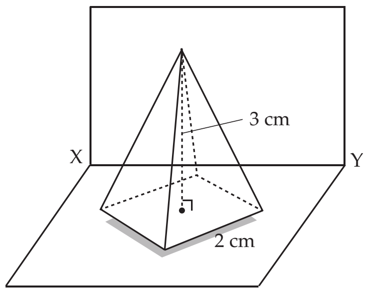


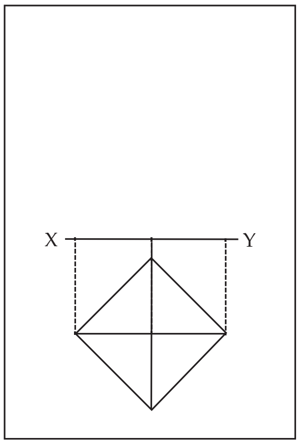
Teaching and Learning Activities

2. Sketch the solids in the projections below.



3. You have a regular quadrangular pyramid that is 3 cm high and has a square base with 2 cm sides. Complete the projection on the front view for this regular quadrangular pyramid.





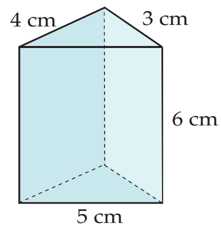
L73: Surface area of prisms and cylinders

Lesson Objective : identify the surface area of solids and calculate the area. **(7.2.2.3)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Enjoy identify surface area of solids and calculate.
Skills	Identify the surface area of solids and calculate the areas.
Knowledge	Surface area.
Mathematical Thinking	Think of ways to view and identify surface area of different.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

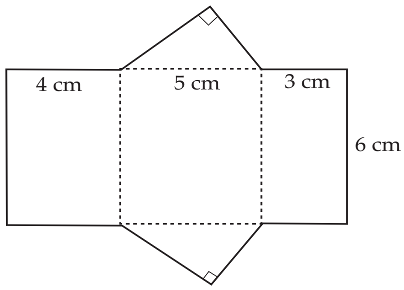
Teaching and Learning Activities

Review
You have a triangular prism like the one on the right.
Find the area of the side faces.



The side faces of the triangular prism in the above became a rectangle in the net. This tells us that the lateral area is:

$$6 \times (4+5+3) = 72\ (\text{cm}^2)$$



Example.1 **Lateral area of a cylinder**

Lateral area of a cylinder 10 cm high with a bottom face whose radius is 4 cm.

As you can see in the figure on the right, the side face of a cylinder is a rectangle in the net. This means that:

Length of rectangle

= height of cylinder

= 10 (cm)

Width of rectangle

= circumference of the circle on the bottom face

= $2\pi \times 4$ cm

Therefore, the lateral area is

$10 \times 2\pi \times 4 = 80\pi\ (\text{cm}^2)$

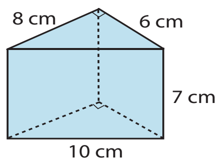
Teaching and Learning Activities

Key Ideas

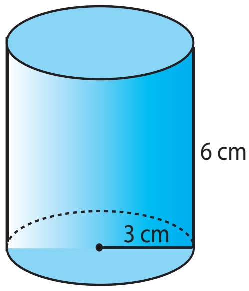
The area of the entire surface of a solid is called the **surface area.** The area of one of the bottom faces is called the **base area**, and the total surface area of the side faces is called the **lateral area**

Exercises

1. Find the surface area of the triangular prism on the right.



2. Find the surface area of the cylinder in *Example.1.*
3. Find the lateral area and surface area of the cylinder below.



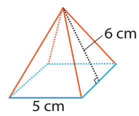
L74 : Surface area of pyramids and cones

Lesson Objective: Identify and calculate the surface area of pyramids and cones. **(7.2.2.2/3)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Confidently calculate surface area of pyramids and cones.
Skills	Identify and calculate the surface area pyramids and cones
Knowledge	Surface area of pyramids and cones
Mathematical Thinking	Think of how to calculate surface area of pyramid and cones.
Assessment	use the ASK-MT to assess the students learning progress during the lesson.

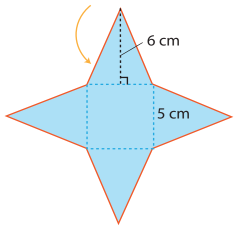
Teaching and Learning Activities

Example.1 **Surface area of a regular quadrangular pyramid**



Find the lateral area of a cone whose bottom face has a radius of 6cm and whose generatrix has a length of 9 cm.
The bottom face is a square with 5-cm sides, so the base is

$$5 \times 5 = 25 \text{ (cm}^2\text{)}$$

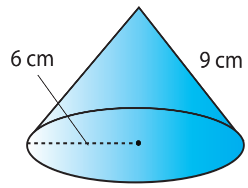


The four side faces are congruent isosceles triangles with 5-cm bases and heights with 6cm, so the lateral area is

$$(\frac{1}{2} \times 5 \times 6) \times 4 = 60 \text{ (cm}^2\text{)}$$

Therefore, the surface area is

$$25 + 60 = 85$$



Example.2: **Lateral area of cones**
Find the lateral area of a cone whose bottom face has a radius of 6cm and whose generatrix has a length of 9 cm.

Approach: Think about the net for the side face of a cone. It is a sector with a radius of 9 cm, and the length of the arc is the equivalent to the circumference of the circle that forms the base face of the cone.

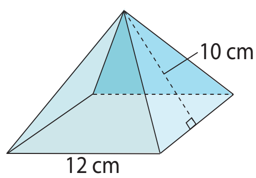
Solution: The net of the side face is a sector with radius of 9cm.
Let $x°$ be a central angle, $(2\pi \times 6) : (2\pi \times 9) = x : 360$
If we solve it , we get

$$x = 240$$

Therefore, the lateral area is

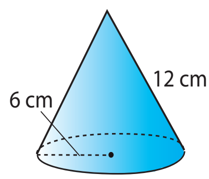
$$(\pi \times 9^2) \times \frac{240}{360} = 54\pi \qquad (54\pi \text{cm}^2)$$

Exercises



1. Find the surface area of the regular quadrangular pyramid on the right

2. Find the lateral area of a cone whose bottom face has a radius of 8 cm and whose generatrix has a length of 12 cm.



3. Find the surface area of the cone on the right.

L75 : Volume of prisms and cylinders

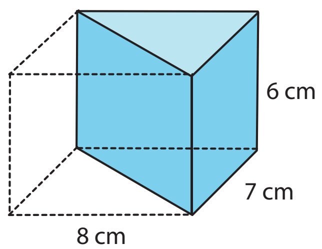
Lesson Objective : Find the volume of prisms and cylinders. **(7.2.2.3)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Share their ideas on how to find volume of prisms and cylinders.
Skills	Calculate the volume of prisms and cylinders.
Knowledge	Calculating Volume of prisms and cylinders.
Mathematical Thinking	Think of ways to calculate volume of prisms and cylinders.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

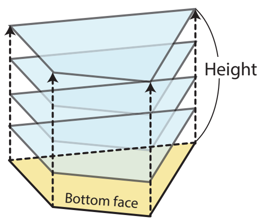
Review

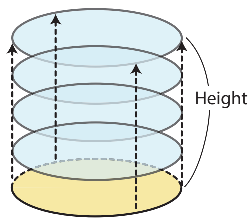
The figure on the right shows a cuboid cut into two triangular prisms. Find the volume of the prism.



Find the volume of the solid.

Example.1 **volume of prisms and cylinder**
You can find the volume of a prism or cylinder using Base area × height





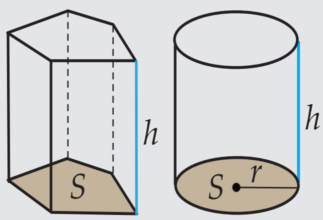
Key Ideas

Volume of prisms and cylinders.
For a prism or cylinder with a base area S, height h, and volume V

$$V = Sh$$

For cylinder in particular , if the radius of the circle that forms the bottom face is r,

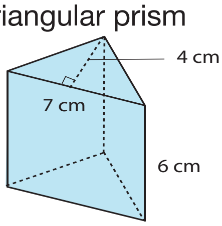
$$V = \pi r^2 h$$



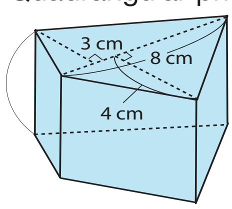
Exercises

Find the volume of the solid.

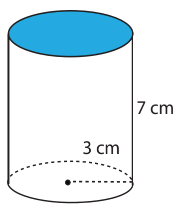
(a) Triangular prism



(b) Quadrangular prism



(c) Cylinder



L76 : Volume of pyramid and cones

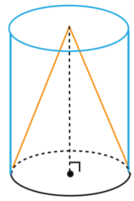
Lesson Objective: Find volume of pyramids and cones. **(7.2.2.3)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Share their ideas on how to find volume of pyramid and cones.
Skills	Calculate the volume of pyramid and cones.
Knowledge	Calculating volume of pyramids and cones.
Mathematical Thinking	Think of ways to calculate volume of pyramids and cones.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introductory

The figure on the right shows a cylindrical and conical container with congruent bottom faces and equivalent heights. How many cones full of water will fit into the cylindrical container?



Example.1 **Pyramid and cone**

If we do an experiment like the one in the pictures below, we find that when their bottom faces and heights are equivalent, three cones full of water fit into the cylindrical container.

This tells us that the volume of the cone is equal 1/3 to the volume of the cylinder. We can say the same thing about a prism and pyramid that have congruent bottom faces and equal heights.

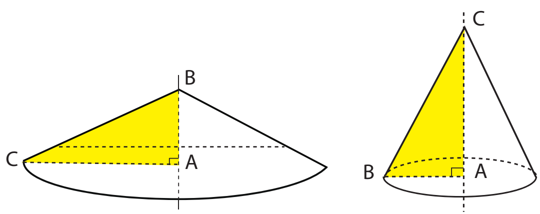
Example.2 **Volume of solids revolution**

The right triangle ABC shown on the right is used to make the two solids of revolution below. Which solid has a larger volume?

(a) A solid created by one revolution along axis AB

(b) A solid create by one revolution along axis AC

Approach: The solids described in (A) and (B) above are the cones shown below. Consider their heights as well as each radii of the circles that form the bottom faces in each.



Teaching and Learning Activities

Key Ideas

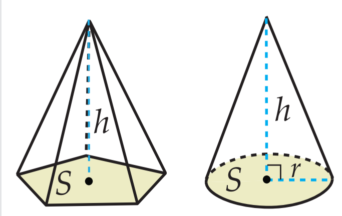
Volume of pyramids and cones

For a pyramid or cone with a base area S, height h, and volume V

$$V=\frac{1}{3}Sh$$

For cones in particular , if the radius of the circle that forms the bottom face is r,

$$V=\frac{1}{3}\pi r^2 h$$



Exercises

1. Find the volume of the solid.
   (a) A regular quadrangular pyramid with a height of 15 cm and where the bottom face is a square with 8-cm sides

   (b) A cone with a height of 20 cm and a bottom face with a 4-cm radius.

2. Find the volumes of solids (a) and (B) in *Example 2*. Which is larger?

L77 : Volume and surface area of spheres

Lesson Objective : Find the volume and surface area of spheres. **(7.2.2.3)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Share their ideas on how to calculate volume and surface areas of spheres
Skills	Calculate volume and surface areas of spheres
Knowledge	Calculating volume and surface areas of spheres
Mathematical Thinking	Think of ways to calculate volume of spheres
Assessment	Use the ASK-MT to assess the students learning progress during the lesson

Teaching and Learning Activities

Introductory

You have two containers like the ones show in the next page. One is a hemisphere with a radius of 5cm, and the other is a cylinder with a height of 10cm and whose bottom face has a radius of 5cm. How many full (A) containers will fit into container (B)?

If we do an experiment using containers (A) and (B) in the above, we find that three (A) containers full of water fit into container (B).

This tells us that the volume of hemisphere (A) is equal to $\frac{1}{3}$ the volume of cylinder (B)

The volume of cylinder (B) is

$$\pi \times 5^2 \times 10 \text{ (cm}^2)$$

So the volume of a hemisphere is

$$\frac{1}{3} = \pi \times 5^2 \times 10 \text{ (cm}^2)$$

which means that the volume of a sphere with a radius of 5 cm is

$$2\times \frac{1}{3} \times \pi \times 5^2 \times 10 = 2 \times \frac{1}{3} \pi \times 5^2 \times 2 \times 5$$
$$= \frac{4}{3} \times \pi \times 5^2 \text{ (cm}^2)$$

Example.1 **Volume of a sphere with a 6-cm radius**

$$\frac{4}{3} \pi \times 6^3 = 288\pi \text{ (cm}^3)$$

Example.2 **Surface area of a sphere with a 6-cm radius**

$$4\pi \times 6^2 = 144\pi \text{ (cm}^3)$$

Teaching and Learning Activities

Key Ideas

Volume of spheres

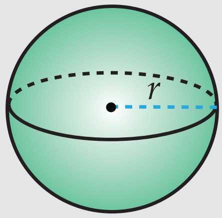
For a sphere with radius r, and volume V

$$V = \frac{4}{3}\pi r^3$$

Surface area of spheres

For a sphere with radius r, and surface area of S

$$S = 4\pi^2$$



Exercises

1. Find the volume of the sphere.

   (a) Radius 3 cm (b) Diameter 8cm

2. Find the surface area of the sphere.

   (a) Radius 3 cm (b) Diameter 8cm

1. Find the volume of the sphere.

   (a) Radius 3 cm (b) Diameter 8 cm

2. Find the surface area of the sphere.

   (a) Radius 3 cm (b) Diameter 8 cm

Unit Checkpoint

Review on Space figures

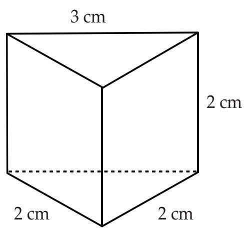
Review on Space Figure

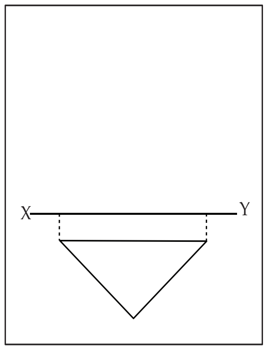
1. Use the triangular prism on the right.
   State the word or symbol that goes in ☐.
   (a) Line BE and line Ac have a(n) ☐ positional relationship
   (b) Plane ☐ is parallel to line CF.
   (c) Plane ☐ is parallel to plane ABC.

2. Fill in the ☐ with the appropriate word.

   (a) We can see a triangular prism as being created by parallel movement of ☐ for a certain distance in direction perpendicular to that face.
   (b) A solid created by rotating a rectangle once around one of its sides as an axis is a ☐.

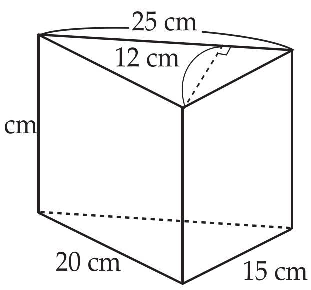
3. You have a triangular prism like the one in the figure below. Complete the projection on the right by adding the front view for this triangular prism.



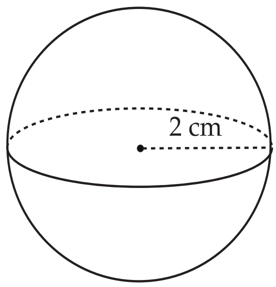


4. Find the surface area and volume for each of the solids in the figure below.

(a) Triangular prism



(b) Sphere with a 2-cm radius



L78 : Frequency distribution table

Using data trends

Lesson Objective : Organizing data in to a frequency distribution table and explain information gathered **(7.4.1.1/2)**

Materials: Sample of data

ASK-MT and Assessment	
Attitudes/Values	Share information and ideas on the frequency table.
Skills	Read and interpret information on the frequency distribution table.
Knowledge	Organize, read and explain data.
Mathematical Thinking	Think of how to organize, read and interpret the information on the frequency table.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introductory

Tables 1 and 2 below show the result of the experiment on the previous lesson.
Use them to find out the following information.

(a) How many seconds was the longest flight time? The shortest flight time?

(b) How many flight lasted 3 seconds or longer?

State anything else you noticed.

Paper helicopter flight time

Table 1: 7cm blades

Trial No	Flight time (seconds)	Trial No	Flight Time (seconds)
1	3.07	26	2.60
2	3.04	27	2.62
3	3.08	28	3.04
4	2.96	29	2.96
5	2.77	30	3.19
6	2.91	31	2.86
7	2.94	32	3.17
8	2.87	33	2.91
9	2.96	34	2.99
10	3.27	35	3.35
11	2.58	36	2.85
12	2.33	37	3.18
13	2.86	38	2.77
14	3.04	39	2.62
15	3.06	40	2.95
16	2.79	41	2.78
17	2.79	42	2.99
18	3.11	43	3.13
19	2.93	44	3.24
20	2.74	45	2.92
21	2.78	46	2.66
22	2.93	47	2.93
23	3.02	48	2.73
24	2.91	49	3.32
25	2.97	50	3.09

Table 2: 5cm blades

Trial No	Flight time (seconds)	Trial No	Flight Time (seconds)
1	2.25	26	2.73
2	2.43	27	2.57
3	2.39	28	2.46
4	2.41	29	2.37
5	2.52	30	2.34
6	2.25	31	2.49
7	2.35	32	2.50
8	2.61	33	2.35
9	2.38	34	2.14
10	2.47	35	2.55
11	2.66	36	2.54
12	2.32	37	2.41
13	2.63	38	2.39
14	2.59	39	2.17
15	2.37	40	2.25
16	2.43	41	2.45
17	2.56	42	2.30
18	2.38	43	3.22
19	2.41	44	3.29
20	2.59	45	2.51
21	2.28	46	2.37
22	2.61	47	2.29
23	3.64	48	2.55
24	2.36	49	2.49
25	2.45	50	2.42

Teaching and Learning Activities

You can find the information in table 1 and 2 above. What difference do you notice between the two tables overall?

When we want to use data to find out something, we first have to organize it according to what we want to do with it.

Example.1 **Frequency distribution table**

The table on the right organizes the data from Table 1 from the previous lesson, dividing each flight time into groups of 0.15 second interval.

There are eight classes listed in Table 3, each with an interval of 0.15 seconds.

Flight Time (Second)	**Frequency** (number of items
(at least) (less than)	1
2.30 – 2.40	1
2.45 – 2.60	6
2.60 – 2.75	10
2.75 – 2.90	18
2.90 – 3.05	9
3.05 – 3.20	3
3.20 – 3.35	2
3.35 – 3.50	
Total	50

Key Ideas

- Each intervals in a distribution table is called a Class.
- The number of data items in each class is called the frequency of the class, and a table like the one above, which shows frequencies according to class is called a frequency distribution table

1. Organize the data in Table 2 into frequency distribution table.

2. Use frequency distribution tables 3 and 4 to find out the following information;

   (a) Which class has the highest frequency?

   (b) How many flights lasted at least 2.60 seconds? What percentage of the total number of flights does this represent?

Tabl.4: 5cm blades

Flight Time (Second)	**Frequency** (number of items
(at least) (less than)	
2.00 – 2.15	
2.15 – 2.30	
2.30 – 2.45	
2.45 – 2.60	
2.60 – 2.75	
Total	

L79 : Histograms

Lesson Objective: Read and understand data from a frequency distribution table and use the data to draw a histogram. **(7.4.1.1/2)**

Materials: blackboard

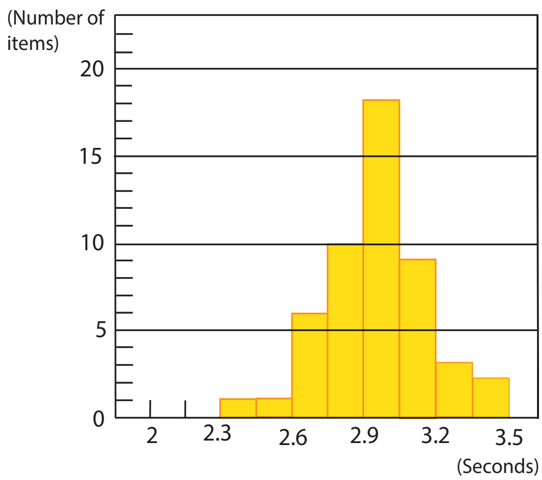
ASK-MT and Assessment	
Attitudes/Values	Appreciate the usefulness of a histogram.
Skills	Read data from a frequency distribution table and use the data to draw a histogram .
Knowledge	Frequency distribution table use to draw a histogram.
Mathematical Thinking	Think of how to draw a histogram using the information from a frequency distribution table.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1 **A frequency distribution table as a Graph**

Displaying a frequency distribution table as a Graph makes it even easier to read. Figure 1 on the right shows the data from Table 3 on the previous page as a graph, with flight time on horizontal axis and frequency on the vertical axis. The graph shows a series of rectangles whose height represents frequency. The area of these rectangles is proportional to the frequency of each class.

Figure 1 : Flight time (7-cm blades)



Key Ideas

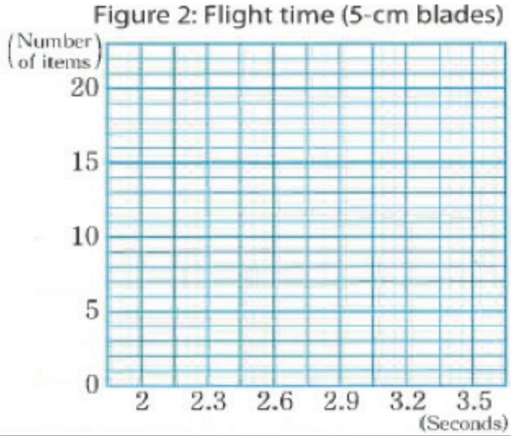
Graphs like this are called **histogram**

Exercises

Use the frequency distribution table below to draw a histogram in figure 2.

Flight Time (Second)	Frequency (number of items
(at least) (less than) 2.00 – 2.15 2.15 – 2.30 2.30 – 2.45 2.45 – 2.60 2.60 – 2.75	
Total	

Figure 2: Flight time (5-cm blades)



L80: Frequency distribution polygons

Lesson Objective : Compare the features of histogram and a frequency distribution polygon and explain **.(7.4.1.1/2)**

Materials: blackboard

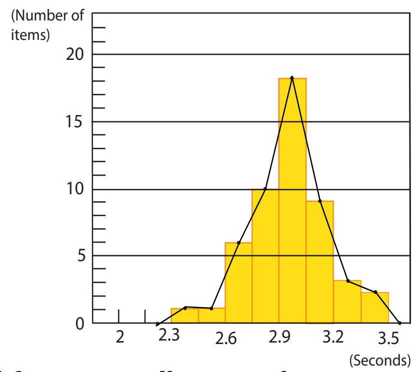
ASK-MT and Assessment	
Attitudes/Values	Enjoy drawing and reading a frequency distribution polygon.
Skills	Draw a frequency distribution polygon from data collect from the previous lesson.
Knowledge	Frequency distribution polygon.
Mathematical Thinking	Think of how to read and interpret information from a frequency distribution polygon.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson

Teaching and Learning Activities

Example.1 **Frequency distribution polygons**

Compare the histograms in figure 1 and 2 on the previous page. We can also take the midpoint of the top side of each rectangle in a histogram and connect them with segments. On each end, we extend a segment out to the horizontal axis, thinking of it as a class with a frequency of zero. Doing so creates a line graph as the one on the right

Figure 1 : Flight time (7-cm blades)



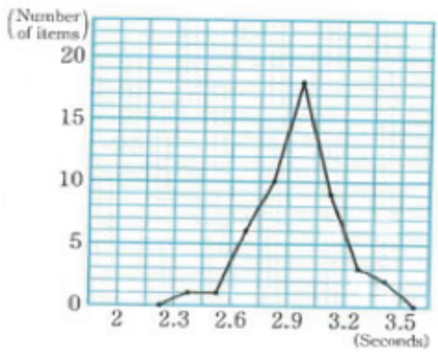
Important! Frequency distribution polygons are sometimes called **frequency line graphs,**

Key Ideas

- Graphs like this one are called Frequency distribution polygons.
- If we overlap two frequency distribution polygons, we can easily compare the two sets of data.

Exercises

The figure on the right shows a Frequency distribution polygon that was created using the data in Figure 1on the previous lesson. Use the data from Figure 2 from the same lesson to draw another frequency distribution polygon.



L81: Relative frequency

Lesson Objective: Explain relative frequency and how to find the relative frequency data **(7.4.5)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Confidently calculate relative frequency using the formula using a given data
Skills	Calculate relative frequency using a give data
Knowledge	Relative frequency
Mathematical Thinking	Think of how to find relative frequency using a given data
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introductory

Paper helicopter flight time

Flight time (seconds)	5 cm Frequency (number of items)	6 cm Frequency (number of items)
(at least) - (less than)		
2.00 - 2.15	1	4
2.15 - 2.30	8	11
2.30 - 2.45	19	30
2.45 - 2.60	16	54
2.60 - 2.75	6	37
2.75 - 2.90	0	11
2.90 - 3.05	0	3
Total	50	150

Another class did an experiment where they dropped paper helicopters with 6 cm blades from a height of 2 m. you want to compare their results with yours, so you organize the data into a frequency distribution table like the one on the right. However, because the total number of trials is different, it's difficult to compare the results. What should you do so that you can compare data? When you have a situation like the one above, you can find out what proportion of the total number of data items is in each class, and then compare those proportions.

Example.1 **How to find relative frequency**

If we want to find the relative frequency of the class 2.00 – 2.15 seconds for the 6-cm paper helicopter in the table above, we can do the following. We can then round to the second decimal place.

$$\frac{4}{150} = 0.0\overset{3}{\cancel{2}}\cancel{6}\ldots$$

Key Ideas

The number of data items (frequency) of each class relative to the total number of data item is called the relative frequency of class.

$$\textbf{Relative frequency} = \frac{\textbf{Frequency of each class}}{\textbf{Total Frequency}}$$

Exercises

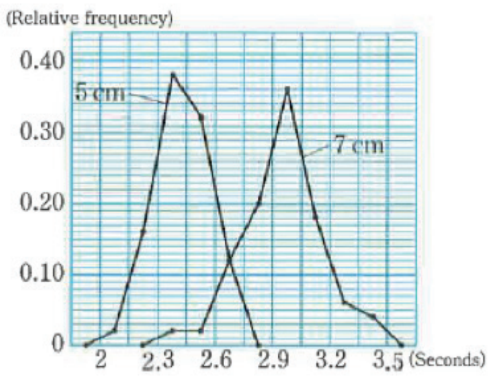
1. Find the relative frequency of the class 2,30 – 2.45 seconds for the 5 cm paper helicopter in the above.

Teaching and Learning Activities

2. The table below summarizes the relative frequency of flight times for 5 cm, 6 cm and 7 cm paper helicopters. Fill in the blanks to complete the table.

Flight time (seconds)	5cm		6cm		7cm	
(at least) (less than)	Frequency (number of items)	Relative frequency	Frequency (number of items)	Relative frequency	Frequency (number of items)	Relative frequency
2.00 - 2.15	1		4	0.03	0	0.00
2.15 - 2.30	8		11		0	0.00
2.30 - 2.45	19	0.38	30		1	0.02
2.45 - 2.60	16		54		1	0.02
2.60 - 2.75	6		37		6	0.12
2.75 - 2.90	0	0.00	11		10	0.20
2.90 - 3.05	0	0.00	3		18	0.36
3.05 - 3.20	0	0.00	0	0.00	9	0.18
3.20 - 3.35	0	0.00	0	0.00	3	0.06
3.35 - 3.50	0	0.00	0	0.00	2	0.04
Total	50	1.00	150		50	1.00

3. The figure on the right uses the the data in the table above to show frequency distribution polygons.
Indicating the relative frequencies for 5 cm paper helicopters. Add a frequency distribution polygon for the 6 cm paper helicopter to the figure.



L82 : Mean

Lesson Objective : To find mean using a given formula. **(7.4.1.2)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Share ideas on how to find the mean of a set of data
Skills	Find mean of a set of data using a given formula
Knowledge	Mean of a set of data
Mathematical Thinking	Think of how to calculate the mean of a set of data
Assessment	Use the ASK-MT to assess the students learning progress during the lesson

Teaching and Learning Activities

Introductory

A bowling team wants to pick one player to compete in an Individual tournament. Table 1 on the right shows the bowling scores for two potential candidates over the course of 20 games. Table 2 below organizes the data into a frequency distribution table. Which player do you think? should compete in the tournament?

Table 1: Bowling scores

Player A	Player B
178	168
173	174
178	173
175	172
166	188
170	181
182	165
176	169
179	174
176	178
193	189
181	193
185	168
164	170
188	177
167	162
182	204
174	179
177	184
171	173

Table 2: Bowling scores

Class (points)	Player A Frequency (number of items)	Player B Frequency (number of items)
(at least) - (less than)		
160 - 165	1	1
165 - 170	2	4
170 - 175	4	6
175 - 180	7	3
180 - 185	3	2
185 - 190	2	2
190 - 195	1	1
195 - 200	0	0
200 - 205	0	1
Total	20	20

One of the values that often used to characterize an entire set of data is the mean. The mean can be found using the following formula.

Example.1 **How to find mean of a set of data**

We can find the mean score for player A in table 1 above in the following way. We can then round to the first decimal place.

$$(178 + 173 + 178 + \ldots + 171) \div 20 = 176.75$$
$$= 176.8 \text{ (2d.p)}$$

The individual data items in any given class in a frequency distribution table could have many different values. However, we can calculate the mean of the entire data set if we consider each value to the middle value of its class.

Teaching and Learning Activities

Example.2: **Class Value**

For example, in Table 2 the class value for the class with scores between 170 and 175 is
$\frac{170+175}{2}$ =172.5(points)

TN: People often represent a set of data using on overall value to help them think about it or make judgments. In these situations, a value that represents the entire data set called a representative value.
The mean is commonly used as representative value. But other values described below are selected depending on data characteristics and the purpose of investigation.

L83 : Median and mode

Lesson Objective: To find and explain median and mode in a set of data. **(7.4.1.2)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Share and communicate their ideas with others.
Skills	To be able to find and explain the median and mode of a set of data.
Knowledge	Be able to Understand median and mode and how find median and mode in a set of data
Mathematical Thinking	Be able to think of how to state the mode and median score from a frequency distribution table or set of data.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1 **Median**

The table on the right lists the data items from Table 1 in the previous lesson in size order. The median score for player A is

$$\frac{177+176}{2} = 176.5$$

If there are an odd number of data items, the middle items the median. If there is an even number of data items, themedian is the mean of the two middle values

Player A	Player B
193	204
188	193
185	189
182	188
182	184
181	181
179	179
178	178
178	177
177	174
176	174
176	173
175	173
174	172
173	170
171	169
170	168
167	168
166	165
164	162

Example.2 **Mode**
Twenty –four grade 7 school boys recorded their sneakers sizes
25, 24, 24, 25, 26, 26, 27, 25, 24, 25, 24, 23,
25, 25, 26, 25, 26, 25, 25, 26, 24, 23, 25, 26

What is the most common size among the students?
TN: In the frequency distribution table, the mode is considered to be the class value for the value with the highest frequency.

Key Ideas

- The median is the middle value of a data set when it is arranged in size order.
- The value that appears most frequently in a set of data is called the mode

Exercises

1) 15 members of the junior high school track team ran the 50- m dash and recorded their times (In seconds) below. Find the media and the mean of their times

7.2, 7.8, 8.2, 7.7, 8.1, 7.0, 7.5, 7.3, 8.3, 7.9, 7.0, 7.4, 8.1, 7.1

2) State the mode score for players A and B. Use the frequency distribution table on the right

Bowling scores

Class (points) (at least) - (less than)	Player A Frequency (number of items)	Player B Frequency (number of items)
160 - 165	1	1
165 - 170	2	4
170 - 175	4	6
175 - 180	7	3
180 - 185	3	2
185 - 190	2	2
190 - 195	1	1
195 - 200	0	0
200 - 205	0	1
Total	20	20

L84 : Data Distribution and representative value

Lesson Objective : Explain relationship between mean, median, and mode and explain how the data are distributed. (**7.4.1.2)**

Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Confidently explain relationship between mean, mode and median
Skills	Explain how the data are distributed
Knowledge	Relationship between mean, median, and mode
Mathematical Thinking	Think of how to explain the distribution of data and relationship between mean, median, and mode
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

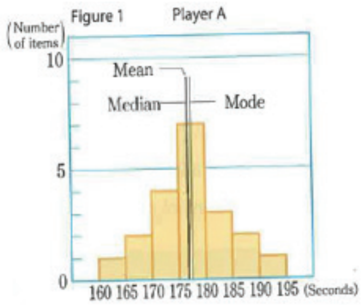
Teaching and Learning Activities

Discuss

Let's look at the relationship between the mean, median, and mode and how the data are distributed.

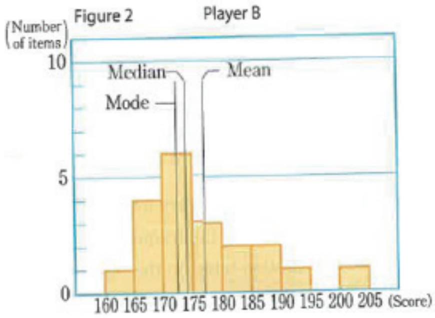
Figure 1 shows the score distribution for player A on the first lesson as histogram. You can see that it is shaped like a mountain with a relatively symmetrical sides.

When the data look like this, the mean, median and mode are all relatively close in value.



If we do the same thing for player B

We get Figure 2. Player B's mean score is about the same as player A's, but the histogram is skewed to the left, and the median and mode are smaller than the mean.



It is important to pay attention to differences like these when determining which representative value best suits your purpose.

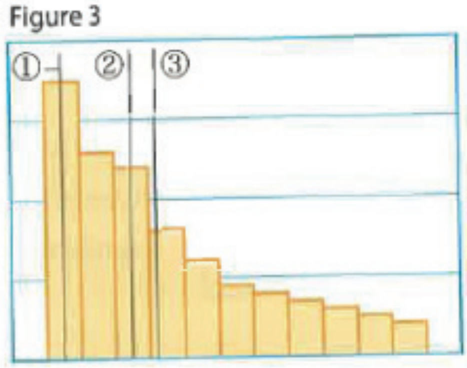
Exercises

Figure 3 on the right shows a histogram labelled 1, 2 and 3.

Which position represents the mean, median and mode?

Match each term to a number.

Figure 3



L85 : Dispersion

Lesson Objective: Explain the difference between the largest and smallest value in a data set. **(7.4.1.2)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Share their ideas on spread of data
Skills	Find the Spread of data distribution
Knowledge	Spread of data distribution
Mathematical Thinking	Think about how to find the Spread of data distribution
Assessment	use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introductory

Containers A and B each have 10 eggs in them.

The table on the right shows the weight of the individual eggs in each container.
The mean and median of the data sets are shown below.
Container A mean: 50.5g, Median: 50.6g
Container B Mean: 50.5g, Median: 50.6g

Egg weights

Class (g) (at least) - (less than)	Container A Frequency (number of items)	Container B Frequency (number of items)
42 - 44	0	1
44 - 46	0	2
46 - 48	1	0
48 - 50	2	0
50 - 52	4	3
52 - 54	3	2
54 - 56	0	1
56 - 58	0	1
Total	10	10

Can we conclude that the distribution of egg weights in containers A and B is about the same?
In the above, containers A and B both have the same mean and median value. However, the largest and smallest values in the each data set are different.

Example.1 **Spread of data**
The range of eggs weights in containers A and B are given below
Container A : 53.3- 47.8 = 5. 5 (g)
Container B : 57.1- 43.2 = 13. 9 (g)

Even if two sets of data have the same mean and median value, their distributions are different if they have different ranges.

Egg weights

Class (g) (at least) - (less than)	Container A Frequency (number of items)	Container B Frequency (number of items)
42 - 44	0	1
44 - 46	0	2
46 - 48	1	0
48 - 50	2	0
50 - 52	4	3
52 - 54	3	2
54 - 56	0	1
56 - 58	0	1
Total	10	10

The frequency distribution table for containers A and B from above is shown on the right. You can see that the weights of eggs in container A are clustered around the mean, while the weight of the eggs in container B are more scattered, or dispersed .When looking for trends in data, we need to consider how it is dispersed as well as the representative value that best suits our purpose.

Key Ideas

- The difference between the largest and smallest value in a data set is called the range of the distribution. **Range = largest value – smallest value**

L86 : Approximate values

Lesson Objective : To express data collected through measurement and other means. **(7.4.1.3)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Share and communicate their ideas with others.
Skills	To be express and explain significant figure and approximate value.
Knowledge	Be able to understand how to express data collect through measurement and other means.
Mathematical Thinking	Be able to think of how express and explain data collected through measurement using terms such as significant figures and approximate values.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introductory
Use a ruler to measure the length and width of the rectangle on the right. Make your measurements in mm.

The value we get when we measure the length is called measure value. If we measure the length of the rectangle above, we get 20mm. we can also express this as 2 cm, but if we want to make it clear that we measured it in millimetres, we would write it as 2.0 cm.

In the 2.0 cm measurement above, the 2 and the 0 are significant figures. In the 3.40- seconds experimental result on page 188, the 3, 4 and 0 are significant figures.

When we want to make it clear which digits are significant figures, we express a number as product of decimal with a digit integer portion and 10 multiplied to some power.

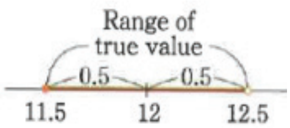
Example.1: **How to express significant figures**

The diameter of the earth is 12750 km.
To express this as a three digit significant figure
1.28×10^4 km
To express this as a two digit significant figure
1.3×10^4 km
The width of the rectangle above is around 42 cm, but it is not exactly 42mm.
No matter how precisely we measure it, the value will deviate somewhat from the true value. Measured values are approximate values. Other approximate values include using the number 3.14 for pi.

Example.2: **Range of true value**
If an approximate value of a decimal number a is 12 after rounding it by the first decimal place, the range of a is

$$11.5 \leq a < 12.5 .$$

In this case, the absolute value of the error is less than or equal to 0.5.
In the example above, the absolute value of the error represents the size of difference between the approximate value and true value. The absolute value is also referred to as error.



Teaching and Learning Activities

Key Ideas

- When we have a value obtained through measurement, a number that has meaning is called a significant figure. The number of digits that are significant figures is the number of significant digits
- A value that is close to a true value is called an approximate value.
- The difference between an approximate value and a true value is called **error.**

Error = approximate value – true value

Exercises

1. The area of a baseball field is 46755 m. Express this as a three digit significant figure.

2. The measured values below are measured to which place?

   (a) 81×10^2 m (b). 2.00×10^3 g

3. A number a is rounded by the second decimal place. The approximate value is 1.6. Use inequality symbols to express the range of numbers a.

Unit Checkpoint

Review on using data

Review on Using Data

Frequency distribution tables

1. The table on the right is a frequency distribution table showing the heights of 30 members of junior high school soccer team.

Height table (soccer team)

Height (cm) (at least) - (less than)	Frequency (number of people)
145.0 - 150.0	2
150.0 - 155.0	7
155.0 - 160.0	10
160.0 - 165.0	6
165.0 - 170.0	4
170.0 - 175.0	1
Total	30

Mean and Median

(a) What is the total member of students who are shorter than 155.0 cm

(b) Find the mean height in the frequency distribution table.

(c) What class contains the median?

Relative frequency

2. The table below is a frequency distribution table showing the results of a study measuring the grip of the first grade boys at junior high schools R and S

Grip (kg)	R Junior High		S Junior High	
	Frequency (number of people)	Relative frequency	Frequency (number of people)	Relative frequency
(at least) - (less than)				
15 - 20	1	0.03	8	0.04
20 - 25	3	$\square$	27	0.13
25 - 30	6	0.16	48	0.23
30 - 35	10	$\square$	59	0.28
35 - 40	8	0.21	45	0.21
40 - 45	7	$\square$	14	0.07
45 - 50	2	0.05	7	0.03
50 - 55	1	0.03	2	0.01
Total	38	1.00	210	1.00

(a) Fill in the blanks in the table above.

(b) state which aspects of the grip data distribution are similar between junior high schools R and S. Which aspects are different?

Approximate values

3. State the number that goes in the $\square$.

(a) The measure value 3.52×10^3 m is measured to the $\square$.

(b) The distance between the earth and the sun is 149597870 km. If we express this as a four –digit significant figure, we get $\square$ $\times10^3$ km.

Assessment, Recording and Reporting

Assessment, recording and reporting is an integral part of the delivery of any curriculum used in the schools.

The primary purpose of assessment is to improve students' learning and teachers' teaching as both respond to fulfilling the following:

- inform and improve students' progress and achievements in learning.
- provide valuable information that enable teachers, schools and Department of Education to make decisions about how to improve the quality of teaching and learning in the education system.
- inform teachers of the progress of students learning in order to adjust teaching and planning to improve student learning.
- inform parents and guardians, about their children's progress and achievements.
- schools and systems, about teaching strategies, resource allocations and curriculum; and other educational institutions, employers and the community, about the achievements of students in general or of particular students.

Effective and meaning assessment must be maintained at all times. The content standards written in the syllabus are expected curriculum for this grade prescribed by units and sets the basis for planning and conducting on - going assessment.

Ongoing classroom assessment is done to:

- Support students learning
- Monitor students learning needs
- Diagnose students learning needs
- Evaluate teaching program and
- Inform students reporting process.

Assessment Strategies and Methods

1. Assessments Methods

Assessment is an integral part of students learning and can be done using different methods.

Teachers are encouraged to use two or more types of assessment when assessing students learning. Standards Based Curriculum specially promotes three types of assessment. These are;

- Assessment *As* Learning **(AAL)**
- Assessment *For* Learning **(AFL)**
- Assessment Of Learning **(AOL)**

2. Assessment Strategies

2.1 Answering Questions and Tasks during the Lesson

The use of questioning or an activity during the teaching of every lesson is an example of AAL that assesses student's achievement (key concepts) of the lesson. It allows student the opportunity to reflect on their own learning and identify areas of their strength and weakness during the lessons.

2.2 Exercises

All exercises given in every lesson are example of AFL. It is not used to evaluate learning but to help learners learn better. It helps the students to see the learning in relation to the goals.

2.3 Checkpoints

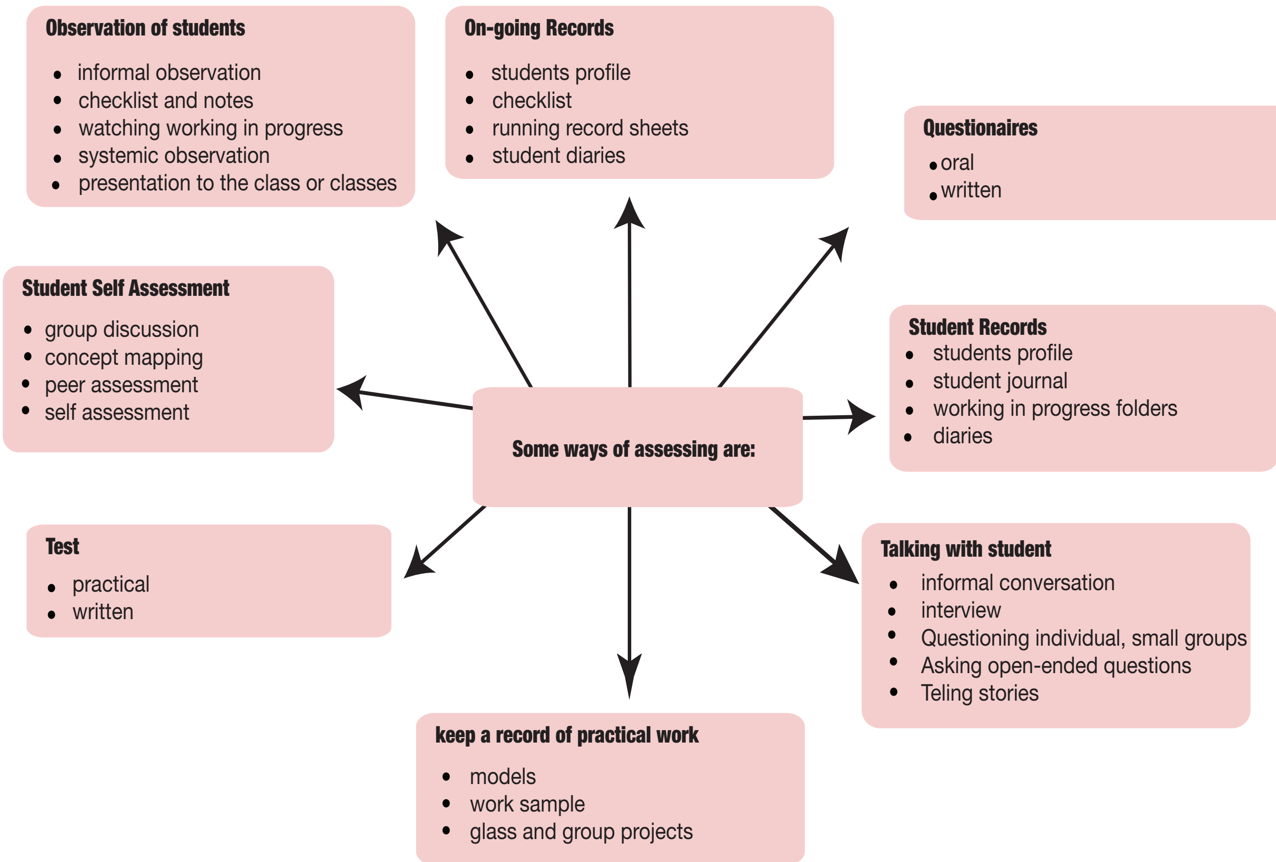
In the Teacher guide there is a check point. It is a Unit Review exercise that appears at the end of the all guided lessons of a particular unit. The exercises are based on basic key concepts to recheck the understanding of students. Teachers are encouraged to use these as classroom assessments apart from tests and assignments. This Unit Review exercises is an example of AOL that teachers can use to measure, record and report on a student's level of achievement in regards to specific learning expectations.

2.4 Written Test/Assignment/Homework

These assessment strategies are used to assess students' performance of their learning formatively or summative. Class teachers prepare these with careful considerations of;

- the knowledge and skills to assess the students
- the level of language to be used
- the construction of questions - clear and precise
- the intended content to be assess
- how much each question is worth and how to award scores for each questions.

Below are some other ways of assessing apart from what has been give above.



Assessment Plan and Tasks

It is important to plan assessment for the whole year using the content overview and the yearly or term plans. Assessment.

Assessment tasks form the basis of the assessment processes of assessing each learner in relation to the content standards. Assessment tasks are learning activities created from the benchmarks. These are written and specifically designed and planned before administering. This particular activity assessing has key concepts (ASK-MT) that must be achieved at the end of each Content Standards.

Assessment Plan

To plan assessment tasks, teachers must decide which type of assessment methods will be used to demonstrate to achieve the content standards. Standard statements are the point of reference in the process of identifying and planning assessment tasks.

In the process of writing and planning assessment tasks, the following are some points to consider;

- choose assessment methods suitable for the assessment task.
  (Refer to sample in Appendix.5)
- develop assessment rubric based the Key Concepts (ASK-MT).
  (Refer to sample in Appendix.5)
- consulting the Bloom's Taxonomy as per the student's cognitive level.

Assessment Tasks

Sample assessment tasks given are examples for teachers to use and plan their own to suit their context and the learning needs of the grade six students in the classroom. The tasks are very specific and direct the teacher to the content of learning stated in the syllabus.

Teachers should be ensured that all assessment tasks are;

- clearly stated in simple language that students can easily interpret
- link to the content standards
- balanced, comprehensive, reliable and fair
- should create assessment criteria (rubrics) to be demonstrated in each tasks and are made known to students.

Grade (7) Yearly Assessment Plan and Task Overview

According to the grade (7) content and yearly plan, a suggested yearly assessment plan and tasks overview has been planned and placed. Teachers are encourage to utilize this according to the school overall assessment program.

Strand	Unit	Content Standard	Assessment Task
Number and Operation	Positive and Negative numbers	**7.1.1**	• Calculate positive and negative numbers in various situations
Geometrical, Measurement and Transformation	Plane Figures	**7.2.1**	• Use basic methods for constructing plane figure in drawings
	Space figures	**7.2.2**	• Explain the relationship of the solid and space figures from different view of point and components of space figures • Calculate surface area and volume of various solids
	Algebraic expression	**7.3.1**	• Represent quantity using letter and apply the rules for writing algebraic expressions.
Patterns and Algebra	Linear Equation with one unknown	**7.3.2**	• Simplify linear expressions
	Proportional Functions	**7.3.3**	• Identify ways to represent functional relationships • Solve problems involving functional relationships
Statistics and Probability	Distribution of data and representation values	**7.4.1**	• Collect and organize data and represent in frequency distribution table and interpret the data. • Represent data on histogram, and use the results to interpret the trends in the data. • Find the trend of central tendency of data and approximate value of data

Recording and Reporting

Recording and reporting of students achievements in the classroom is very important. Teachers should use a range of tasks to ensure that commended standards statements are equally assessed and reported. This helps the teachers to reflect the effectiveness of their teachings.

Recording

Teachers must keep accurate records of students' achievement of their learning. They must report these achievements in fair and accurate ways to parents, guardians, teachers and students. Teachers should records the evidence of students' demonstrations of achievements of standards using assessment instruments that are manageable.

Examples of recording methods include;

- Anecdotal notes in a journal or dairy
- Checklist
- Portfolios of students' work
- Progressive records
- Work samples with comments written by the teacher.

Reporting

Reporting is important in assessment and should be done effectively. Teachers should report what students have done well and how they can improve further. Students' reports should be based on assessment information collected from students learning progress and other related areas such as behaviours. Schools will decide on how reports will be presented according to the needs of their communities.

Methods will include interviews and written reports. Written reports should include;

- A written record of content standards achieved by students since the previous report
- A written record of the content standards the student is now working towards
- Information about students' attitudes, values and other additional information that is specific to individual students.

Evaluation

- Evaluation is part of the process of continuously raising standards of student achievement in PNG. Assessment information used for evaluation purposes should be used in ethical and constructive ways.

- Teachers will use assessment information to evaluate the effectiveness of their teaching, learning to make improvements to their teaching practice in order to improve student learning. Evaluation tools such as written records, questionnaires, logs and diaries, submissions or records of meetings and discussion with general staff members, teaching staff, parents and other community members.

Resources

Mathematics lessons require resources both for teachers and students. The recommended list of resources in this Teacher Guide are vital for making the teaching and learning meaningful and to understand concepts more precisely and clear.

1. List of Teaching Aids /Materials

- Set squares
- Protractors (Full circle and semi of various sizes)
- Compass (various sizes)
- Wall Clock
- Rulers (1m & 30cm)
- Tape measure (various sizes)
- Multiplication table chart
- Fraction wall chart (to be Improvised by the teacher)
- Stop watches
- Base ten materials (cubes, long, flat and blocks)
- Thermometer
- Models of 2-D and 3-D shapes
- Scissors/blades
- Square coloured papers
- A grid square papers

2. Teaching Resources

- Melanesian School Mathematics Dictionary
- Community School Mathematics 6A & 6B
- Grade 6 Mathematics TV Students Workbook
- Grade 6 Mathematics TV Teacher resource book
- Tingting Maths Grade 6 Teacher Resources book
- Tingting Maths Grade 6 Students book
- Mathematics Essential Skills 6 Outcomes Edition for PNG

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Appendices

Appendix 1: STEAM or STEM

- By exposing students to STEAM and giving them opportunities to explore STEAM-related concepts, they will develop a passion for it and, hopefully, pursue a job in a STEAM field.
- Providing real life experiences and lessons, e.g., by involving students to actually solve a scientific, technological, engineering, or mathematical, or Arts problem, would probably spark their interest in a STEAM career path. This is the theory behind STEAM education.
- By integrating STEAM content and real life learning experiences at different levels of the curriculum process (e.g., Curriculum frameworks, content standards, benchmarks, syllabi, teachers' guides and students' books, curriculum design and development, annual and term school programs and lesson plans, teaching methodologies.
- Teaching methodologies – Problem and project-based learning, partnerships with external stakeholders e.g., high education institutions, private sector, research and development institutions, and volunteer and community development organizations.
- They underpin STEM education. They are the main enablers of STEM education.
- The 21st century skills movement, which broadly calls on schools to create academic programs and learning experiences that equip students with the most essential and in-demand knowledge, skills, and dispositions they will need to be successful in higher-education programs and modern workplaces.
- The term 21st century skills refers to a broad set of knowledge, skills, work habits, and character traits that are believed - by educators, school reformers, college professors, employers, and others - to be critically important to success in today's world, particularly in collegiate programs and contemporary careers and workplaces.
- Generally speaking, 21st century skills can be applied in all academic subject areas, and in all educational, career, and civic settings throughout a student's life.
- The skills students will learn will reflect the specific demands that will be placed upon them in a complex, competitive, knowledge-based, information-age, technology-driven economy and society.

Appendix 2: The 21st Century Skills, Knowledge, Attitudes and Values

The following list provides a brief illustrative overview of the knowledge, skills, work habits, and character traits commonly associated with 21st century skills:

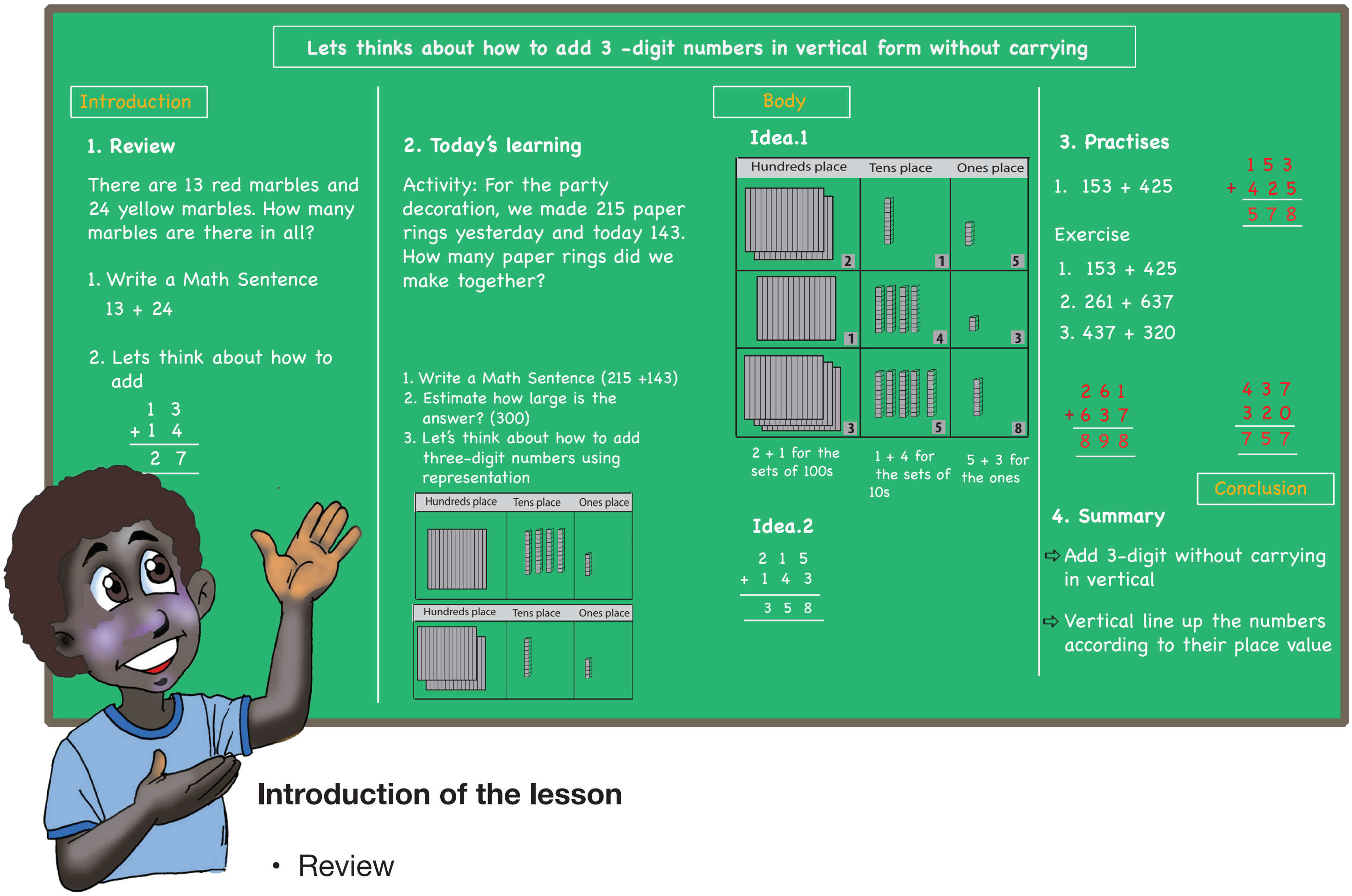
- Critical thinking, problem solving, reasoning, analysis, interpretation, synthesizing information
- Research skills and practices, interrogative questioning
- Creativity, artistry, curiosity, imagination, innovation, personal expression
- Perseverance, self-direction, planning, self-discipline, adaptability, initiative
- Oral and written communication, public speaking and presenting, listening
- Leadership, teamwork, collaboration, cooperation, facility in using virtual workspaces
- Information and communication technology (ICT) literacy, media and internet literacy, data interpretation and analysis, computer programming
- Civic, ethical, and social-justice literacy
- Economic and financial literacy, entrepreneurialism
- Global awareness, multicultural literacy, humanitarianism
- Scientific literacy and reasoning, the scientific method
- Environmental and conservation literacy, ecosystems understanding
- Health and wellness literacy, including nutrition, diet, exercise and public.

Appendix 3: The Blooms Taxonomy

BLOOM'S REVISED TAXONOMY

Remembering	Recalling information, Recognizing, listing, describing, retrieving, naming, finding. *E.g* How many ways can you travel from one place to another? List and draw all the ways you know. Describe one of the vehicles from your list, draw a diagram and label the parts. Collect "transport" pictures from magazines- make a poster with info.
Understanding	Explaining ideas or concepts, Interpreting, summarizing, paraphrasing, classifying, explaining. *E.g*. How do you get from school to home? Explain the method of travel and draw a map. Write a play about a form of modern transport. Explain how you felt the first time you rode a bicycle. Make your desk into a form of transport.
Applying	Using information in another familiar situation Implementing, carrying out, using, executing. *E.g* Explain why some vehicles are large and others small. Write a story about the uses of both. Read a story about "The Little Red Engine" and make up a play about it. Survey 10 other children to see what bikes they ride. Display on a chart or graph.
Analysing	Breaking information into parts to explore understandings and relationships Comparing, organizing, de-constructing, interrogating, finding. *E.g* Make a jigsaw puzzle of children using bikes safely. What problems are there with modern forms of transport and their uses- write a report. Use a Venn Diagram to compare boats to planes, or helicopters to bicycles.
Evaluating	Justifying a decision or course of action, Checking, hypothesizing, critiquing, experimenting, judging. *E.g* What changes would you recommend to road rules to prevent traffic accidents? Debate whether we should be able to buy fuel at a cheaper rate. Rate transport from slow to fast etc..
Creating	Generating new ideas, products, or ways of viewing things, Designing, constructing, planning, producing, inventing *E.g* Invent a vehicle. Draw or construct it after careful planning. What sort of transport will there be in twenty years time? Discuss, write about it and report to the class. Write a song about traveling in different forms of transport.

Appendix: 4 - Sample Black board Plan



Introduction of the lesson

- Review
- If the lesson happens to be first lesson of a new unit or chapter. Should have an introductory activity related to establish new ASK-MT

Body of the lesson

- Today's learning activity based on lesson objective
- Practices

Conclusion of the lesson

- Summary

Appendix 5: Sample timetable

Here is the sample timetable for you to adopt and adjust to your need,

Start		End		Sessions	Minutes
8 : 00	~	8 :25	0:25	ASSEMBLY	25
8 : 25	~	9:05	0:40	1st Class	40
9 : 05	~	9:10	0:05	break	
9 : 10	~	9:50	0:40	2nd Class	40
9 : 50	~	10:25	0:45	RECESS BREAK	30
10 :25	~	11:05	0:20	3rd Class	40
11:05	~	11:10	0:45	break	
11:10	~	11:50	0:05	4th Class	40
11:50	~	12:20	0:45	LUNCH BREAK	30
12:20	~	13:00	1:00	5th Class	40
13:00	~	13:05	0:25	break	
13:05	~	13:45	0:45	6th Class	40
13:45	~	13:50	0:05	break	
13:50	~	14:30	0:45	7th Class	40
				Daily T/L Minutes	280
				Weekly T/L Minutes	1575
				T/L Minutes without Assembly	1350

Note: 5 minutes break in the above time table sample is the preparation time for students / teachers for the next lesson.

‘FREE ISSUE - NOT FOR SALE’