Mathematics

Teacher Guide

Grade 8

Standards Based

'FREE ISSUE NOT FOR SALE'

Papua New Guinea
Department of Education

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Issued free to schools by the Department of Education

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Contents

Secretary's Message

This Mathematics Teacher Guide for Grade 8 was developed as a support document for the implementation of Mathematics Syllabus for grades 6, 7 and 8. It contains sample guided lessons and assessment tasks and rubrics with suggested teaching and learning strategies that teachers can use to work towards the achievement of content standards and benchmarks in the syllabus.

The importance of mathematics curriculum is to ensure that all students will achieve mathematical skills and competencies of the 21st century that will serve them well in their lives and help them to compete locally and globally. The curriculum will engage learners, who are mathematically literate and can think differently and creatively. It is therefore vital for the mathematics curriculum to support every learner to reach their full potential.

The Teacher Guide reflects the essential knowledge; skills and values that students are expected to know and be able to do at the end of Grade 8. It is designed to promote a firm understanding of practical everyday mathematical concepts, thus raising the standards in mathematics. It also provides an excellent vehicle to train the mind, and to develop its capacity to think logically, abstractly, critically and creatively.

This Teacher Guide is to be used with the grade 6,7 and 8 syllabus for the teaching of mathematics in Grade 7. It is a guide for teachers to deliver the Mathematics content outlined in the syllabus. There are lessons for each day and teachers are expected to follow these throughout the year to ensure students meet the required standards

I encourage teachers to read each section of the guide carefully and become familiar with the content of the subject specified in the teaching and learning and other sections of the guide. I also encourage teachers to try out your own ideas and strategies that you believe will be effectively work in your schools for your students.

I commend and approve this Grade 8 Mathematics Teacher Guide to be used in all Primary Schools throughout Papua New Guinea.

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DR. UKE W. KOMBRA, PhD
Secretary for Education

Introduction

Purpose

This Teacher Guide must be used in conjunction with the Grades 6, 7 & 8 Syllabus. The main purpose is to implement the syllabus in the classroom.

The Teacher Guide provides you with guidelines and directions to help you plan and develop teaching and learning activities for the achievement of Content Standards and Benchmarks. It provides you with information and processes to:

- understand and expand on the relevant Knowledge, Skills, Attitudes and Values (KSAVs) provided in this guide
- develop teaching programs based on your school contexts
- plan and develop daily lesson activities
- plan and conduct assessments to monitor students' achievements.

Teachers are required to read carefully and use the guidelines in the Teacher Guide to plan and develop teaching and learning programs. The guide contains the following main components:

- yearly and termly overview which consists of all strands, units, topics and lesson titles
- sample weekly program or timetable
- suggested daily plans which consists of guided lessons and KSAVs
- assessment tasks and rubrics
- support resources for use when planning and programming.

How to use this Teacher's Guide

You are encouraged to use this Teacher Guide to help you design your teaching programs, lesson and assessment plans. Therefore, you need to:

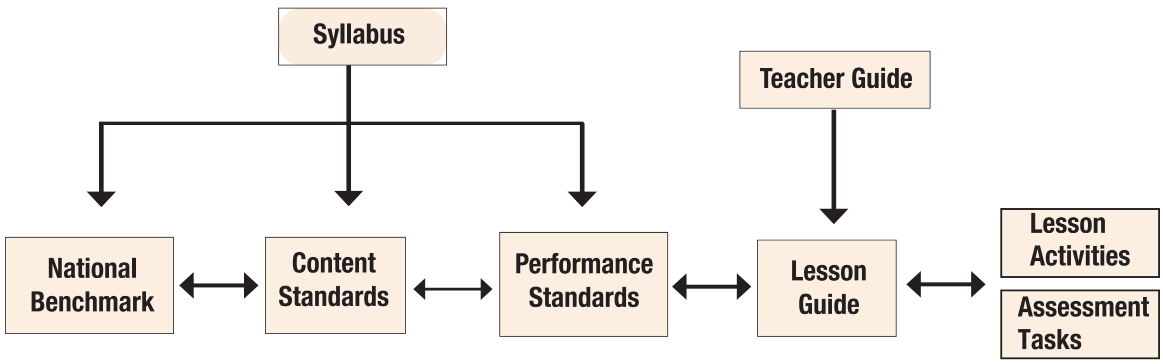
- read this teacher guide and the syllabus carefully
- to understand the content and what you will require for your classroom teaching
- become familiar with the syllabus strands, units, topics and lesson topics
- read and understand the content standards and benchmarks
- read and understand how the assessment plans and tasks are structured so that you can design appropriate assessment plans
- read and understand the structure and content of sample guided lessons and background information to support you in modification of your lesson.

Link to the syllabus

The teacher guide illustrates key parts of the mathematics syllabus. It provides practical ideas about how to use the syllabus and why the teacher guide and syllabus should be used together.

The teacher guide explains ways you can plan and develop teaching, learning and assessment programs. It includes recommended knowledge, processes, skills and attitudes for each of the content standards in the syllabus and examples of assessment tasks and how to record and report students' achievements.

You are encouraged to select and adapt the strategies and processes illustrated in the guide to meet the needs of your students and their communities.



Time Allocation

Mathematics week for grade 8 is to be timetabled for 240 minutes per. Teachers can use the time allocation to do their timetable or programme according to their school program. Topics and activities may vary in length however; you can plan for double periods of more than 40 minutes.

Key Features

The key features found in this Mathematics teacher guide are unique and important in planning, development and implementation of this subject.

Mathematical Process Skills

The five Mathematical process skills that can help the students improve their mathematical thinking.

1. Mathematical Problem Solving

- Understand the meaning of the problem and look for entry points to its solution
- Analyse information (givens, constrains, relationships, goals)
- Make conjectures and plan a solution pathway
- Monitor and evaluate the progress and change course as necessary
- Check answers to problems and ask, “Does this make sense?

2. Mathematical Communication

- Use definitions and previously established causes/effects (results) in constructing arguments
- Make conjectures and use counter examples to build a logical progression of statements to explore and support their ideas
- Communicate and defend mathematical reasoning using objects, drawings, diagrams, actions
- Listen to or read the arguments of others
- Decide if the arguments of others make sense and ask probing questions to clarify or improve the arguments.

3. Mathematical Reasoning

- Make sense of quantities and relationships in problem situations
- Represent abstract situations symbolically and understand the meaning of quantities
- Create a coherent representation of the problem at hand
- Consider the units involved
- Flexibly use properties of operations.

4. Mathematical Connections

- Look for patterns or structure, recognizing that quantities can be represented in different ways
- Recognize the significance in concepts and models and use the patterns or structure for solving related problems

- View complicated quantities both as single objects or compositions of several objects and use operations to make sense of problems
- Notice repeated calculations and look for general methods and short cuts
- Continually evaluate the reasonableness of intermediate results (comparing estimates) while attending to details and make generalizations based on finding

5. Mathematical Representation

- Apply prior knowledge to solve real world problems
- Identify important quantities and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas
- Make assumptions and approximations to make a problem simpler
- Check to see if an answer makes sense within the context of a situation and change a model when necessary.

Mathematical Activities

Mathematical activities are various activities related to mathematics where students are actively engage in by and for themselves to discover the properties of Number and geometrical figures based on what they have learned and apply them in their life and further studies.

Mathematical activities are usually done through problem solving with rich mathematical thinking which includes various questioning on problem situations such as for finding methods and better ideas in solutions. it also includes explanations for sharing ideas with various representations such as changing / translating representations to find the beautiful and reasonable pattern.

Mathematical activities are easily done if students acquire the fluency for operations and reasoning. They are necessary for developing mathematical thinking and proficiency, providing opportunities for students to feel the joy of thinking and learning, utilizing and appreciating mathematics in their lives.

Through the reflection of mathematical activities student are possible to appreciate the value of mathematics such as simple, easier, reasonable, general, and beautiful and harmony.

You can incorporate these activities into your lessons to have the mathematics lessons become;

- More students centred activities and more proactive with rich content.
- More fun to students.
- Easier to understand by students.
- More compelling and elaborative.
- More innovative with various discussions
- Creative and exploratory.
- Connected to daily life and natural phenomena.
- Easier to think about activities that relate to other subjects and Integrated Study.

Grade 8 Mathematical Activities

Activities / Experience	In learning each content of "Number, Operation and Computation , Geometry, Measurement and Transformation, Patterns and Algebra and Statistics and Probability", and the connections of these contents students should be provided opportunities doing mathematical activities for;
Performance Activities	(a) Finding out and developing the properties of numbers and geometrical figures based on previously learned mathematics (b) Making use of mathematics in daily life and society (c) Explaining and communicating each other in an evidenced, coherent and logical manner by using mathematical representations

Teaching and Learning Strategies

Teaching Strategies

Teaching strategies guide the teacher to teach the lesson content with appropriate learning strategies. Effective learning and acquisition of knowledge, skills, attitudes and values by students in a lesson is achieved through demonstrating appropriate teaching strategies.

It is therefore required that teachers identify and apply the best teaching strategies to deliver the content in the classrooms

KWL Chart

To help students to build on what they already know, ask them to use a KWL (Know, Want, Learned) strategy when they work on a topic or theme. An example is given below for the theme Traditions, Customs and Festivals.

K (what I already know)	W (what I want to know)	L (what I have learned)
What I know about traditions, customs and festivals	What I want to know about traditions, customs and festivals	What I have learned about traditions, customs and festivals

Apply the following steps when using the KWL strategy:

1. Organize the students into small groups.
2. Tell the students the lesson topic.
3. In small groups ask the students to list what they already know about the topic.
4. Get the groups to share their ideas with the class as a whole.
5. Ask the students to list what they want to know about the topic.
6. Students complete the first two columns before they start the topic and the third column is completed at the closure of the unit, topic or lesson.

Way of teaching and delivery of Mathematics lessons

The new curriculum promotes the competencies in Interest/Motivation/Attitude, Knowledge and Understanding, Skills and Mathematical Thinking that is specified clearly each guided lessons.

The method of delivery of Mathematics is student oriented through the process of problem-based approach. In this SBC teaching method of mathematics the students are given the opportunity to think and derive ideas by themselves about the task (problem) and be able to improve their way of thinking mathematically.

Delivery Methods of Mathematics Lessons

1. Situation oriented lesson conceptualizing is situation oriented or problem based

2. Encourage to deriving operations/ideas opportunity is given for students to derive ideas or operations by themselves

3. Allow for Thinking of ways to calculate
   Students to think of ways to calculate by themselves

4. Conclusion using students' ideas
   Lesson conclusion should come from student's viewpoints, and use these ideas for understanding new concepts

5. Practice for consolidation
   Practice similar exercises for affirmation of their understanding of the concept learned in the lesson.

Learning Strategies

The students should develop the ability to recognize and categorize situations critically, provide rationale reasoning, constructively solve problems, apply knowledge intelligently, and communicate effectively. Special consideration and more emphasizes must be given to identifying suitable learning strategies which encourage high student participatory learning.

Students' way of learning Mathematics

Steps of conceptual understanding through problem solving approach.

i. Emphasis on understanding meanings
   Students should look at task (problem) given, understand the task, and try to express what the task is.

ii. Thinking how to calculate
   Students think how to solve the tasks, and share ideas of how to solve the problem, and obtain the answer of the problem.

iii. Emphasis on expression and explanation (Reasoning)
   Students conclude and understand new concept from the way they solved the problem.

Planning and Programming

Yearly and termly overview

Teachers are encouraged to use this yearly and termly overview and yearly lesson overview to develop the weekly plans.

Term	Strand	Units	Topics
1	**Number, Operation and Computation**	Calculation of algebraic expressions	Addition and subtraction with expressions
			Calculation with various polynomial expressions
			Multiplication and division with Monomial expressions
			Using algebraic expression
2		Simultaneous equations	Simultaneous equations
			Using simultaneous equation
	Geometry, Measurement and Transformation	Properties of Parallel lines and angles	Angles and parallel lines
			Angles of polygons
			Congruent triangles
			Proof and proof set ups
			Making a proof
3		Congruence of plane figures	Isosceles triangles
			Congruent right triangles
			Properties of parallelograms
	Patterns and Algebra	Linear Functions	Linear functions
			Graphing linear functions
4			Graphing linear functions
			Finding expressions for linear Functions
			Using linear functions
	Statistics and Probability	Probability	Meaning of probability
			How to find probability

Yearly Lesson Overview

Yearly and termly overview

Teachers are encouraged to use this yearly and termly overview and yearly lesson overview to develop the weekly plans.

Strand	Units	Topics	Lesson No.	Lesson titles
Numbers Operation and Computation	Operation of algebraic expressions	Addition and subtraction with expressions	1	Monomials and polynomials
			2	Addition and subtraction with expression
		Calculation with various polynomial expressions	3	Calculate various polynomial expressions
			4	Value of expressions (with two or more letters)
		Multiplication and division with monomial expressions	5	Multiplying and division with monomial expressions
		Using Algebraic expression	6	Use Algebraic expressions to solve problems
			7	Even and odd numbers
			8	Transforming equalities
		Checkpoint		Review on Calculation of algebraic expressions
Patterns and Algebra	Simultaneous Linear equations	Simultaneous equations	9	Solving simultaneous equations
			10	Addition and subtraction method (1)
			11	Addition and subtraction method (2)
			12	Substitution method
			13	Various simultaneous equations
		Using simultaneous equation	14	Solve real problems using simultaneous equations (1)
			15	Solve real problems using simultaneous equations (2)
		Checkpoint		Review on Simultaneous equation
Geometry, Measurement and Transforma-tion	Properties of Parallel lines and angles	Angles and parallel lines	16	Exploring Lines that meet at intersections
			17	Parallel lines and corresponding and alternate interior angles
			18	Parallel lines and corresponding and alternate interior angles
		Angles of polygons	19	Interior and exterior angles in triangles
			20	Sums of interior angles in polygons
			21	Sums of exterior angles of a polygons
		Congruent triangles	22	Congruent triangles
			23	Conditions for congruent figures

Strand	Units	Topics	Lesson No.	Lesson titles
Geometry, Measurement and Transformation		Proof and proof set ups	24	Proof-Making a kite
			25	Setting up a proof
		Making a proof	26	Using the conditions for congruence
		Checkpoint		Review on Properties of Parallel lines and angles
	Congruence of plane figures	Isosceles triangles	27	Properties of Isosceles triangles
			28	Triangles with two equal angles.
			29	Equilateral triangles
		Congruent right triangles	30	Congruent right triangles
		Properties of parallelograms	31	Properties of parallelograms and prove
			32	Condition of parallelograms
			33	Special parallelograms
		Parallel lines and area	34	Shapes of figure without changing the area
		Checkpoint		Review on Congruence of plane figures
Patterns and Algebra	Linear Functions	Linear functions	35	Two functions that change together
			36	Linear functions
			37	Values that change in linear functions
		Graphing linear functions	38	Characteristics of linear functions graphs
			39	line slopes
			40	How to draw graphs of linear functions
		Finding expressions for linear functions	41	Finding expression using the slope and intercept
			42	Finding expression using the slope and coordinates of a point
		Equations and graphs	43	Finding expression using the coordinates of two points on a graph
			44	Equations and graphs
			45	Solutions to simultaneous equations and graphs
		Check point		Review on Linear Functions
Statistics and Probability	Probability	Meaning of probability	46	
			47	Usage of numbers to express probability
			48	Probability of various events
		How to find probability	49	Probabilities when tossing two coins.
			50	Probabilities when rolling two dices.

Content Background

Strand: Number, Operation and Computation

1. Calculation of Algebraic Expressions

In grade 8, students are expected to learn to carry out the four arithmetic operations with polynomials with more than one variable. They are also expected to understand that algebraic expressions with letters can be used to capture and explain quantities and quantitative relationships, as well as further develop their ability to write and interpret algebraic expressions with letters. The goal is to help students experience the merits of using algebraic expressions with letters.

Addition and subtraction of polynomials and multiplication and division of monomials
The goals here are for students to understand the meaning of monomials and polynomials, to carry out simple addition and subtraction of polynomials, such as $(3x - 2y) - (2x + 5y)$, to multiply polynomials by numbers, such as $2(4x - 5y)$, and to multiply and divide monomials.

Addition and subtraction of polynomials, focus on helping students develop proficiency in carrying out simple calculations necessary to solve simultaneous linear equations, such as $2(3x - 2y) - 3(2x + 5y)$. Also, consider providing opportunities for students to re-learn in connection with grade 7 study of calculations of algebraic expressions with letters. For example, the error below is probably because students did not understand the idea of terms.

$(4x+5)-(2x+3)$ (4

$= 4x-2x+5-3$
$= 2x+2$
$= 4x$

Through such studies, students will develop their abilities to identify and represent quantitative relationships in phenomena and interpret algebraic expressions in contexts. They will also discover new relationships through induction and analogy and understand the necessity of explaining those discoveries generally using algebraic expressions with letters. They can also develop their ability to use algebraic expressions with letters. These ideas must be learned over time; therefore, instruction should aim at gradual development, keeping in mind the use of algebraic expressions with letters to be studied in grade 3 of lower secondary school.

Strand: Geometrical, Measurement and Transformation

The study of coherent and logical reasoning starts with the study of properties of vertical angles and properties of parallel lines.

1. Two properties of parallel lines

The study of coherent and logical reasoning starts with the study of properties of vertical angles and properties of parallel lines.
Typically, the following two properties of parallel lines are discussed:
Corresponding angles formed by a transversal intersecting a pair of parallel lines are congruent.

If corresponding angles formed by a line intersecting a pair of lines are congruent, then the two lines are parallel. Students learn about parallel lines in grade 4 of elementary school. For example, 2 lines are defined to be parallel when they are perpendicular to another line. Then, through activities such as drawing parallel lines, the two ideas above are intuitively and empirically recognized. In lower secondary schools, these ideas become the basis of reasoning.
From the proposition, "vertical angles are congruent", and the two ideas above, the following propositions can be deduced:
Alternate interior angles formed by a transversal intersecting a pair of parallel lines are congruent.
When alternate interior angles formed by a line intersecting a pair of lines are congruent, then the pair of lines are parallel.
Then, enable students to deduce that, "the sum of angles in a triangle is 180° ", or the angles in a parallelogram can be determined when one angle measurement is given.
As for deducing new ideas, considerations should be given to the fact that elementary school mathematics provided students with some foundational experiences. In grade 2 of lower secondary school, it is important to help students become able to explain their ideas in an orderly manner using their own words instead of demanding formal written proofs. For example, while thinking about the congruence of vertical angles, instead of just relying on measurements, it is necessary to incorporate activities to verify this relationship in a coherent manner, making clear what the basis of the argument is.

2. Properties of angles in polygons.

Based on the properties of angles in a triangle that gives the sum of interior angles and the sum of exterior angles of polygons.
The sum of interior of angles in polygons can be found by dividing polygons into basic figures such as triangles. This is an example of mathematical thinking, "relating to what was learned previously." For the sum of exterior angles, enable students to find the sum using what they learned about the sum of interior angles. It is also important to reflect on the methods and results of the sum of interior angles in triangles and quadrilaterals, which were discussed in primary school. Students are to experience deducing properties of angles in polygons using properties of parallel lines and explaining their thinking in a coherent manner while making the reasons explicit. Here, the major goal is to foster students' abilities to verify,

examine, and represent properties of geometrical figures deductively, using the congruence conditions of triangles.

The relationship of inscribed angles and central angles has been moved to grade 3 from grade 2 of lower secondary school in this current revision. The reason is to continue the study of properties of geometrical figures in grade 2 and to examine the relationship between inscribed angles and central angles using those properties and through mathematical reasoning. The use of the relationship between inscribed angles and central angles in concrete situations is also emphasized lines are parallel.

Strand: Patterns and Algebra

1. Simultaneous equations with two unknowns

In grade 7 the students are expected to understand the meaning of letters and solutions of linear equations with one variable, and to learn to solve those equations. In grade 8, based on those studies, students are expected to understand linear equations with two variables and the necessity and the meaning of simultaneous linear equations with two variables. They are expected to be able to solve those equations. Moreover, it is aimed that students will develop the ability to use simultaneous linear equations in concrete situations.

Meaning of linear equations with two variables and their solutions
Help students understand that the solution to a linear equation with two variables is a pair of values for the two letters, x and y, which will satisfy the given equation. In other words, there is no fundamental difference from the meaning of solutions for linear equations with one variable. The two letters in a linear equation with two variables are both variables, and the pair of values from the range of numbers for each variable that satisfies the equation is its solution.

For example, with the equation, $2x + y = 7$, if the range of values for variables x and y are natural numbers, then there are a finite number of solutions, namely (1; 5), (2; 3), and (3; 1). If the range of variables is all integers, then there are an infinite amount of solutions. In this way, solutions to linear equations with two variables differ from solutions to linear equation with one variable because there may be more than one solution with linear equations with two variables.
The necessity and the meaning of simultaneous linear equations with two variables and the meaning of their solutions
A pair of simultaneous linear equations with two variables represents two conditions, each represented by an equation, to be satisfied. To solve a system of simultaneous linear equations with two variables means to find a pair of values that will satisfy both equations. To help students understand the meaning of the solution to a pair of simultaneous linear equations with two variables, it is possible to restrict the range of values for each valuable to the set of natural numbers and have students list solutions for each equation so that they can find a common solution. This way of solving simultaneous equations is not efficient in the process of problem solving. Those methods will be useful in the context of concrete problem solving situations. It is also possible to deepen students' understanding of the meaning of solutions to simultaneous equations by connecting linear functions and graphs of linear equations with two variables.

Solving simultaneous linear equations with two variables
A way to solve simultaneous linear equations with two variables is to eliminate one of the letters so that we can use the methods to solve linear equations with one variable. This way of finding the solution is an example of thinking that will connect new problem solving situations to what has already been learned previously. In addition to enabling students to solve simultaneous linear equations with two variables, help students recognize this way of thinking on their own so that they can understand solution by elimination or by substitution.

Consideration should also be given to provide opportunities to re-learn what it means to, ”solve an equation,” by making connections to the study of linear equations with one variable from grade 7

The study of solving simultaneous linear equations should aim at developing sufficient proficiency to solve simultaneous linear equations from concrete problem situations and enabling students to use simultaneous linear equations.

Use of simultaneous linear equations with two variables
While using linear equations with one variable, we can use only one variable to represent quantitative relationships in phenomena. However, in concrete situations, it is often easier to represent relationships using two variables than with only one variable. By using simultaneous linear equations with two variables, the range of problems that can be solved will expand, as well as problem solving becomes easier. To use simultaneous linear equations, the step of setting up the equations is important. In the process of instruction, having students capture and represent quantitative relationships in algebraic expressions by focusing on a specific quantity such as relationships of length, time or weight. It is also useful to make the relationships more explicit by representing the quantities in a table or a segment diagram.

Moreover, as students attempt to solve concrete problems, help them realize that the number of equations and the number of variables must be the same in order to have a unique solution. Help students use linear equations with one variable and simultaneous linear equations with two variables with a good prospect in mind.

2. Linear Functions

Through activities of investigating changes and correspondences of two quantities in concrete phenomena, students will examine linear functions. The study fosters students’ abilities to identify, represent and examine functional relationships.
Phenomena and linear functions
Students are expected to find the following relationships between two quantities, and y, that are in a functional relationship from concrete phenomena. As x increases by k,y increases by ak.

In general, linear functions can be represented as $y = ax + b$, where a and b are constants.

Building on the study of direct and inverse proportional relationships, students identify two quantities that change simultaneously and examine what functional relationships might exist between them and how they may be represented as algebraic expressions and graphs.

Direct proportional relationships are special cases of linear functions, $y = ax + b$.
Linear equations with two variables and linear functions
Given the linear equation with two variables, $ax + by + c$ = 0, as a representation of the relationship between the two variables x and y, then, the combinations of values of x and y become the objects of examination. When determining the pairs of values for x and y that will satisfy this equation, if b 6= 0, then once the value of

x is fixed the corresponding value of y is also fixed. Therefore, we can consider the algebraic expression $ax + by + c = 0$ represents a functional relationship between x and y.

For example, the linear equations with two variables, $x - 2y + 6 = 0$, can be considered as an algebraic expression representing a functional relationships. Moreover, if we transform this expression to $y = 1/2x + 3$, we can see that y is a linear function of x. Since the graphs of linear equations with two variables are lines, the solution of simultaneous linear equations with two variables can be solved by locating the point of intersection of the two lines in the coordinate plane. Therefore, it is possible to understand visually the meaning of solutions of simultaneous linear equations with two variables.

Strand: Statistics and Probability

1. Probability

In elementary school mathematics, how to count the number of all possible events systematically in concrete situations is taught in grade 6.
In grade 1 of lower secondary school, students learn that relative frequencies indicate the ratios of frequency in individual classes to the total number of data, and they can be considered as frequency of each class.
Based on these studies, in lower secondary school mathematics, students are to know that numbers that have been used to describe definitive phenomena may also be used to describe uncertain phenomena such as the tossing of a die in grade 2. Also, enable students to explain uncertain phenomena using the idea of probability.
The necessity and the meaning of probability
In mathematics lessons, definitive phenomena are typically discussed. However, in reality, uncertain phenomena from our daily life and society are also objects of mathematics, and to describe and understand the likelihood of occurrence, we need the idea of probability
We cannot predict the result of rolling a die. However, when we roll a die many times and organize the data, the ratio of the number of times each number rolled to the total number of tosses tends to reach a fixed value.
Students are to understand that probability is used to describe the likelihood of the occurrence of an event based on," the law of large numbers."
For example, make the total number of tosses, n, larger and larger. Then, count the number of times, rs,"1" is rolled and calculates the ratio r/n. As n becomes larger, r also becomes larger. However, the value of r/n becomes closer to a certain value. This fixed value to which the ratio tends toward is called the probability of rolling"1" when you toss a die.
Also, if we roll a fair die, we should expect the same likelihood of rolling any number. Therefore, for each number, if we roll a die many times, we anticipate the ratio of rolling it will approach 1/6 . In reality, the value to which the ratio r/n discussed above approaches when we toss a die many times is indeed 1/6.
As with this example, when we can expect each event to occur at the same frequency, that is, it is "equally likely," we can determine the probability by counting the number of ways events may occur.
To determine probability, we can save much time and effort by counting the number of outcomes instead of actually conducting experiments many times. However, the true meaning of probability, that is what can be learned about uncertain events, may get lost .

For example, if a student thinks, "because the probability of rolling a"1" is 1/6," "if we roll a die 6 times, we will get a "1" one time," then it indicates that their understanding is incomplete.
In teaching this topic, have students conduct experiments many times so that they can actually experience and understand that the ratio of a particular event occurring will approach a particular value. When probability is determined by counting possible events, it is also important to have students not only check whether or not the answers are correct but also what was understood about the likelihood of the occurrence of events by actually conducting experiments and investigations.

2. Determining probability for simple cases

To determine probability based on the counting of all possible outcomes, it is necessary to count accurately the number of equally likely outcomes for the event. Based on the study in grade 6, help students learn to count accurately by organizing outcomes systematically. The events should be simple enough so that students can count outcomes correctly using tools such as tree diagrams and 2-dimensional tables.

An example of a simple case is the tossing of two coins simultaneously and recording the outcomes of each coin.
There are four different outcomes possible: (head, head), (head, tail), (tail, head), and (tail, tail), and these outcomes are equally likely. Of these four outcomes, there is only one outcome that will result in both coins landing as heads. Therefore, the probability of getting two heads is 1/4," as discussed earlier, in this example, it is necessary to note that the "probability is 1/4 " does not mean that if we toss two coins simultaneously four times, we will definitely get two heads at one time.
Also in the example above, a common error is to think that the outcomes are (head, head), (head, tail), and (tail, tail), and the probability of getting two heads is 1/3. Help students to count all outcomes without any omission or duplication. It is also important that students can understand experientially. Have students actually conduct experiments many times and compare the results obtained with their counts.

3. Grasping and explaining uncertain phenomena

We can grasp and explain uncertain phenomena using the idea of probability. Probability is effective as the basis for grasping and explaining uncertain phenomena.
In teaching this topic, the focus should not be solely on determining probability, but an emphasis should be placed on problem solving involving uncertain phenomena. It is important that students can explain phenomena using probability as the basis of their reasoning. It is necessary to incorporate phenomena from our daily life and society so that it becomes clear to students that those phenomena can be explained using probability as the basis of reasoning.
For example, to determine whether or not the order in which you draw will influence the likelihood of winning a lottery, that is, whether or not the lottery is fair, we can explain using the idea of probability. It is also used to instruct that the rules of the lottery have to be clearly stated since the judgment may be influenced by the way the rules are stated.
By grasping and explaining uncertain phenomena using probability, help students understand that numbers can be used to make decisions involving those situations where we cannot say, "this will certainly be _____."
Help students experience the relationships between mathematics and real-life and in society. It is important to emphasize the meaning of probability when teaching this idea.

Mathematics Grade 8

Guided Lessons

Lessons No: 1 to 50

LO1 : Monomials and polynomials

Addition and Subtraction with expressions

Lesson Objective: To identify the meaning of monomials and polynomials expression **(8.1.1. 1/3)**
Materials: Blackboard

ASK-MT and Assessment	
Attitudes/Values	Show interest in learning about monomials and polynomials expression.
Skills	Identify and express monomials and polynomials expressions.
Knowledge	Monomials and polynomials expressions.
Mathematical Thinking	Think of ways to identify and represent monomials and polynomials expressions.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example 1: **Terms of a polynomial expression**

$3a^2 - 2a + 1$ can be written as $3a^2 + (-2a)$, so the terms of the polynomial expression $3a^2 - 2a+1$ are $3a^2$, $-2a$, and 1

TN: When a term in an expression is the product of a number and letters, the number is the coefficient of the letter. In Example 1 above, the coefficient of a^2 is 3, and the coefficient of a is -2.

The degree of $4x$ and $-2a$ is 1, while the degree of $5ab$ and $3x^2$ is 2. In a polynomial expression the degree of the expression is highest degree among all of the terms in the expression.

Example 2: **Degrees of polynomial expressions**

The degree of $3x^2 - 4x + 6$ is 2.
The degree of $2x + 5$ and $-7a + 6$ is 1.

Example 3: **Combining like terms** (1)

$$\begin{aligned} 6a - 2b + 3b - 4a &= 6a - 4a - 2b + 3b \\ &= (6a - 4a) + (-2b + 3b) \\ &= (6 - 4)a + (-2 + 3)b \\ &= 2a + b \end{aligned}$$

Example 4: **Combining like terms** (2)

$$\begin{aligned} x^2 + 3x + 1 - 4x + 2x^2 &= x^2 + 2x^2 + 3x - 4x + 1 \\ &= (x^2 + 2x^2) + (3x - 4x) + 1 \\ &= (1 + 2)x^2 + (3 - 4)x + 1 \\ &= 3x^2 - x + 1 \end{aligned}$$

TN: The calculation law below can be used to combine like terms into a single term.

$$ma + na = (m + n)a$$

Teaching and Learning Activities

Key Ideas

- Expressions that only show multiplication of numbers or letters as $3a$, ab, and a^2, are called **monomial expressions.** Expressions with only one number or letter, such as a or 500, are also considered **monomial expressions.**

- Expressions that show the sum of monomial expressions, such as $50a + 80b$, are called **polynomial expressions.** Each monomial expression ($5a$, $80b$), is called a **term** of polynomial expression $50a + 80b$.

- The number of letters being multiplied in a monomial expression is called the **degree** of a monomial.

- An expression with a degree of 1 is called a **linear expression,** while an expression with a degree so 2 is called a **quadratic expression.**

- Expressions like $6a - 2b$ - 4a have terms with the same letter portion ($6a - 2b + 3c - 4a$, have terms with the same letter portion ($6a$ and $-4a$,$-2a$ and $3b$). These are called **like terms.**

Exercises

1. State the terms in the polynomial expression $6a - b + 5$. What are the coefficients of a and b?

2. What is the degree of the following expression?

   (a) $-x^2 + 4y + 3$ (b) $a - b + 5$

3. State the like terms in the following expressions.

   (a) $4a + 5b - 6c + 7a - 8c$ (b) $xy + x - 5xy - 2x$

4. Combine like terms and simplify

   (a) $3a - 6b + 8a + b$ (b) $3x - 7y - x + 2y$ (c) $x^2 - 4x + 2 + 3x$ (d) $y^2 - 3y - 3y^2 + 2y$

LO2 : Addition and subtraction with expression

Lesson Objective: To express and apply calculations of addition and subtraction with polynomial expressions.**(8.1.1.1/3)**

Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Enjoy adding and subtracting algebraic expressions.
Skills	Add and subtract monomial and polynomial expressions.
Knowledge	Addition and subtraction of monomial and polynomial expressions.
Mathematical Thinking	Think of how to add and subtract monomial and polynomial expressions.
Assessment	Use the ask-mt to assess the students learning progress during the lesson.

Teaching and Learning Activities

To add or subtract two expressions, we put each expression in parentheses, combine them with a + or – sign, then calculate.

Example 1. **Adding $5a + 3b$ and $2a + 5b$**

$$(5a + 3b) + (2a + 5b)$$
$$= 5a + 3b + 2a + 5a$$
$$= 7a + 8b$$

Example 2. **Subtracting $2a + 5a$ from $5a + 3b$**

$$(5a + 3b) - (2a + 5b)$$
$$= 5a + 3b - 2a - 5b$$
$$= 3a - 2b$$

TN: You can also add or subtract polynomial expressions by putting them vertically and lining up the like terms.

Example 3. **$(3x - 7y) + (2x + 5y)$**

$$\begin{array}{r} 3x - 7y \\ +\ 2x + 5y \\ \hline 5x - 2y \end{array}$$

$3x + 2x = 5x$
$-7y + 5y = -2y$

Example 4. **$(4x + 6y) - (x + 6y - 5)$**

$$\begin{array}{l} 4x + 6x \\ -)\ x + 6y - 5 \\ \hline 3x \qquad +5 \end{array}$$

$4x - x = 3x$
$6y - 6y = 0$
$0 -(-5) = 5$

Exercises

(1) Add the two expressions.

(i) $4x - 7y,\ x + 5y$ (ii) $5a - 2b,\ -a - 3b$

(2) Subtract the expression on the right from the expression on the left.

(i) $5 + 2y,\ 3x + y$ (ii) $3a - 6b,\ 2a + 4b$

(3) Calculate.

(i) $\begin{array}{r} 2x - 3y \\ +\ 4x + 5y \\ \hline \end{array}$ (ii) $\begin{array}{r} x + y \\ +\ x - y \\ \hline \end{array}$

L03 : Different ways to calculate linear expression

Calculating with various polynomial expressions

Lesson Objective: To use distributive law to simplify expressions containing parenthesis **(8.1.1.1/2/3)**
Materials: Blackboard

ASK-MT and Assessment	
Attitudes/Values	Share their ideas on how to use the distributive law.
Skills	Use distributive law to simplify expressions containing parenthesis.
Knowledge	The Use of the distributive law in simplifying expressions.
Mathematical Thinking	Think of ways how to simplify expressions containing parenthesis.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example 1 : **Number × polynomial expression**

$$5(2a + 3b) = 5 \times 2a + 5 \times 3b$$
$$= 10a + 15b$$

Example 2: **Polynomial expression ÷ by number**

$$(9x - 6y) \div 3 = \frac{9x}{3} - \frac{6y}{3}$$
$$= 3x - 2y$$

Example 3: **Calculating with parentheses ①**

$$3(x - 2y) + 2(2x + y)$$
$$= 3x - 6y + 4x + 2y$$
$$= 7x - 4y$$

Example 4: **Calculating with parentheses ②**

$$5(x + 3y) - 3(2x - 5y + 1)$$
$$= 5x + 15y - 6x + 15y - 3$$
$$= -x + 30y - 3$$

Example 5: **Calculating expressions with fractions**

$\frac{1}{3}(2x + y) - \frac{1}{6}(x - 5y)$ This method removes the parenthesis first

$$= \frac{2}{3}x + \frac{1}{3}y - \frac{1}{6}x + \frac{5}{6}y$$
$$= \frac{1}{2}x + \frac{7}{6}y$$

Key Ideas

Use the distributive law $m(a + b) = ma + mb$ to simplify expressions containing parentheses.

Exercises

Calculate.

(i) $-3(x - 2y)$ (ii) $(4a - 6b) \times \frac{1}{2}$ (iii) $(-8x + 6y) \div 2$ (iv) $(5A - 15B) \div (-5)$

Calculate.

(i) $2(3x - y) + 3(x + 2y)$ (ii) $3(5a - b) - 2(2a - 2b)$ (iii) $4(a + 1) + 2(2a + b - 3)$

LO4 : Value of expression

Lesson Objective: To find the value of an expression by using substitution method. **(8.1.1.2/3/4)**
Materials: Blackboard

ASK-MT and Assessment	
Attitudes/Values	Share their ideas on the use of the substitution method.
Skills	Use substitution method to simplify and find solutions of the expression.
Knowledge	Solutions of expression.
Mathematical Thinking	Think of how to substitute and simplify the expressions.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1: **Calculating the value of an expression**

Find the value of the expression when $x = \frac{1}{2}$ and $y = -\frac{1}{3}$

$(3x + 5y) - (7x + 2y)$

Approach: Simplify the expression, then substitute.
Solution:

$(3x + 5y) - (7x + 2y)$
$= 3x + 5y - 7x - 2y$
$= -4x + 3y$

Substituting $x = \frac{1}{2}$ and $y = \frac{1}{3}$ in this expression

$-4x + 3y = -4 \times \frac{1}{2} + 3 \times (-\frac{1}{3})$
$= -2-1$
$= -3$

Key Ideas

Expressions can be simplified and when given values for each term, substitutions can be done

Exercises

Find the value of the expressions when $a = -\frac{1}{11}$ and $b = \frac{1}{7}$

(1) $3a + 2b + 5b - 4b$

(2) $8(2a + b) - 5(a - 4b)$

L05 : Multiplication and division of algebraic expression

Multiplication and division of monomial expressions

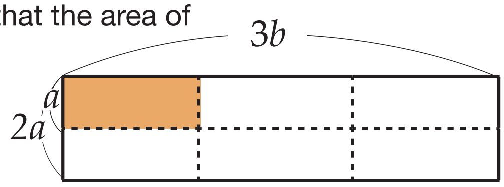
Lesson Objective: Apply multiplication and division with monomials expressions. **(8.1.1.1/2)**
Materials: Blackboard

ASK-MT and Assessment	
Attitudes/Values	Be interested in multiplying and dividing with monomials expressions.
Skills	Calculate monomial expressions by multiplication and division.
Knowledge	Multiplication and division of monomials and polynomials.
Mathematical Thinking	Think of ways on how to multiply and divide monomials and polynomials.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introductory

You have several tiles a cm long and b cm wide. They are arranged as shown in the figure on the right to form a rectangle $2a$ cm long and $3b$ cm wide. Compare the area of the rectangle with the number of tiles. What do you notice? In the figure on the right, you can see that the area of the rectangle $(2a \times 3b)$ cm^2 is equal to $6ab$ cm^2.



This can be deduced as follows:

$$\begin{aligned} 2a \times 3b &= (2 \times a) \times (3 \times b) \\ &= (2 \times 3) \times (a \times b) \\ &= 6ab \end{aligned}$$

Example 1. **Multiplying Monomial Expressions**

a) $4x \times (-2y)$
$= 4 \times (-2) \times x \times y$
$= -8xy$

b) $(-8a) \times 5a$
$= (-8) \times 5 \times a \times a$
$= -40a^2$

Example 2: **Calculating Expressions with exponents.**

$$\begin{aligned} (-5y)^2 &= (-5y) \times (-5y) \\ &= (-5) \times (-5) \times y \times y \\ &= 25y^2 \end{aligned}$$

TN: You can divide monomial expressions in the same was as you divide numbers.

Example 3: **Dividing monomial expressions**

(a) $8xy \div 4x = \dfrac{8xy}{4x} = 2y$

(b) $6a^2 \div 2a = \dfrac{6a^2}{2a} = 3a$

$A \div B = \dfrac{A}{B}$

Example 4: **Dividing with fractions**

$$\begin{aligned} -\frac{3}{2}x^2 \div \frac{3}{4}x &= \frac{3x^2}{2} \div \frac{3x}{4} \\ &= -\left(\frac{3x^2}{2} \times \frac{4}{3x}\right) \\ &= -2x \end{aligned}$$

Example 5: **Calculations with both multiplication and division**

$$\begin{aligned} -xy \times 6x \div (-3y) &= \frac{4xy \times 6x}{3y} \\ &= 8x^2 \end{aligned}$$

Teaching and Learning Activities

Example 6: **Dividing three expressions**

$$12a^2b \div 2a \div (-3b) = -\frac{12a^2b}{2a \times 3b}$$

$$= -2a$$

$$A \div B \div C = \frac{A}{B} \div C = \frac{A}{B \times C}$$

Key Ideas

Multiply monomial expressions by taking the product of the coefficient and multiplying it by the product of the letters.

Exercises

1. Calculate.

(i) $(-4x) \times 5y$ (ii) $(-7y) \times (-3x)$ (iii) $\frac{5}{9}a \times (-3b)$ (iv) $\frac{1}{2}x \times \frac{3}{4}x$

2. Calculate.

(i) $(-7a)^2$ (ii) $\frac{1}{3}x \times (3x)^2$ (iii) $-(4x)^2$ (iv) $(-a)^2 \times 3a$

3. Calculate.

(i) $(-6ab) \div 2a$ (ii) $8x^2 \div x$ (iii) $(-9x^2y) \div (-3y)$ (iv) $5a^2 \div (-10a^2)$

4. Calculate.

(i) $7x^2 \div (-\frac{7}{4}x)$ (ii) $-\frac{5}{18}ab \div (-\frac{10}{9}b)$ (iii) $-\frac{1}{5}x^2y \div \frac{1}{5}x$ (iv) $\frac{2}{3}y^2 \div \frac{3}{2}y^2$

5. Calculate.

(i) $2a \times 3ab \times 4b$ (ii) $6ab \times (-7a) \div 14b$ (iii) $8x^2 \div (-4x) \times (-3x)$ (iv) $16xy^2 \div 4y \div (-2x)$

L06 : Using algebraic expressions to solve problems

Applying algebraic expressions

Lesson Objective: To use algebraic expressions to solve problems **(8.1.1.1/2/3/4)**
Materials: Blackboard

ASK-MT and Assessment	
Attitudes/Values	Share their ideas on how use algebraic expression.
Skills	Interpret word problems and use algebraic expression to solve problems.
Knowledge	Use of algebraic expressions.
Mathematical Thinking	Think of ways to use algebraic expression to solve problems.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introductory
As you can see here,

$24 = 20 + 4 = 10 \times 2 + 4$
$63 = 60 + 3 = 10 \times 6 + 3$
$85 = 85 + 5 = 10 \times 8 + 5$

Any positive two-digit integer can be expressed as 10 x (number in the tens place)+ (number in the ones place). If we use a to represent the number in the tens place and b to represent the number in the one place, we get $10\,a + b$

Example.1: **Two-digit integer problem**
The sum of any two-digit positive integer and the number formed when the numbers in the tens and ones place of that integer are switched is always a multiple of 11. Explain why?.

Approach: We can express any multiple of 11 as 11 x integer.
Solution: If we use a to represent the number in the tens place and b to represent the number in the ones place in our original integer, we can express the integer as $10a$ + b. The number with the tens and ones places switched can then expressed as $10a + a$.

The sum of the two numbers is therefore
$(10a + a) + (10b + a) = 11a + 11b$
$= 11\ (a + b)$
$a + a$ is an integer, so $11(a + b)$ must be a multiple of 11.

Key Ideas
Multiply monomial expressions by taking the product of the coefficient and multiplying it by the product of the letters.

Exercises

What kind of number is the sum of the two numbers in *Example.1* divided by 11?

LO7 : Even and odd numbers

Lesson Objective: To apply addition and subtraction with even and odd numbers **(8.1.1.1/2/3/4)**
Materials: Blackboard

ASK-MT and Assessment	
Attitudes/Values	Show interest in adding and subtracting expressions with even and odd numbers.
Skills	Add and subtract expressions with even and odd numbers.
Knowledge	Even and odd numbers in addition and subtraction.
Mathematical Thinking	Think of how to add and subtract expressions with even and odd numbers
Assessment	Use the ask-mt to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introductory

Even numbers can be divided evenly by 2, so we can express them as 2 x integer. If we use m to represent the integer, we get 2m. We can think of odd numbers as being 1 more than an even number. If we use n to represent integers, we can express them as 2n + 1.

Example.1: **Sum of two odd numbers**

Two odd numbers are expressed as

$$2m + 1 \text{ and } 2n + 1$$

by using integers m and n. The sum of the two integers is then

$$(2m+1)+(2n+1) = 2m+2n+2$$
$$= 2(m+n+1)$$

Since m + n +1 is an integer, 2(m + n +1) must be even. In other words, the sum of any odd numbers is always even.

Key Ideas

When you have one even number and one odd number, the sum is always odd.

Exercises

When you have one even number and one odd number, the sum is always odd.
Explain why?

LO8 : Transforming equalities

Lesson Objective: Calculate values of unknowns by transforming equalities **(8.1.1.2/4)**
Materials: Blackboards

ASK-MT and Assessment	
Attitudes/Values	Share their ideas on how to transform equalities.
Skills	Calculate and use transpose method to transform equalities and solve the unknown values.
Knowledge	Transpose method to transform equalities.
Mathematical Thinking	Think of ways to find solutions to transform equalities.
Assessment	Use the ask-mt to assess the students learning progress during the lesson.

Teaching and Learning Activities

Review

The figure on the right is a track consisting of a rectangle and two semicircles. The perimeter of the track is 200 m. use π =3.14 to find the length of the following.

(a) The length of line segment AB when the radius of the semicircles is 10 m.
(b) The radius of the semicircles when the line segment AB is 50 m.

Example.1: **Transforming equalities**
If the radius of the semicircles is r m and the length of line segment AB is x m, we can set up the following equality for information above.

$2x + 2\pi r = 20$..........................
Once we determine the value of r, we can set up an expression that allows us to find the value of x.
If we transpose 2πr, we get $2x = 200 - 2\pi r$
Divide both sides by 2 $x = 100 - \pi r$............ ▯

Key Ideas

Taking equality (1) and setting it up as expression (2) to find x is called solving for x in the original equality.

Exercises

How many meters is the line segment AB in the diagram above if the radius of the semicircles is 15 m? If the radius is 20m?
Solve for r in the equality (1) above. Next find the radius of the semicircles in meters when line segment AB is 40m in the diagram above.

Solve the equality for the letter in [ ]

(i) $x + y = 6$ [x] (ii) $2x - y = 3$ [y] (iii) $l = 2\pi r$ [r] (iv) $l = 2(a + b)$ [b]

Unit Checkpoint

Review on operation of algebraic expressions

Review on operation of algebraic expressions

1. Calculate.

(a) $3x - 7y + 4x$ (b) $8a - b - 7a + 2$
(c) $-5x + 9y + 3x - 8y$ (d) 3x^2-5x-2x^2+x

2. Add the two expressions.
Next, subtract the expression on the right from the expression on the left.

(a) 3a+2b, a-4b (b) x-4y, -2x+3y

3. Calculate

(a)
$$\begin{array}{r} 3x + 4y \\ +)\ 2x - 2y \\ \hline \end{array}$$
(b)
$$\begin{array}{r} a - 2b \\ -)\ -a - 3b \\ \hline \end{array}$$

4. Calculate

(a) 5(4a-5b) (b) 5x+2(x--2y

(c) 2(2x-y)+(5x-y) (d) 3(x+y)-3(x-y)

5. Calculate

(a) 2a x (-9b) (b) (4x) x (-5y) (c) $(-2a)^2$

(d) 12÷3b (e) $3x^3 \div x$ (f) (□6x□^2)÷2x

6. The following explains why the sum of two even numbers is an even number.
Fill in the □

We can express any two even integers as □ and □ by using integers m and n.
□+ □ = □ (m + n)
Since m + n is an integer, it must be even. In other words, the sun of any two even numbers is always even.

7. Solve for y in the equality 2x + y = 5

L09 : Solving simultaneous equations

Simultaneous equations

Lesson Objective: Solve simultaneous equations with two unknowns **(8.3.3.1/2).**
Materials: Blackboard

ASK-MT and Assessment	
Attitudes/Values	Appreciate and value equations with two unknowns and their solutions.
Skills	Use substitution method to solve simultaneous equations.
Knowledge	Understand the linear equations and simultaneous equations.
Mathematical Thinking	Think about ways of calculating the simultaneous equations and finding their solutions.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introductory

Let x to represent the number of times Mero's uncle said, "OK!" to put fish in the left side cooler and y to represent the number of times when he said, "OK!" to put fish in the right side cooler

Situation

The cooler on the left side gets 2 fish each time
The cooler on the right side gets 1 fish each time
We can then use an equality to show their relationship to the number of fish

(insert diagram of 2 coolers)

The following equality expresses the relationship of the above situation
$2x + y = 21$……………………
This type of equality is also an equation

Example.1: **Pair of values**

Find the value of y in linear equation with two unknowns
1. When the value of x is 0, 1, 2, and so on. Fill in the blanks below;
Hint: In the equation when x is 1, then y is 2x 1 + y =21

x	0	1	2	3	4	5	6	7	8
y	21	19							

(0,21),(1,19), and other pairs of x,y values in the table above are all solutions to linear equation with two unknowns . Pairs like (1/2,20) and (3/2,18) are also solutions to the equation.

Kita's uncle said, "OK!" 13 times for fish to be put into the coolers
We can add this condition by expressing it as;
$x + y = 13$

Use the two tables above (1) and (2), to find the pairs of x,y values that satisfy both linear equations with two unknowns in the equations;

$2x + y = 21$……….1
$x + y = 13$……….2 is (8,5)

Teaching and Learning Activities

Key Ideas

- An equation with two letters is called a linear equation with two unknowns
- A pair of values that fits the letter in a linear equation with two unknown is called a solution of the equation
- Set of two equations are called simultaneous equations
- The pair of x, y values that satisfy both equations are called solutions to simultaneous equations, and finding those solutions is called solving simultaneous equations

Exercises

(1) Fill in the table below with values of y, when value of x is 0,1,2,3 and so on.

x	0	1	2	3	4	5	6	7	8
y	21	19							

(2) Use the above table and the one in Example.1 to find the pair s of x,y values that satisfy both linear equations with two unknowns and .

(3) Which of (a) through (c) below has (3,4) as a solution?

(a) {(x + y = 7 @x + 2y = 8) (b) {(3x - y = 4 @2x - 5y = 7) (c) {(4x + y = 8@-x +3 y = 9)

L10: Using addition and subtraction method (1)

Lesson Objective: To solve simultaneous equations using addition and subtraction method **(8.3.3.1/2).**
Materials: Blackboard

ASK-MT and Assessment	
Attitudes/Values	Confident in solving simultaneous equations using addition and subtraction method.
Skills	Solve simultaneous equations using addition and subtraction method.
Knowledge	Substitution and elimination methods to find the solutions to the simultaneous equation.
Mathematical Thinking	Think about how to solve simultaneous equations using addition and subtraction method.
Assessment	Use the ask-mt to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introductory

Three pencils and one notebook cost K2.50
and one pencil and one notebook cost K1.50

What is the price of each item? Use the figures to help you.

If we let x toea to represent the cost of pencils and y toea to represent the cost of notebooks then we can make a simultaneous equation from the diagram above to find the cost of each item as below

{(3x + y = 250……. ①
x + y = 150……… ②

Think about how you can solve this equation!
If we subtract left side of the equation: ① from ② we get 2x
If we subtract right side of the equation: ① from ② we get 100

$$\begin{array}{r} 3x + y = 250 \\ \underline{(-)x + y = 250} \\ 2x = 100 \end{array}$$

Which is 2x = 100, then x is equal to 50 (x = 50)
To find y toea, we can substitute x toea in equation (2),
Then y is 50 + y = 150, which is y is equal to 100 (y = 100)

Therefore x toea for each pencil is 50 toea and
y toea for each notebook is 100 toea (K1)

The solution to the simultaneous equation is (50,100)

This method is used to subtract the equations so that y value is eliminated and the 3rd equation is made to easily find the x value, then y value can be found when x value is substituted in one of the simultaneous equations.

TN: So you can write solutions for x and y as (x,y) = (50,100) like this

x = 50,y = 100

or like this x = 50
y = 100

Teaching and Learning Activities

Example.1: **Solve simultaneous equations by subtracting left and right sides**

Solve the simultaneous equations below by subtracting the left and right sides

Example 2: **Solve simultaneous equations by adding left and right sides**

Solve the simultaneous equations below by adding the left and right sides

(1) . {(2x+y=7 5x-y=14)   (2) {(-x+3y=4@x-4y=-6)

Key Ideas

- Deriving an equation without y from simultaneous equation with x and y is called **eliminating** y
- Eliminating one of the letters by adding or subtracting the left and right sides in a simultaneous equation is called the **addition/subtraction method**

Exercises

Use the addition /subtraction method to solve the simultaneous equation below.

(1) $\begin{cases} 6x - y = 22 \\ 6x + 5y = -2 \end{cases}$ (2) $\begin{cases} 3x - 2y = 19 \\ 5x + 2y = 21 \end{cases}$ (3) $\begin{cases} x + y = 6 \\ -x + y = 10) \end{cases}$

L11: Using Addition and subtraction method (2)

Lesson Objective: To solve simultaneous equations by multiplying one of the expressions. **(8.3.3.1/2).**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Share their ideas on to solve simultaneous equations by multiplying one of the expression
Skills	Solve simultaneous equations by multiplying one of the expression
Knowledge	Solving simultaneous equations by multiplying one of the expression
Mathematical Thinking	Think about how to solve simultaneous equations by multiplying one of the expression
Assessment	Use the ask-mt to assess the students learning progress during the lesson

Teaching and Learning Activities

Example. 1: **Multiplying expression**

$\begin{cases} x + 2y = 4 \\ 2x + 3y = 5 \end{cases}$ ……..① ……..②

TN: If we add or subtract ① and ②as they are, we won't be able to eliminate on of the letters.
However, if we multiply b of ① by 2 in order to match the x coefficients of ①and ② ,we get

$2x + 4y = 8$ ……..①

If we now subtract ② from ①',we can eliminate x
Subtracting from ① ' give us $y = 3$
If we substitute this value for y in □, we get $x = -2$. The solution to the simultaneous equation is therefore

$(x,y) = (-2,3)$

Example. 2: **Multiply expression to solve** {$4x + 7y = -2$……….①
$6x - 5y = 28$…………②

Approach: We can't eliminate on of the integers by multiplying just one of the expressions by an integer and then adding or subtracting. In this case, we need to multiply both sides of equations in the set to match the coefficients of the letters.

Solution

①× 3 $12x + 21y = -6$………①
②× 2 $12x - 10y = -2$………②
①-②' $31y = -62$
$y= -2$

Substitute $y=-2$ in □
$4x -14=12$
$4x =12$
$X = 3$
$(x,y) = (3,-2)$

Teaching and Learning Activities

Exercises

(1) Use the addition /subtraction method to solve the simultaneous equation below.

(a) { 2x-y =4
5x+3y=-1)

(b) {2x + y = 7
x + 4y =7

(c) {4x - 5y =-9
x - 2y = 0

(2) Solve the simultaneous equation below.

(a) {3x + 2y = 8
5x - 3y = 7

(b) {6x + 4y = 2
7x - 3y =-13

(c) {9x - 2y = 11
4x - 5y =-9

L12: Substitution method

Lesson Objective : To solve simple simultaneous linear equations with two variables by the substitution and transpose method. **(8.3.3.1/2)**

Materials: Blackboard

ASK-MT and Assessment	
Attitudes/Values	Appreciate the importance of using substitution and transpose method to simultaneous equations
Skills	Solve simultaneous equations using substitution and transpose method
Knowledge	Substitution and transpose method to solve simultaneous equations
Mathematical Thinking	Think about how to use substitution and transpose method to solve simultaneous equations
Assessment	Use the ask-mt to assess the students learning progress during the lesson

Teaching and Learning Activities

TN: There is another way to eliminate one of the letters besides the addition/subtraction method.

Example.1: **Substitute to eliminate a letter**

y = x - 2............. ①
5x + 3y =18......... ②

$y = x - 2$
$5x + 3y = 18$
↓ Substitute
$5x + 3(x-2) = 28$

Here we can substitute x –2 in ① for y in ② .
This gives us 5x + 3 (x-2) =18. If we substitute this for x in ①, we get y = 1. Therefore, the solution to the simultaneous equation is (x,y) = (3,1).

Example.2: **transform the expression and substitute to solve**

Solve the simultaneous equation below.

y - x = 6.......... ①
3x + 2y = 17..... ②

Approach: Transpose terms and then eliminate a letter.
Solution:

Transpose $-x$ in ① to get y = 6 + x...........'①
Substitute ①' in ②

3x + 2 (6 + x) = 17

Solve to get x = 7

Substitute x = 1 ① to get y = 7

(x,y) = (1,7)

Key Ideas

- Eliminating a letter by substitution is called a Substitution Method

L13: Various simultaneous equations

Lesson Objective: To solve simultaneous equations with parentheses and fractional coefficients. **(8.3.3.1/2/3).**

Materials: Blackboard

ASK-MT and Assessment	
Attitudes/Values	Share ideas on how to solve simultaneous equations with parentheses and fractional coefficients
Skills	Solve simultaneous equations with parentheses and fractional coefficients
Knowledge	Solving simultaneous equations with parentheses and fractional coefficients
Mathematical Thinking	Think about how to solve simultaneous equations with parentheses and fractional coefficients
Assessment	Use the ASK-MT to assess the students learning progress during the lesson

Teaching and Learning Activities

TN: If you have simultaneous equations with parenthesis or non- integer coefficients, simplify first, and then solve

Example.1 **How to solve simultaneous equation with parentheses**

Solve the simultaneous equations below

{(x-y=4………... 2x=3(1-y…..)

Approach: Remove the parentheses from expression and transpose the term to simplify. Then solve.

Solution:

From ②, we get $2x = 3 - 3y$

$2x + 3y = 3$……… ②

① × 3 $3x - 3y = 12$ ………①

① + ② $x = 3$

Substitute $x = 3$ ①

$3 - y = 4$

$y = -1$

$(x,y) = (3,-1)$

Example.2 **How to solve simultaneous equation with fractional coefficients**

Solve simultaneous equation below

{(x=2y+5…………@@x/3-y/2=2…………)

Approach: Cancel the denominators to simplify the equation

Solution:

② × 6 $2x - 3y = 12$…………………. ②

Substitute ① in ②

$2(2y + 5) - 3y = 12$

Solve to get $y = 2$

Substitute $y = 2$ in ① to get $x = 9$

$(x,y) = 9,2)$

Teaching and Learning Activities

Example.3 **How to solve the equation A=B=C**

Solve the equations 3x-2y=x+y-20=5

Approach: When you have an equation in the form A=B=C, change it into one of the following three formats, and then solve.

(A) $\begin{cases} A = C \dots\dots\dots ① \\ B = C \dots\dots\dots ② \end{cases}$ (B) $\begin{cases} A = B \dots\dots\dots ① \\ A = C \dots\dots\dots ② \end{cases}$ (C) $\begin{cases} A = B \dots\dots\dots ① \\ B = C \dots\dots\dots ② \end{cases}$

Exercises

(A) $\begin{cases} A = C \dots\dots\dots ① \\ B = C \dots\dots\dots ② \end{cases}$ (B) $\begin{cases} A = B \dots\dots\dots ① \\ A = C \dots\dots\dots ② \end{cases}$ (C) $\begin{cases} A = B \dots\dots\dots ① \\ B = C \dots\dots\dots ② \end{cases}$

(a) $\begin{cases} 4x + 7y = 39 \dots\dots\dots ①. \\ 2(x - y) = 3x + 3y \dots ② \end{cases}$ (b) $\begin{cases} 3(x + y) = 2x - 1 \dots\dots\dots ①. \\ x + y = -5 \dots\dots\dots\dots ② \end{cases}$

(c) $\begin{cases} \frac{x}{4} - \frac{y}{5} = 1 \dots\dots\dots ①. \\ 3x + 4y = -52 \dots ② \end{cases}$ (d) $\begin{cases} x + y = 11 \dots\dots\dots ①. \\ \frac{8}{100}x + \frac{9}{100}y = 1 \dots ② \end{cases}$

(e) $\begin{cases} 0.3x + 0.4y = 0.5 \dots\dots\dots ① \\ x - 2y = -5 \dots\dots\dots\dots ② \end{cases}$ (f) $\begin{cases} 0.1x + 0.04y = 15 \dots\dots\dots ①. \\ 3x - 2y = 50 \dots\dots\dots\dots ② \end{cases}$

(2) Change the equation in Example .3, into format (A) and solve it

(3) Solve the equation 5x + 2y= -x-y + 3 = 4

L14: Solve real problems using simultaneous equations

Using simultaneous equations

Lesson Objective : To use simultaneous equations to solve real life problems **(8.3.3.3)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Confidently use simultaneous equation to solve real life problems
Skills	Solve real life problems confidently using simultaneous equation
Knowledge	Solving real life problems using simultaneous equation
Mathematical Thinking	Think about how to solve real life problems using simultaneous equation
Assessment	Use the ASK-MT to assess the students learning progress during the lesson

Teaching and Learning Activities

Example.1 **use simultaneous equations to solve real life problems**

Set up simultaneous equations to find the number of 2 and 3 point shots in the problems.

Problem: A player makes a total of eight 2 and 3 point shots doing so, he scores a total of 19 points. How many of each type of shots did he make?

Steps
You can use the below to set up simultaneous equation for this problem
(1) Determine the quantitative relationships in the problem.

- Relationship involving the numbers of shots

[Number of 2-point shots] + [Number of 3-point shots] = 8 (shots) ……①

- Relationship involving the numbers of points

[Points earned from 2-point shots] + [Points earned from 3-point shots] = 19 (points) ……②

(2) Set up the simultaneous equations using the two letters x and y

Let the number of 2-point shots be x shots and the number of 3-points shots be y shots. Set up an equality using relationships ① and ② above to get the following simultaneous equations.

$$\begin{cases} x + y = 8 \ldots\ldots\ldots\ldots ① \\ 2x + 3y = 19 \ldots\ldots\ldots ② \end{cases}$$

Example.2 **Problems involving money**

Admission to a museum for two adults and one junior high school students is K13.00. Admission for one adult and two junior high school students is (K11.00. What is the price of each type of tickets?

Approach: Here are the quantitative relationships in this problem.

(Price of 2 adult tickets) + (price of 1 student ticket) = (K65)
(Price of 1 adult ticket) + (price of 2 student tickets) = (K40)

Teaching and Learning Activities

Solution

Let x kina represent the price of one adult ticket and y kina represent the price of one student ticket.

$2x + y = 65$ ……….①

$x + y = 40$ …….. ②

②× 2 $2x + 2y = 80$

②-① $y = 15$

Substitute $y = 15$ in ① to get $x = 25$

$(x, y) = (25, 15)$

An adult ticket cost K25 and student ticket cost K15

L15 : Solve real problems using simultaneous equations (2)

Lesson Objective: To solve speed, time, travel distance and ratios using various approaches to solve problems in simultaneous equations. **(8.3.3.3)**

Materials: blackboard, related pictures

ASK-MT and Assessment	
Attitudes/Values	Appreciate using different approaches to solve various problem situations using simultaneous equations.
Skills	Use various approaches to solve simultaneous equations involving quantitative relationships
Knowledge	Solving simultaneous equations involving quantitative relationships within time, speed and travel distances, cost and discounts
Mathematical Thinking	Think about the approaches on solving the different types of problems.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson

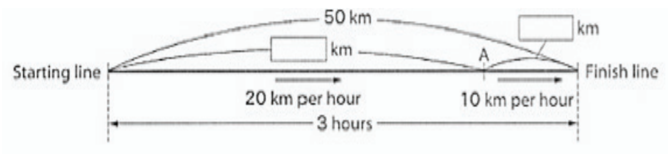
Teaching and Learning Activities

Example.1 **Problems involving speed, time and travel distance**

On a 50-kilometer racecourse, participants ride bicycles until they get to point A. They then get off their bikes and run the rest of the way.

A racer can complete the course in 3 hours if she bikes at a speed of 20km per hour.
Find the travel distance of the biking and running portions of the course.

Approach: Use the diagram below showing the quantitative relationships in this problem



(Biking distance) + (running distance) = 50 (km)
(Biking time) + (running time) = 3 (hours)

Solution:

Let x km represent biking distance and y km represent running distance.
{ x + y = 50x/20+y/10=3)

Solving this gives us (x,y) = (40,10)
Biking distance is 40 km, running distance is 10 km

Time=(travel distance)/speed

Teaching and Learning Activities

Example.2. **Problems involving ratios**

You buy a hat and shirt together.
Regular price for both would have been 31 kinas, but the shirt was 20% off and hat was 30% off. You paid a total of K23.00

What are the regular prices of the shirt and hat?

Approach: A discount of a% is equal to (100-a) % of the regular price, so we can represent the prices paid for the shirt and hat like this:

(Regular price of the shirt) × 80/100, (regular price of the hat) × 70/100

Then the quantitative relationship in the problem will look like this;

	shirt	hat	Total
Regular price kina)	Δ	□	3100
Actual cost (kina)	Δ× 80/100	Δ× 70/100	2300

Solution:

Let x kina represent the regular price of the shirt and y kina represent the regular price of the hat
x + y = 3100 80x/100 x+70/100 y=2300)□
Solving this gives us (x,y) = (1300,1800)
Regular price of shirt is 13 kina
Regular price of hat is 18 kina

Exercises

1. A vehicle travels a distance of 170km from point A to point C, passing through point B along the way. It goes 30km per hour between A and B and 70 km per hour between B and C. the total travel time is 3 hours. Find the distance between A and B as well as between B and C. (What approach and solution will you use to find the distances?)

2. The grade 7 students of a junior high school have 165 students. A total of 29 students (15% of boys and 20% of the girls) participate in volunteer activities. Find the number of boys and girls that participate in volunteer activities.

Unit Checkpoint

Review on Simultaneous Linear Equations

1. (x,y) = (5,□) is the solution to the linear equation with two unknowns x+2y=9. Fill in the □

Solving linear Equations with two unknowns

2. Which of the simultaneous equations (A) through (D) below.

Solving simultaneous equations

(A){(x+y = 6. 2x+y=10) (B){(x+3y=-2.x-y=2)

(C){(x =2. y-x=-2) (D){(x+2y=10.y =x+2)□

3. Use the addition and subtraction method to solve the simultaneous equations below.

Solving simultaneous equations with the addition and subtraction method

(a){(x+4y=16. x+y=13) (b){(5x-y=11. 3x+2y=4)

4. Use the substitution method to solve the simultaneous equation below.

Solving simultaneous equations with the substitution method

(a){(y=2x.@x+y=12)□ (b){(2x+y=6.@y=x+3)□

5. Solve the equations below

Various simultaneous equations

(a){(x+2(y-1)=3.@x-3y=0)□ (b)x+y=4x+3y=1

6. You buy a total of 10 apples and peaches for K12.00. the apples cost K1.00 each and The peaches cost K1.50 each. How many of each did you buy?

Using simultaneous equations to solve real world problems

L16 : Two functions that change together (1)

Linear function

Lesson Objective: To investigate the relationship between two quantities that change together. **(8.4.1.2)**
Materials: Blackboard

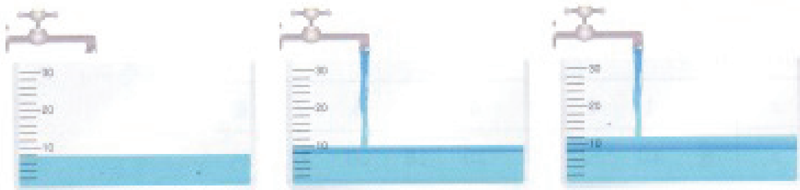
ASK-MT and Assessment	
Attitudes/Values	Show interest in finding the relationship between two functions.
Skills	Explore the relationship between two quantities that change together
Knowledge	Relationship between two quantities that change together
Mathematical Thinking	Think about to determine the functional relationship between two quantities when the value of one quantity is determined
Assessment	Use the ASK-MT to assess the students learning progress during the lesson

Teaching and Learning Activities

Review

Students fill an empty tank with water at a rate of 2cm per minute.
If the tank fills at a constant rate, the height of the water from the bottom of the tank to the surface will change as time passes.

If we let x minutes represent the quantity of time that the tank has been filling and y cm the height of the water, we can find out how the value changes.



(a) How does the value of y change as the value of x changes by 1?

(b) How does the value of y change as the value of x doubles, triples and quadruples?

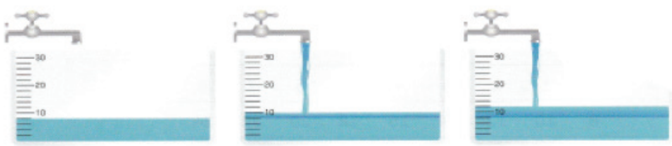
(c) Use an expression to show the relationship between x and y.

Key Ideas

A functional relationship exists between two quantities when the value of one quantity is determined, the value of the other quantity will also be fixed to one and only one value.

Exercises

A tank is filled with 8 cm of water. The students start filling it at a rate of 2 cm per minutes. If we let x minutes represent the quantity of time that the tank has been filing and y cm the height of the water, we can find out how the two values change.



L17 : Linear functions

Lesson Objective: : To identify the relationship between two quantities that change together. **(8.4.1.2)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Shares ideas on how to express function of y as a linear function of x.
Skills	Express the relationship between y and x.
Knowledge	Linear function.
Mathematical Thinking	Think about how to express the relationship between y and x .
Assessment	Use the ASK-MT to assess the students learning progress during the lesson

Teaching and Learning Activities

Introductory

From the previous lesson, the relationship between the amount of time since the students started filling the tank (x minutes) and the height from the bottom of the tank to the water surface (y cm) is shown in the table below.

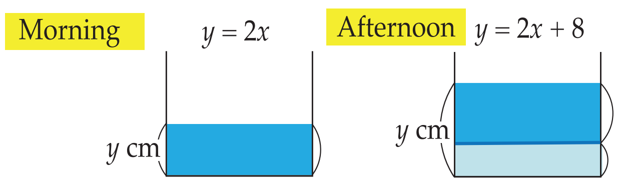
Morning

x	0	1	2	3	4	5	6	7	8
y	0	2	4	6	8	10	12	14	16

Afternoon

x	0	1	2	3	4	5	6	7	8
y	8	10	12	14	16	18	20	22	24

The table shows that the y value corresponding to each value of x is 8 more in the afternoon than it is in the morning. We can therefore express the relationship between x and y using the expression below.



Linear functions are expressed in the form: $y = ax + b$ where a and b are constants
The linear function $y = ax + b$ indicates the sum of a portion that is proportional to x (ax) and a constant (b). When $b = 0$, the function becomes a proportional relationship in the form $y = ax$.

Key Ideas

When y is a function of x that can be represented as a linear expression like $y = 2x + 8$, we say that y is a **linear function** of x
This linear function can also be written as $f(x) = 2x + 8$
A proportion is a particular kind of linear function.

Exercises

The expressions below indicate that y is a function of x. Which are linear functions? For all linear functions shown, state the portion that is proportional to x and the constant.

(a) $y = 8x - 1$ (b) $y = 4/x$ (c) $y = 1/3\ x$ (d) $y = 5 - 7x$

L18: Changing values in linear functions

Lesson Objective : To determine how y values change as x values change in linear functions. **(8.4.1.2)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Appreciate calculating rate of change.
Skills	Apply and calculate rate of change in a linear function.
Knowledge	Rate of change in a linear function.
Mathematical Thinking	Think about how to determine the rate of change between two quantities in a linear function relationship.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson

Teaching and Learning Activities

Introduction

The corresponding y and x values in the linear function y = $2x$ + 1 are shown below.

(Increases in x marked above the table: 3, 1, 3; increases in y below the table: □, □, □)

x	…	-3	-2	-1	0	1	2	3	4	…
y	…	-5	-3	-1	1	3	5	7	9	…

Figure out the quantity of increase in y as the x value changes and fill in the □.
What do you notice?
When the x value goes from 1 to 4 in $y = 2x + 1$,
The increase in x is 4 - 1 = 3, and the increase in y is 9 - 3 = 6
The increase in y is therefore twice the increase in x.

(Increase in x: 3; increase in y: 6)

x	1	4
y	3	9

$\frac{6}{3} = 2$

In the linear function $y = 2x + 1$, the rate of change is always 2. This is true not only when the x value goes from 1 to 4 or 5 to 9, but also in any case where there is an increase. A rate of change of 2 indicates the increase in y when the increase is in x is 1.

Example.1 **Rate of change in linear function**
Complete the table below to find the rate of change for the linear function y= -2 x + 7

x	…	−3	−2	−1	0	1	2	3	4	…
y	…									…

(a) Figure out the increase in y when the value of x changes from 1 to 4. Then find the rate of change.
(b) Pick 2 values for □ and ❍ when the value of x changes from □ and□. Figure out the increase in y, then find the rate of change.
(c) Find the increase in y when the increase in x is 1.

We have shown that the increase in y is a when the increase in x is 1. We can therefore say the following about $y = ax + b$

When a > 0, the value of y increases as the value of x increases.
When a< 0, the value of y decreases as the value x increases.

Teaching and Learning Activities

Example.2 **The rate of change for the inversely proportional relationship y=6/x**

When the value of x changes from 1 to 2,

x	1	2
y	6	3

Rate of change = $\frac{2 - 6}{3 - 1}$ =-3

When the value of x changes from 2 to 3,

x	2	3
y	3	2

Rate of change = $\frac{2 - 3}{3 - 2}$ =-1

Inversely proportional rates of changes are not fixed

Key Ideas

The ratio of the quantity of increase in y to the quantity of increase in x is called the **rate of change**

Rate of change = $\frac{\text{increase in } y}{\text{increase in } x}$

For any linear function $y = ax + b$, rate of change is a fixed quantity equal to a.

Rate of change = $\frac{\text{increase in } y}{\text{increase in } x} = a$

Exercises

1. When the x value goes from 5 to 9 in the linear function $y = 2x + 1$, how many times the increase in x is the increase in y?

2. For the linear function $y = 2/3\ x + 5$, find the increase in y under the following conditions;

   (a) When the increase in x is 1
   (b) When the increase in x is

3. For the inversely proportional relationship y = 6/x, find the rate of change as the value of x changes from -3 to -1. Compare the result with *Example.2*

L19: Characteristics of linear function graphs

Graphing linear functions

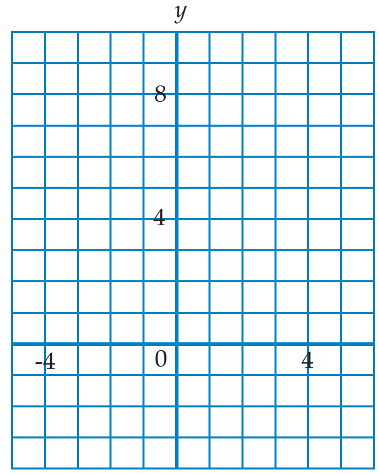
Lesson Objective: To identify the characteristics of linear function graphs by drawing. **(8.4.1.2)**
Materials: blackboard, graph paper

ASK-MT and Assessment	
Attitudes/Values	Appreciate the advantages of expressing linear function as a graph.
Skills	Express linear function as a graph.
Knowledge	Characteristics of linear function graphs.
Mathematical Thinking	Think about how to relate graph of proportion to that of a linear function graph.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introduction

Think about how to express the linear function $y = 2x + 3$ as a graph.



(a) Complete the table for the linear function $y = 2x + 3$

x	…	-3	-2	-1	0	1	2	3	4	…
y	…									…

(b) Plot the points having the coordinates for the corresponding pairs of x and y values from the table in the figure.

(c) Draw the graph of the proportional relationship $y = 2x$

(d) What can you say about the graph of the linear function $y = 2x + 3$?

TN: We can expect the graph of the linear function $y = 2x + 3$ to be a line parallel to the graph of the proportional relationship $y = 2x$.
We can check if this is true based on the graph of proportional relationship $y = 2x$.

If we compare

$y = 2x$................ ①

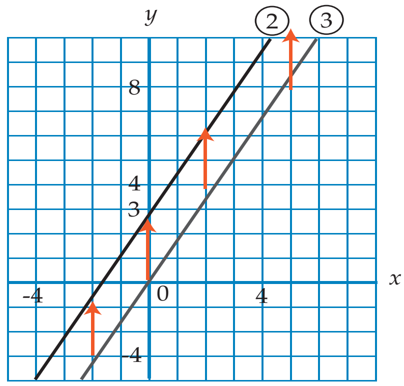
$y = 2x + 3$..........②

we get the corresponding values below.

x	…	-3	-2	-1	0	1	2	3	…
① $2x$	…	-6	-4	-2	0	2	4	6	…
② $2x + 3$	…	-3	-1	1	3	5	7	9	…

Teaching and Learning Activities

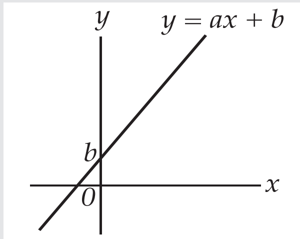
Example.1 **characteristics of linear function graphs**



If we compare ① and ②, we see that the y value corresponding to each value of x is always 3 more in ② than ①.

If we think about this graphically, it means that the graph of ② is a parallel line 3 above the graph of ①.
The graph of the linear function $y = 2x + 3$ is therefore a straight line that is parallel to the graph of the proportional relationship $y = 2x$ and passes through the y axis at the point (0,3).

Key Ideas



The graph of $y = 2x + 3$ is called the line $y = 2x + 3$
The graph of the linear function $y = ax + b$ is the line which is parallel to the line $y = ax$ and passes through the y axis at the point (0,b).
The y coordinate b from the point (0,b) where the line $y = ax + b$ Intersects the y axis is called the y- intercept of the line.

The figure on the right shows the graphs of $y = 2x$ and $y = -2x$. Use this information to graph the linear functions below in the same figure.

(a) $y = 2x - 2$ (b) $y = -2x + 4$ (c) $y = -2x - 3$

2. State the intercept of line $y = 3x + 5$

L20: Line Slopes/ Gradients

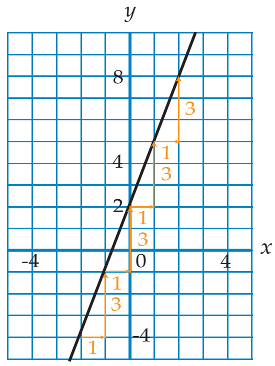
Lesson Objective : To investigate and identify the properties of line slopes in linear function graphs. **(8.3.1.2/4)**

Materials: Blackboard, graph paper

ASK-MT and Assessment	
Attitudes/Values	Share ideas on line slopes on a linear function graph.
Skills	Compare and explain line slopes in a linear function graph.
Knowledge	Properties of a line slopes and their graphs
Mathematical Thinking	Think about how to examine the constant in a linear expression as the slope in a linear function graph.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson

Teaching and Learning Activities

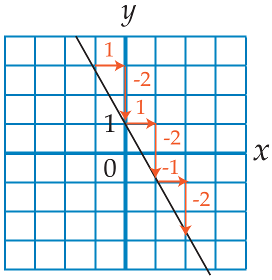
Review



We can look at the slope of 3 in the line $y = 3x + 2$ in the following way $y = ax + b$.

In the figure on the right, the line $y = 3x + 2$ rises 3 every time it goes1 to the right. The line slopes up to the right, and the slope of 3 indicates that y increases 3 for every 1 increase in x.

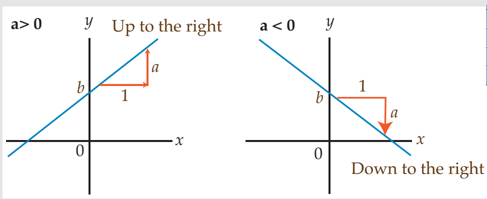
Example.1 **A line with a slope of -2**



The slope of the line $y = -2x + 1$ is -2. This line rises -2 every time it goes 1 to the right. In other words, it drops by 2. The line slopes down to the right.

Key Ideas

- The value of a is called the slope of the line $y = ax + b$
- The graph of the linear function $y = ax + b$ is a line with a slope a and intercept b
- The value of a tells us the following.



- Graphically, we express “rate of change a in the linear function $y = ax + b$” as “slope a in the line $y = ax + b$”

Exercises

1. State the slope and intercept of the lines below.

(a) $y = 3x - 2$ (b) $y = 3x + 6$ (c) $y = \frac{4}{5}x - 1$ (d) $y = \frac{3}{2}x + 1$

L21: How to draw graphs of linear functions

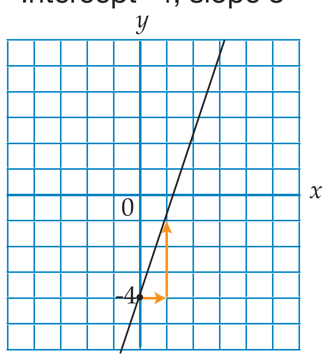
Lesson Objective: To draw linear function graphs. **(8.3.1.2/4)**
Materials: Graph paper, blackboard

ASK-MT and Assessment	
Attitudes/Values	Share ideas in drawing a linear function graph.
Skills	Plot coordinates and draws graphs of linear function.
Knowledge	Graphing a linear function.
Mathematical Thinking	Think about how to graph a linear function.
Assessment	Use the ask-mt to assess the students learning progress during the lesson.

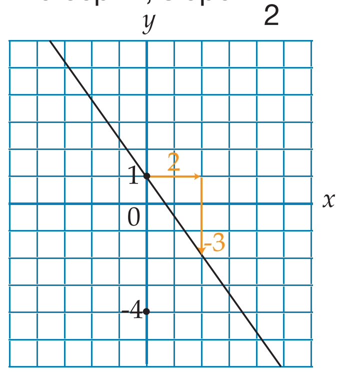
Teaching and Learning Activities

Example.1 **Graphing a linear function**

(1) Graph of $y = 3x - 4$
Intercept -4, slope 3



(2) Graph of $y = -\frac{3}{2}x + 1$
Intercept 1, slope $-\frac{3}{2}$



TN: You can graph the linear function $y = ax + b$ by first determining the intercept b at the y - axis and then drawing a line that has a slope a

Exercises

Graph the linear functions below;

$y = x - 3$
$y = -4x + 1$
$y = \frac{2}{3}x - 3$
$y = \frac{1}{3}x + 2$

L22: Finding expression using graph of linear function

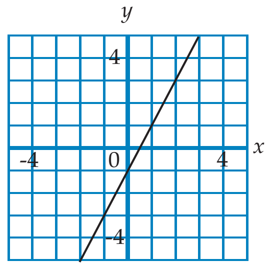
Finding expressions for linear functions

Lesson Objective : To use linear function graphs to determine the expression. **(8.3.1.2/4)**
Materials: Grid papers, blackboard

ASK-MT and Assessment	
Attitudes/Values	Show interest in finding the expressions of a linear function
Skills	Use the graph of linear function to identify the expression
Knowledge	Expression of linear function using slope and interception
Mathematical Thinking	Think about how to determine the expression of a linear function using the graph
Assessment	Use the ask-mt to assess the students learning progress during the lesson

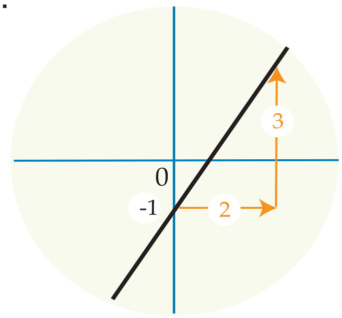
Teaching and Learning Activities

Example.1 **Expression using slope and intercept**



Graphs of functions are drawn by determining the intercept and slope from the expression. The figure on the right shows a graph of a linear function. How can we find the expression by looking at the graph?

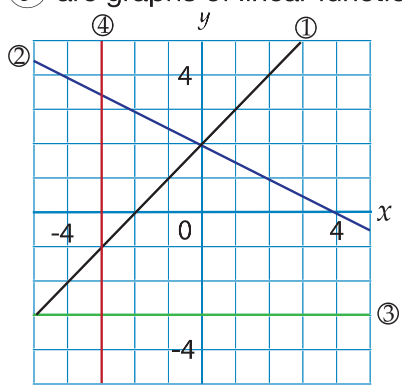
The line(above graph) passes through the point (0,-1), so the intercept is -1.
Since the line rises 3 every time it goes 2 to the right, the slope is $\frac{3}{2}$.
Therefore, the line is a graph of the linear function
$y = \quad x - 1$.
If you can read the slope a and intercept b from the graph of a linear function, You can find the expression $y = ax + b$.



TN: if we can figured out the slope, intercept, and/or a point along a graph of a linear function, we can found the expression for the function.

Exercises

Lines showing slopes ①, ② and ③ are graphs of linear functions. Find the expression for each.



L23: Finding expression using the slope and coordinates of a point.

Lesson Objective: To use the slope and coordinates of a point on a graph to find the expression for linear functions. **(8.3.1.2/4)**

Materials: Graphs papers, blackboards

ASK-MT and Assessment	
Attitudes/Values	Share their ideas on how to find the expression of a linear function using the slope and the coordinates of a point
Skills	Identify the expression of a linear function using the slope and the coordinates of a point
Knowledge	Expression of linear function using slope and coordinates of a point
Mathematical Thinking	Think about how to determine the expression of linear function using slope and coordinates of a point
Assessment	Use the ask-mt to assess the students learning progress during the lesson

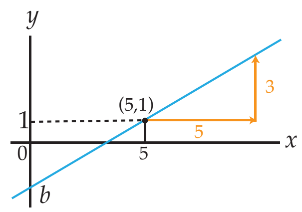
Teaching and Learning Activities

Example.1: **Expression using the slope and coordinates of a point**

y is a linear function of x. The graph of the function is a line that passes through point (5,1) and has a slope of $\frac{3}{5}$. Find the expression for this linear function.

Approach: The slope is $\frac{3}{5}$, so we know the expression for linear function is in the form $y = \frac{3}{5}x + b$.

We can find the value of b using the fact that the line passes through the point (5,1).



Solution:

Since the slope is $\frac{3}{5}$, we know the expression for a linear function is

$y = \frac{3}{5}x + b$...............①

Since this line passes through point (5,1), substitute $x = 5$ and $y = 1$ in ①

$1 = \frac{3}{5}x \times 5 + b$

so $b = -2$

We get the expression $y = \frac{3}{5}x - 2$

Exercises

y is a linear function of x. The graph of the function is a line that passes through point (1,2) and has a slope of -3 . Find the expression for this linear function.

L24: Finding expression using the coordinates of two points on a graph

Lesson Objective : To use the coordinates of two points on a graph to find the expression of a linear function. **(8.3.1.2/4)**

Materials: Graphs papers, blackboard

ASK-MT and Assessment	
Attitudes/Values	Share their ideas on how to find the expression of a linear function using the coordinates of two points.
Skills	Find the linear expression of a linear function graph using the coordinates of two points.
Knowledge	Expression of linear function using the coordinate of two points on a graph.
Mathematical Thinking	Think about how to determine the expression of linear function using the coordinate of two points on a graph.
Assessment	Use the ask-mt to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1: **Expression using the coordinates of two points on a graph**

y is a linear function of x. The graph of the function is a line that passes through point (1,2) and point (5,-6). Find the expression for this linear function.

Approach: To find the expression for linear function $y = ax + b$, find slope a from the coordinates of two points that the line passes. Find intercept b using the same method from the previous lesson.

Solution:

Find the expression for linear function $y = ax + b$.

The graph passes through two points, (1,2) and (5,6).

Therefore, slope a is

a = (-6)-2)/(5-1)=(-8)/4=-2

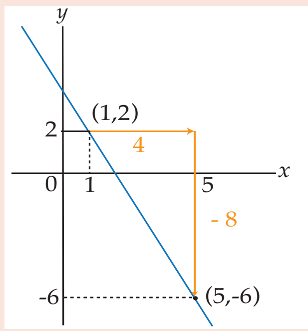
This gives you y = 2 x 1 + b.

Since the graph passes through point (1,2),

2 = -2 x 1 + b

so b = 4.

We get the expression y = -2x + 4



Example.2: **Determining a and b** for $y = ax + b$ using simultaneous equations

Solve the simultaneous equations to find the expression for linear function

y = 2 when x =1, so 2 = a + b.............①

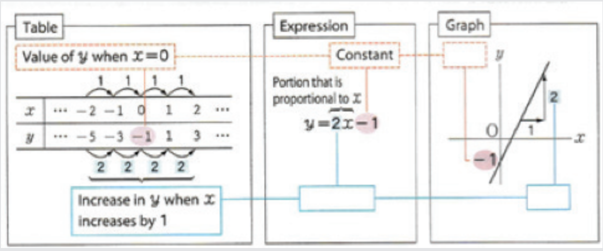
y =-6 when x = 5, so -6 = 5a + b.........②

Set up ① and ② as simultaneous equations and solve for a and b

Teaching and Learning Activities

Key Ideas

Relationship between the table, expression, and graph of linear function



1. y is a linear function of x. The graph of the function is a line that passes through point (-1,-4) and has a point of 3,8. Find the expression for this linear function.
2. y is a linear function of x. y = -1 when x = -2 and y = 8 when x = 4. Find the expression for linear function.

L25: Equations and graphs

Linear functions and equations

Lesson Objective: To draw graphs and find solution to linear equations with two unknown $ax + by = c$. **(8.3.1.2/4)**

Materials: Graph papers, blackboard

ASK-MT and Assessment	
Attitudes/Values	Enjoy drawing a linear function graph using the solution of two unknowns.
Skills	Draw graphs of linear function using the solutions of two unknowns in a linear expression as points.
Knowledge	Graphing the solution of linear equations for function $ax + by = c$
Mathematical Thinking	Think about how to determine graphing the solution of linear equations for function $ax + by = c$
Assessment	Use the ask-mt to assess the students learning progress during the lesson

Teaching and Learning Activities

Introduction

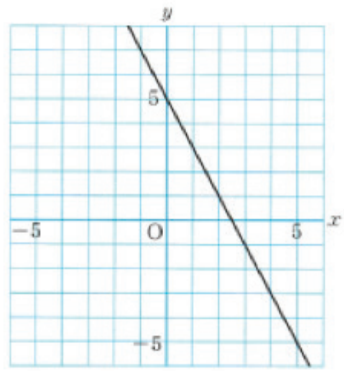
If we solve for y in the linear equation with two unknowns

$2x + y = 5$............... ①

we get $y = -2x + 5$......... ②

where y is a linear function of x.

① and ② express the same relationship, so all of the points that take solutions of ① as coordinates should be in a line that is identical to the graph of linear function ②.

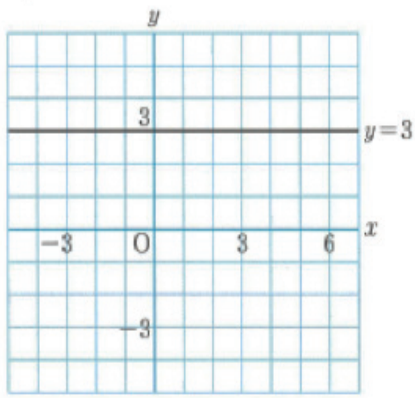


TN: This line is called the graph of the equation $2x + y = 5$. $2x + y = 5$ is also called the expression of the line. We can draw the graph of the linear equations with two unknowns $ax + by = c$ by solving for y and then using the slope and intercept.

Example. 1 **Graphing by finding two points on the line.**

Graph of $2x - 3y = 12$
Since $y = -4$ when $x = 0$
$x = 6$ when $y = 0$,

The graph is a line that passes through two points (0,-4) and (6,0).



Example.2: **Graph of the equation ax + by = c when a = 0.**

Think about the graph of the equation $ax + by = c$ when $a = 0$.

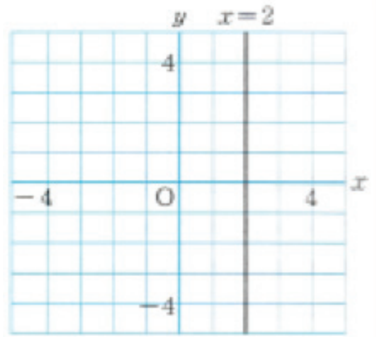
For example, when $a = 0$, $b = 1$ and $c = 3$, the equation becomes $y = 3$
In this case, solutions to the equation include (-1,3),(0,3),(1,3) and so on.
This means that $y = 3$ no matter what the value of x.
Therefore, the graph of $y = 3$ is a line that passes through the point (0,3) and is parallel to the x axis.

Teaching and Learning Activities

Example.3: **Graph of the equation ax +by = c when b = 0**

Think about what happens to the graph of the equation $ax + by$ = c when b =0?
When a = 1,b = 0, and c = 2, in the equation $ax + by = c$, it becomes $x = 2$

What is the graph of this equation?
The solutions to the equation $x = 2$ include (2,-1),(2,0),(2,1) and so on.
This means that $x = 2$ no matter what the value of y.
Therefore, the graph of $x = 2$ is a line that passes through the point (2,0) and is parallel to the y axis.



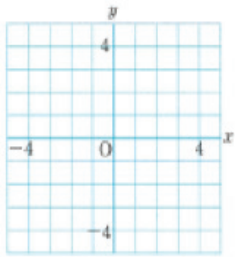
Key Ideas

- The graph of the linear equation with two unknowns ax + by = c is a line.
  More specifically,
  The graph of y = k is a line parallel to the x axis.
  The graph x = h is a line parallel to the y axis.

1. Solve for y in the linear equations with two unknowns below. Then draw their graphs in the figure above.
(a) x- 2y = 6 (b) 4x + 3y = 0
2. Graph the equations below.
(a) x- y = 5 (b) x + 4y = -2
3. Graph the equations below.
(a) y = 2
(b) 2y = -6

4. Graph the following equations in the figure in example .3

(a) x =-2 (b) 3x = 12



L26: Solutions to simultaneous equations and graphs

Lesson Objective : To explain the relationship between the intersection of two lines and solutions to simultaneous equations. **(8.3.1.2/4)**

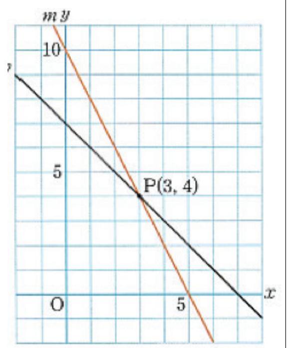
Materials: Graphs papers, blackboard

ASK-MT and Assessment	
Attitudes/Values	Appreciate to know the relationship between the intersection of lines and solution to simultaneous equations.
Skills	Solve two linear equations by graphing.
Knowledge	Relationship between the intersection of two lines and their solutions on the graph.
Mathematical Thinking	Think about how to determine coordinates of intersection as the solution to linear functions.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1: **Relationship between the intersection of two lines**

Graph the two equations $x + y = 7$, $2x + y = 10$
and find the coordinates of the point where the two lines intersect.
Treat the two equations as simultaneous equations and solve. What do you notice?



If we graph the two equations

$x + y = 7$..........①

$2x + y = 10$.........②

We get lines l and m in the figure on the right.
The intersection of l and m at P(3,4) is a point along both l and m.This means that $(x,y) = (3,4)$ satisfies both ①and ② .Therefore, coordinates (3,4) intersection P are the solution to simultaneous equations ① and ②. Coordinates of the intersection between two lines can be found by solving simultaneous equations that includes the expressions for those lines.

Example.2: **Finding the coordinates of the point where two lines intersect**

Look at the figure on the right. Find the coordinates of point P where lines l and m intersect

Approach: Find the expression for the two lines. Set them up as simultaneous equations and solve.

Solution:

The expressions for lines l and m are

$y = -2x + 3$...............①

$y = x + 1$..................②

set up ①and ②as simultaneous equations and solve to get $(x,y) = \left(\frac{2}{3}, \frac{5}{3}\right)$

Therefore, $P = \left(\frac{2}{3}, \frac{5}{3}\right)$

Teaching and Learning Activities

Example.1: **Relationship between the intersection of two lines**

Graph the two equations $x + y = 7$, $2x + y = 10$
and find the coordinates of the point where the two lines intersect. Treat the two equations as simultaneous equations and solve. What do you notice?

If we graph the two equations
$x + y = 7$ ……..□
$2x + y = 10$ ……□

We get lines l and m in the figure on the right.
The intersection of l and m at P(3,4) is a point along both l and m.This means that (x,y)=(3,4) satisfies both □ and □.Therefore, coordinates (3,4) intersection P are the solution to simultaneous equations □ and □. Coordinates of the intersection between two lines can be found by solving simultaneous equations that includes the expressions for those lines.

Example.2: **Finding the coordinates of the point where two lines intersect**

Look at the figure on the right. Find the coordinates of point P where lines l and m intersect

Approach: Find the expression for the two lines. Set them up as simultaneous equations and solve.

Solution:

The expressions for lines l and m are
y = $-2x$ +3 ……..□
y = x + 1 ……..............□
set up □ and □ as simultaneous equations and solve to get
(x,y) = (2/3,5/3)
Therefore, P=(2/3,5/3)

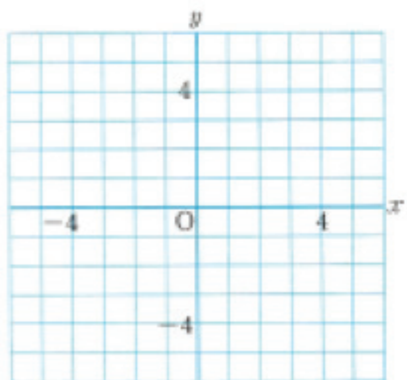
Key Ideas

Solutions to the simultaneous equations and graphs
{(ax+by=c…….□@a^' x+b^' y=c^'…..□)□
is the same as the coordinates of the point where lines □ and □

Exercises

1. Use a graph to solve the simultaneous equations below.
{(x+2y=2@2x+y=-2)□
Use calculations to find the solution and see if it matches

2. Look at the figure on the right. Find the coordinates of point P where lines l and m intersect.



L27 : Use linear functions to solve problems (1)

Using linear function

Lesson Objective: Apply linear functions to solve real life situtaions. **(8.3.1.2/4)**
Materials: Graphs papers, blackboard

ASK-MT and Assessment	
Attitudes/Values	Enjoy solving real life problems using linear function.
Skills	Solve real life problems using linear function.
Knowledge	Solving real life problems using linear function.
Mathematical Thinking	Think about how to solve real life problems using linear function.
Assessment	Use the ASK-MT to asse ss the students learning progress during the lesson.

Teaching and Learning Activities

Review

x	0	1	2	3	4	5
y	20.0	25.8	32.8	39.2	46.0	52.2

Relationship of number values collected during an experiment can sometimes be viewed as a linear function From the experiment "heating water to find the relationship between heating time and temperature", the corresponding points were nearly on a straight line. This means we can consider the relationship between y and x to be a linear function.

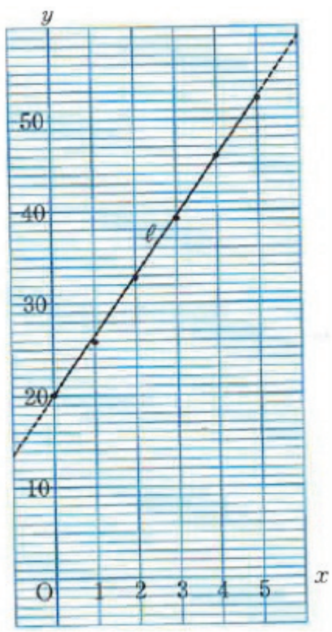
If we draw a line l that passes as close to these points as possible, we get the figure on the right.

If we find an equation by assuming that this line l passes through two points (0,20) and (4,46), we get

$$y = 6.5x + 20 \ (0 \leq x \leq 5)$$

If we assume in the experiment that the water temperature will continue to change in the same way once it goes beyond 5 minutes of heating time, we can use the expression above to predict the temperature after 6 minutes, like this,

$$y = 6.5 \times 6 + 20 = 95(C°)$$



Example.1 **Time after departure and travel distance to the destination**
Mori leaves her house in West town and travels to her uncle's house in East Town. On the way, she stops at a store.

Example.1 **Time after departure and travel distance to the destination**
Mori leaves her house in West town and travels to her uncle's house in East Town. On the way, she stops at a store.

Teaching and Learning Activities

Solution:

(a) The part of the graph where the value of y is fixed indicates the time when Mori was at the store.
The value of y is 3 here, so the answer is 3km

(b) The slope of the graph before Mori arrives at the store is steeper than the slope of the graph after she leaves.
She traveled faster before she got to the store

(c) 18 minutes after she left her house, Mori had still not reached the store. The graph is a line with a slope of $-\frac{1}{10}$ and an intercept of 5. The expression that indicates the relationship between x and y is,

$$y = -\frac{1}{10}x + 5 (0 \leq x \leq 20)$$

$$y = -\frac{1}{10} \times 18 + 5 = \frac{16}{5}$$

Ans: $\frac{16}{5}$ km

Exercises

1. Think about the temperature of the water after 10 minutes in the experiment in the review above.

   How many minutes will it take from the time the students start heating the water until it reaches a temperature of 72°C

2. How many km was it to Mori's uncle's house 50 minutes after Mori left her house?

Unit Checkpoint

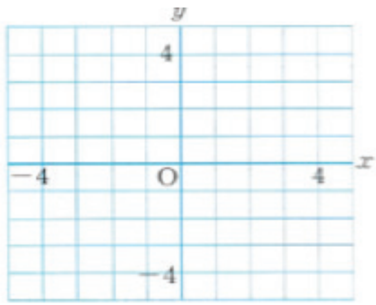
Review on linear functions

1. In which of the following is y a linear function of x?

   a) The remaining quantity of milk is (y mL) after drinking x mL out of 500mL total.
   b) The length (y cm) of a rectangle with an area of 30 cm² and a width of x cm.
   c) The perimeter (y cm) of an equilateral triangle with x cm sides

2. Answer the following questions about the linear function
   $y = -2x + 5$

   a) Find the increase in y when the increase is 1.
   b) Find the increase in y when the increase is 3.



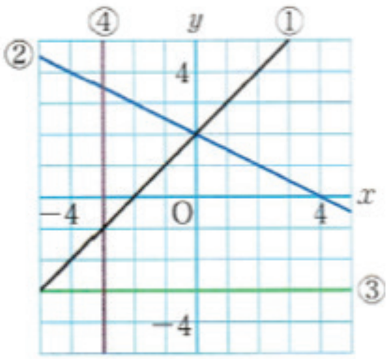
3. Graph the linear functions below.

   a) $y = 2x - 1$
   b) $y = -x + 4$

4. Find the expressions for the linear functions below
   a) The graph is a line that has a slope of 3 and intercepts 4.
   b) The rate of change is -2 and $y = 2$ when $x = 1$.
   c) the graph is a line that passes through points (1,1) and (2, 3).

5. Look at the graph on the right. Select the number of the line the express each equation below.

   a) $x + 2y = 4$
   b) $x - y = 2$
   c) $y =-3$



L28: Exploring lines that meet at intersections

Properties of angles and parallel lines

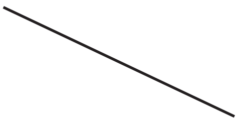
Lesson Objective : To draw lines that meet at an intersection to form vertical angles that are equal in size and calculate the missing angles. **(8.2.1.1/2)**

Materials: ruler and protractor

ASK-MT and Assessment	
Attitudes/Values	Work collaboratively to draw lines of intersection.
Skills	Draw lines of intersection, measure the angles and explain the properties of lines that meet at intersection.
Knowledge	Lines that meet at intersection.
Mathematical Thinking	Thinking about ways of measuring the angles that meet at the point intersection.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

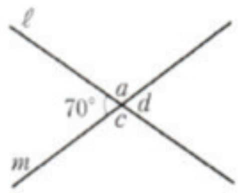
Teaching and Learning Activities

Introduction



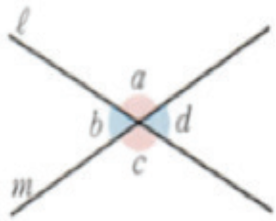
Draw a line intersecting the line on the left and measure the 4 angles that are formed as a result. What do you notice?

Example.1 **Vertical angles**



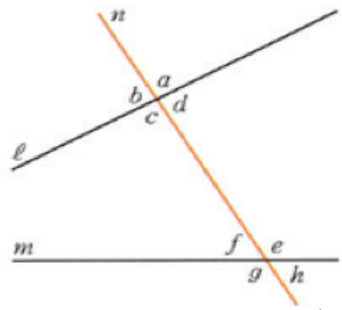
When $\angle b = 70°$, $\angle a$ and $\angle c$ are both 180°-70°, then we know that $\angle a = \angle c$
This relationship can also be expressed as;
$\angle a = 180° - \angle b$, $\angle c = 180° - \angle b$,
And this holds true no matter what the measure of ∠ b can be.

The angles opposite each other $\angle a$ and $\angle c$ are called **vertical angles**, $\angle b$ and $\angle d$ are also called **vertical angles.**



Example.2 **Corresponding angles**

In the figure like the one on the right, where two lines l and m intersect line n any two angles in positions like those in $\angle a$ and $\angle e$ are called **corresponding angles**
Pairs of corresponding angles are $\angle b$ and $\angle f$: $\angle c$ and $\angle g$: $\angle d$ and $\angle k$



Example.3 **Alternate angles**

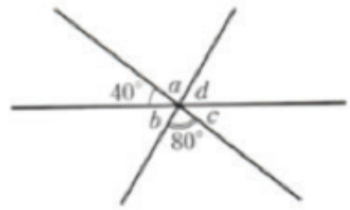
Any two angles in the positions of $\angle c$ and $\angle e$ these are called **alternate interior angles** $\angle d$ and $\angle f$ ar

Key Ideas

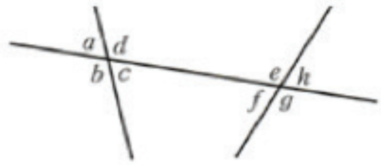
- When two lines intersect as on the figure on theright, four angles are formed around the point of intersection.
- The vertical angels are always equal.
  alternate interior angles and corresponding angles are all equal.

Teaching and Learning Activities

1. The figure on the right shows three lines intersecting at a single point. Find the measure of $\angle a$, $\angle b$, $\angle c$ and $\angle d$



2. State the corresponding angle to $\angle a$ in the figure on the right. state the alternate interior angle to $\angle e$.



L29 : Parallel lines and corresponding and alternate interior angles

Lesson Objective: To identify and explain the relationship of angles in the parallel lines with an intersection line that forms corresponding and alternate interior angles. **(8.2.1.1/2)**

Materials: set square and ruler

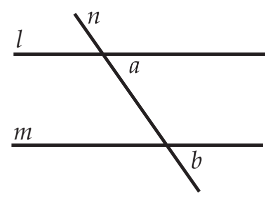
ASK-MT and Assessment	
Attitudes/Values	Show confident to explain the relationship of parallel lines with an intersection line that forms the corresponding and alternate interior angles.
Skills	Explain the relationship parallel lines with a n intersection line that forms the corresponding and alternate interior angles.
Knowledge	Relationship of parallel lines and corresponding or alternate interior angles.
Mathematical Thinking	Think about how to explain the relationship of parallel lines with an intersection line that forms the corresponding and alternate interior angles.
Assessment	Use the ask-mt to assess the students learning progress during the lesson.

Teaching and Learning Activities

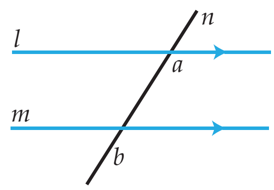
Review

Use a set square to drew a line parallel to line l

Example.1 **Parallel lines and corresponding angles**

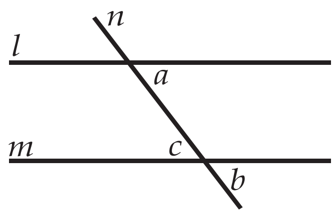


When we use the set square to measure the corresponding angles $\angle a$ and $\angle b$ on the figure on the right the lines $l \mathbin{//} m$ are also equal in distance. In other words,
If $\angle a = \angle b$, then $l \mathbin{//} m$



In addition, no matter how a line intersecting the lines l and m the corresponding angels $\angle a$ and $\angle b$ are always equal.
If $l \mathbin{//} m$, then $\angle a = \angle b$

Example.2 **Parallel lines and alternate interior angles**
Use the relationship between the parallel lines and corresponding angles to figure out the relationship between the parallel lines and alternate interior angles.



When line n intersects the lines $l \mathbin{//} m$, what is the measure of corresponding angle to $\angle$ b in line m on the figure on the right.
What can you say about $\angle$ a, $\angle$ c and $\angle$ d ?

Key Ideas

We can summarize what we've learned so for like this. Properties of parallel lines
When a line intersects two others lines, it is true that

- Corresponding angles are equal if the two lines are parallel
- Alternate interior angles are equal if the two lines are parallel

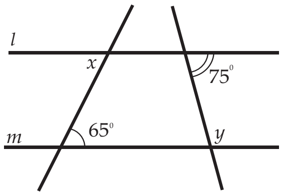
Conditions for parallel lines

When a line intersects two others lines, it is true that

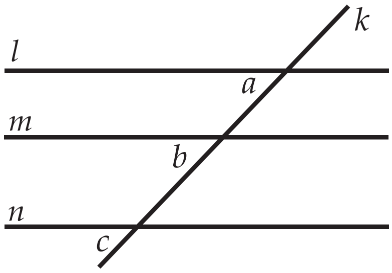
- The two lines are parallel if two corresponding angles are equal
- The two lines are parallel if two alternate interior angles are equal

Teaching and Learning Activities

1. Find the angles of $\angle x$ and $\angle y$ when lines $l \parallel m$ in the figure on the right.



2. For the figure below, use angle relationship to explain why $l \parallel n$,when $l \parallel m$, and $m \parallel n$



L30: Interior and exterior angles in triangles

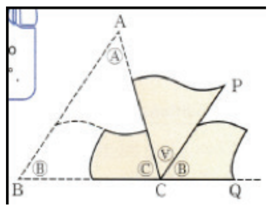
Lesson Objective : To explain the relationship of interior and exterior angles in triangles and how the angles are measured. **(8.2.1.3)**

Materials: ruler, protractor and set square

ASK-MT and Assessment	
Attitudes/Values	Enjoy learning the properties about interior and exterior angles in triangles.
Skills	Explain the angle relationship in the triangle for interior and exterior angles and measure their angles.
Knowledge	Relationship between interior and exterior angles in triangles.
Mathematical Thinking	Think of how to explain the angle relationship of interior and exterior angles in a triangle and measure their angles.
Assessment	Use the ask-mt to assess the students learning progress during the lesson.

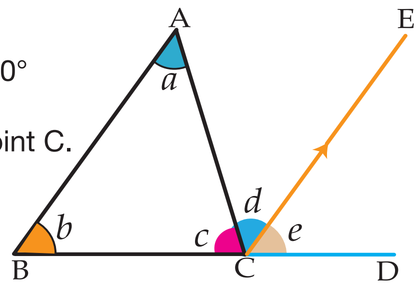
Teaching and Learning Activities

Review



In previous grade you used figures like the one on the right to learn that the sum of the three angles in a triangles is always 180°.

Example.1: **Interior and exterior angles in triangles**



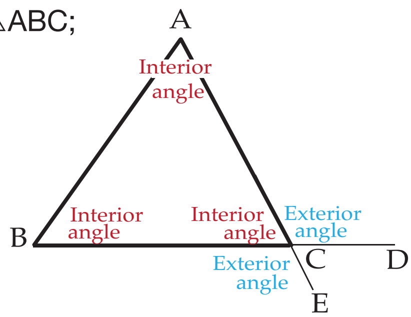
What are the positional lines BA and CP in the figure on the right?
Use this information to make sure that the three angles add up to 180°
In the next figure on the right, D is on the line extending the line BC of angle △BAC. Line CE is drawn parallel to side BA and through Point C.

In this case;

The alternate angles of parallel lines are equal. So $\angle a = \angle d$ (1)
The corresponding angles of parallel lines are equal. So $\angle b = \angle e$ (2)
Knowing (1) and (2) allows us to find the sum of the three angles of △ABC;

$$\angle a + \angle b + \angle c = \angle d + \angle e + \angle e$$
$$= \angle BCD$$

The three points B, C, D are on the same line so $\angle BCD = 180°$.
This means that the sum of the three angles of a triangle is 180°.

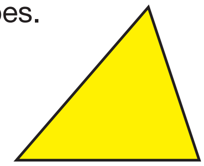


D is on the line formed by extending side BC of △ ABC.
In this case □ACD is known as **exterior angle** at vertex C.

If another angle ∠BCE is formed from extending AC, this is also an exterior angle at vertex C
We can also think about the exterior angles at vertices A and B in the same way.
On the other hand, ∠A, ∠B and ∠C in ΔABC are all **interior angles.**
TN: If we look at the interior angles of triangles, we can categorize them into the following three types.

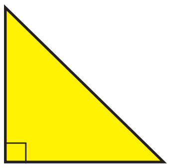
Teaching and Learning Activities

TN: If we look at the interior angles of triangles, we can categorize them into the following three types.



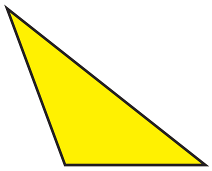
Acute triangle

A triangle in which all three interior angles are acute angles



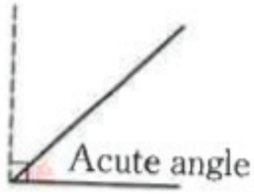
Right triangle

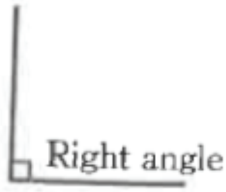
A triangle in which all one interior angle is a right angle

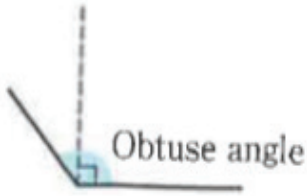


Obtuse triangle

A triangle in which all one interior angle is an obtuse angle

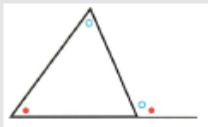






Key Ideas

Properties of interior and exterior angles



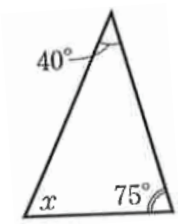
- The sum of the three interior angles of a triangle is 180°
- The measure of an exterior angle of a triangle is equal to the sum of the two non- adjacent interior.

- Angles larger than 0° and smaller than 90° are called **acute angles**
- Angles larger than 90° and smaller than 180° are called **obtuse angles**

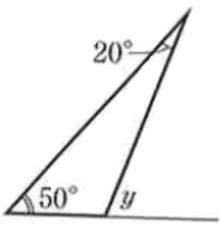
Exercises

Find the angles of $\angle x$,$\angle y$ and $\angle z$ in the figures below.

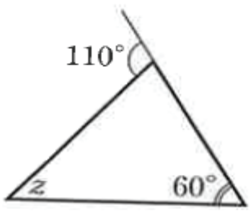
(a)



(b)



(c)



L31: Sums of interior angles in polygons

Lesson Objective: To calculate the sum of interior angles in polygons using the angle sum of triangles in a given polygon and write an expression. **(8.2.1.3)**

Materials: ruler and protractor

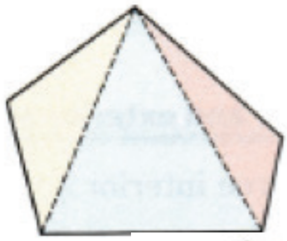
ASK-MT and Assessment	
Attitudes/Values	Show interest in finding out the sum of interior angles in polygons.
Skills	Calculate the angle sum of interior angles for n sided polygons.
Knowledge	Sums of interior angles in polygons.
Mathematical Thinking	Think about how to calculate the angle of interior angles of polygons.
Assessment	Use the ask-mt to assess the students learning progress during the lesson.

Teaching and Learning Activities

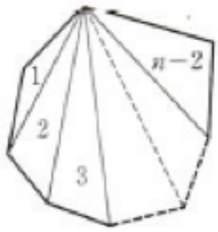
Example.1 **Sums of interior angles in polygons**

What are the sum angles of quadrilaterals, pentagons and hexagons?

If we draw diagonals from a single vertex in polygons, like quadrilaterals and pentagons we can divide them into several triangles.



If we draw diagonals from a single vertex of an n sided polygon we can divide the polygon into (n - 2) triangles. This means that the sum of interior angles of n sided polygon can be expressed as; 180°× (n –2)



Key Ideas

Sum of interior angles of a polygons
The sum of the interior angles of an n-sided polygon is 180°× (n – 2).

1. The table below will help you to find the sums of interior angles by dividing them into triangles Draw diagonals from a single vertex in a polygon and fill in □ above.

2. Find the sum of the interior angles of a decagon (10-sided polygon) .What is the measure of each interior angle in a regular decagon?
3. You have a polygon whose interior angles add up to the following. What kind of polygon is this?

   (a) 900°   (b) 1800°

L32: Sums of exterior angles of a polygon

Lesson Objective : To calculate and write an expression for the sum of exterior angles in polygons using the angle sum of triangles in a given polygon.**(8.2.1.3)**

Materials: ruler and protractor

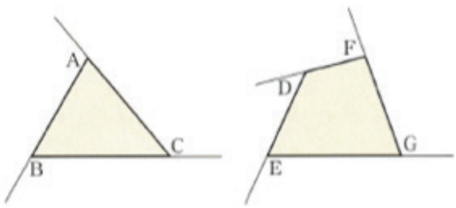
ASK-MT and Assessment	
Attitudes/Values	Enjoy calculating the sum of exterior angles in polygons.
Skills	Explain the sums of exterior angles in polygons and use ideas to calculate the sum angles using expressions.
Knowledge	Sums of exterior angles in a polygon.
Mathematical Thinking	Think of how to explain and calculate the sums of exterior angles in polygons.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example: **Sums of exterior angles in a polygon**

Now let's look at the sum of exterior angles of a polygon. This means adding up one exterior angle for each vertex.

What is the sum of exterior angles in a triangle and quadrilateral below?
Draw various polygons in your note books and find the sum of their exterior angles. What do you notice?



Based on what we find out in the above sum of exterior angles of the polygons,
it seems that the sum of any exterior angle add up to 360°
What would be the sum of exterior angles of a pentagon?
For example when we look at the sum of exterior angles for a pentagon,
The sum of interior and exterior angles is $180° \times 5 = 900°$
The sum of interior angles is $180° \times (5 - 2) = 540°$
Therefore the sum of exterior angles is $900° - 540° = 360°$
We can say that for all sums of exterior angles for n sided polygons,
(Sum of interior angles of n sided polygons) – (sum of exterior angles of n sided polygons) $= 180° \times n$

Based on what we find out in the above sum of exterior angles of the polygons, it seems that the sum of any exterior angle add up to 360°
What would be the sum of exterior angles of a pentagon?
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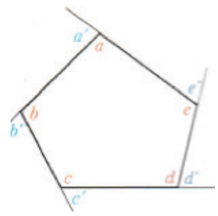
The sum of interior angles is $180° \times (5 - 2) = 540°$
Therefore the sum of exterior angles is $900° - 540° = 360°$
We can say that for all sums of exterior angles for n sided polygons,
(Sum of interior angles of n sided polygons) – (sum of exterior angles of n sided polygons) $= 180° \times n$



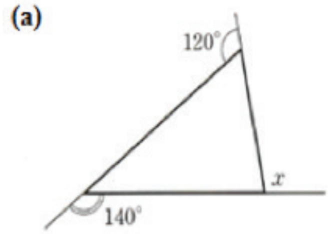
Teaching and Learning Activities

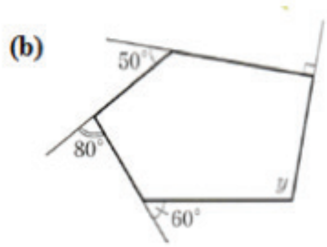
Key Ideas

Sum of interior angles of a polygons
The sum of the exterior angles of polygon is 360°

Exercises

1. Find the angles of $\angle x$ and $\angle y$ in the figures below;





2. What is the measure of each exterior angle in a regular dodecagon (12-sided polygon). What is the measure of each interior angle?

L33 : Congruent Triangles

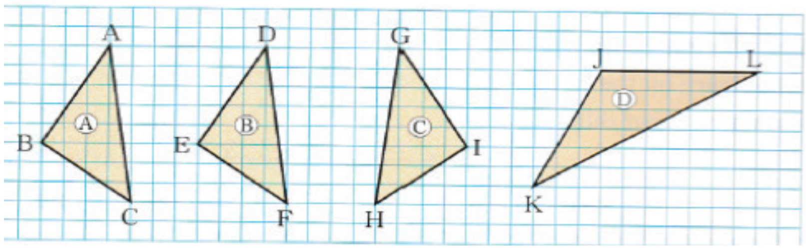
Lesson Objective: To investigate and find the meaning of congruency in triangles **(8.2.2.1)**
Materials: ruler set square, protractor

ASK-MT and Assessment	
Attitudes/Values	Show interest in learning about congruency of figures.
Skills	Investigating the segments and angles of congruent figures.
Knowledge	Congruent figures.
Mathematical Thinking	Think about the meaning of the congruent figures s and express using the symbols.
Assessment	Use the ask-mt to assess the students learning progress during the lesson.

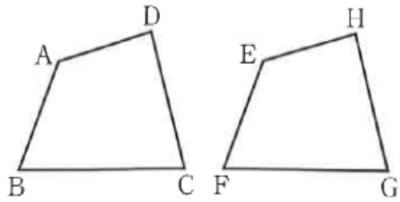
Teaching and Learning Activities

Review

Which of the triangles below would line up exactly, if we laid it on top of △ABC (try flipping them)? Say which sides would match up.



If triangles (A) and (B) line up exactly then they are congruent.
The triangles (A) and (C) are also congruent. For example for figures on the right;



Example.1 **how to determine congruencies of figures**

We can express the fact that quadrilateral ABCD is congruent to quadrilateral EFGH using the symbols ≡ : Quadrilaterial ABCD ≡ quadrilateral EFGH
When we express congruency this way, we list the vertices in corresponding order

Key Ideas

The properties of congruent figures

- The corresponding segments of congruent figures are equal in length.
- The corresponding angles of congruent figures are equal in measures.

Exercises

Use the symbol ≡ to express the fact that triangles (A) and (B) are congruent and that (A) and (C) are congruent in review activity.

L34: Conditions for congruent triangles

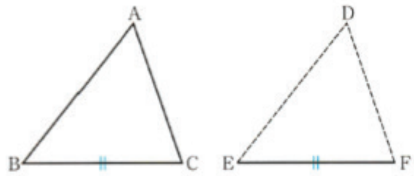
Lesson Objective : To investigate the condition of congruent of triangles and draw them using the measuring instruments. **(8.2.2.1)**

Materials: ruler, protractor, compass

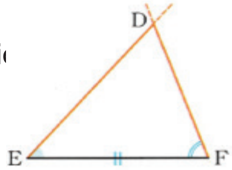
ASK-MT and Assessment	
Attitudes/Values	Enjoy working with others to draw congruent triangles.
Skills	Investigate the conditions of congruent triangles and draw them.
Knowledge	Conditions of drawing congruent triangles.
Mathematical Thinking	Thinking about how to draw congruent figures.
Assessment	Use the ask-mt to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1 : **Conditions for congruent triangles**



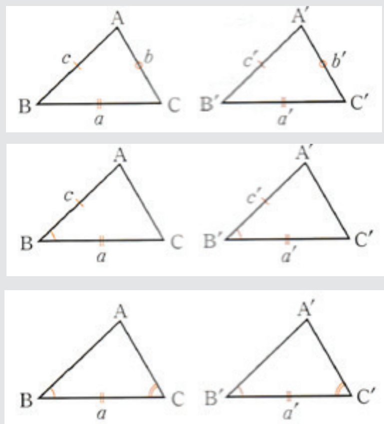
What should we do?
How can we draw △DEF, which is congruent to △ABC.
We start by drawing the side EF, which is the same length as BC.
Where can we determine the vertex D?



As the figure on the right shows , we can determine a unique △ DEF, if we positi
point D so that in addition to EF = BC, ∠ E= ∠ B,∠ F = ∠C.
If we line up side EF with side BC of ΔABC, then vertex D lines up with A. This tells us that △DEF is congruent to △ABC

Key Ideas

Conditions for congruent triangles
Two triangles are congruent if any one of the following conditions holds true,



- All three pairs of sides are equal in length
  $a = a', b = b', c = c'$
- Two pairs of sides and the angle between them are equal.
  $a = a',\ c = c',\ \angle B = \angle B$
  One pair of sides and the angles on both sides are equal
  $a = a'$
  $\angle B = \angle B',\ \angle C = \angle C'$
- One pair of sides and the angles on both sides are equal
  $a = a'$
  $\angle B = \angle B',\ \angle C = \angle C'$

Teaching and Learning Activities

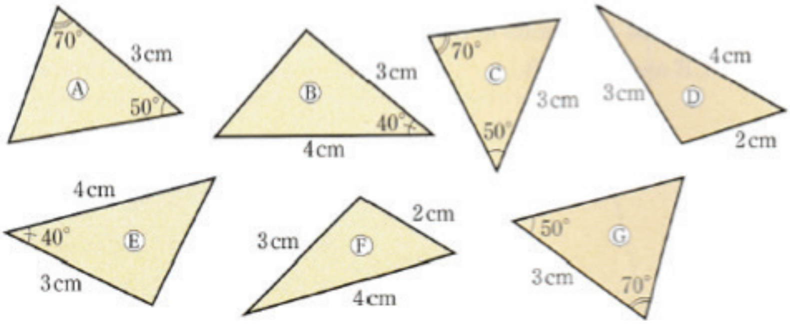
Exercises

1. Refer to the diagram on right

(a) Draw △DEF by determining point D so that EF = BC and ∠ E= ∠B, and DE = AB

(b) Draw △DEF by determining point D so that in addition to EF = BC and DE = AB and DF = AC

2. Sort the triangles below into pairs of congruent figures. Then state the conditions for congruence that you used.



L35: Proofs

Lesson Objective: To proof to demonstrate that the angles and segments have equal measures from the known and are concluded to be true. **(8.2.2.2)**

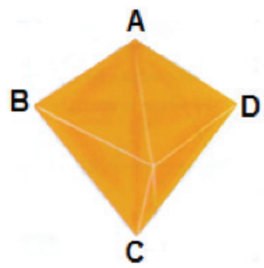
Materials: ruler, compass and protractor

ASK-MT and Assessment	
Attitudes/Values	Show confident in proofing of properties of congruent figures
Skills	Demonstrate the proof of properties and conditions of congruent figures and learn how to show that hypothesis is true.
Knowledge	Proof of the properties and conditions of congruent figures
Mathematical Thinking	Thinking about how to prove properties and conditions of congruent figures and learn how to show that hypothesis is true
Assessment	Use the ASK-MT to assess the students learning progress during the lesson

Teaching and Learning Activities

Review

You decide to make a kite like the one on the right, where

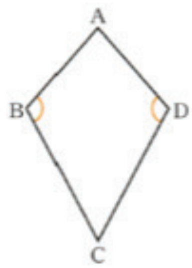


$AB = AD$ and $BC = DC$

Use the compass to drew two circles with different radii, one from point A and one from point C. Mark the points where they intersect B and D to construct the quadrilateral $ABCD$.

Example.1 **How to proof**

In a quadrilateral that you have constructed above;



∠ABC = ∠ADC

We can use the conditions for congruent triangles to demonstrate this in the following way.

We can connect A and C to form △ABC and △ADC

In △ABC and △ADC,

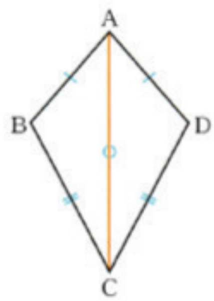
AB = AD............①

BC = DC...........②

Since AC is common to both triangles, AC = AC.............③

①, ② and ③tells us that all three pairs of sides are equal, so that we know that △ABC = △ADC

Since corresponding angles in the congruent figures are equal, we can conclude, △ABC= △ADC



Teaching and Learning Activities

TN: In the proof from the previous lesson above, we demonstrated that,
If $AB = AD$ and $BC = DC$, then △ABC = △ADC
…………………………………………..(B)
Or that (B) is that conclusion deduced from (A)
Is what is given or known
Is the conclusion we deduce from (A)

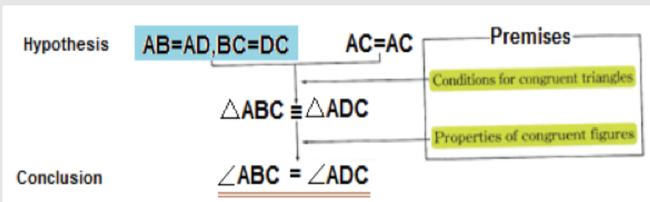
Key Ideas

- A proof demonstrates that something is true in a coherent and logical manner. The statements from the example.1
- If (A), then (B);

When we use this kind of statement
(A) portion is called the hypothesis, and the (B) portion is called the conclusion
In order to deduce the conclusion from the hypothesis in the proof in example.1, we use the following accepted statements as premises.

- Two triangles with three pairs of equal sides are congruent.
- The corresponding angles in congruent figures are equal.

If we focus on the premises as outlined the steps in the proof we the following line of reasoning.
For △ABC and △ADC

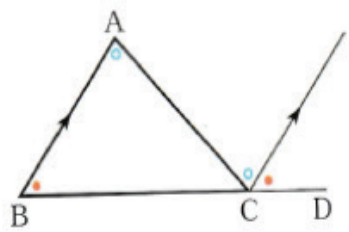


- Proofs are generally structured in the following way.

1. Start with the hypothesis
2. Use accepted statements as premises
3. Follow logic to reach the conclusion

1. State the hypothesis and the conclusion in the statements below;

(a) If △ABC ≡ △ADC , the AB = DE
(b) If $l // m$ and the $m // n$, then $l // n$



2. In lesson # 31, *Example.1* the proof of the statement "the sum of three interior angles of a triangle is 180° is given.
What premises are used in this proof?

L36: Setting up a proof

Lesson Objective : To set the proof from the hypothesis and conclusion of the properties and congruence of triangles. **(8.2.2.2)**

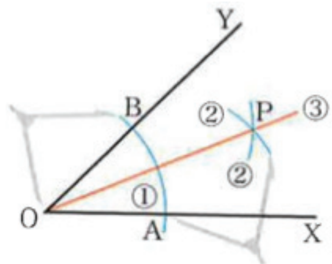
Materials: set square, ruler, protractor

ASK-MT and Assessment	
Attitudes/Values	Show their ideas on how to set up proofs that angles and segments have equal measures
Skills	Set proof, and express hypothesis and conclusion segments and angles.
Knowledge	Proof setting of figures.
Mathematical Thinking	Thinking about how to set the proof figures.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

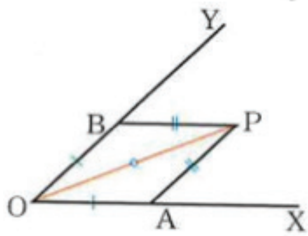
Teaching and Learning Activities

Example.1 **Proof and proof setting**

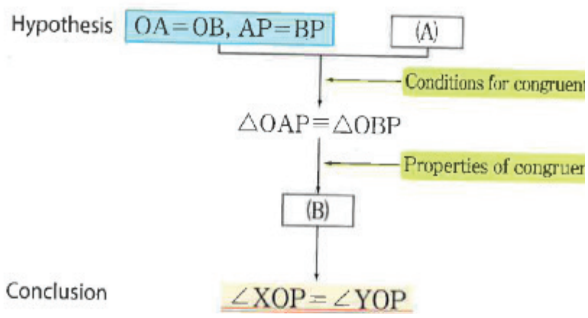
The figure on the right show OP
Constructed as the bisector of ∠ XOY.
We'll prove that
∠ XOY =∠YOP



If we draw the by connecting point P to points O,A and B,
we get the hypothesis and conclusion
Hypothesis OA = OB, AP = BP
Conclusion ∠XOP = ∠ YOP



TN: If we focus on the premises as we outline the steps in the proof, we get the following line of reasoning.



Exercises

Fill in (A) and (B) in the figure above

L37: Using the conditions for congruence

Lesson Objective: To think about how to make a proof using the conditions for congruence. **(8.2.2.3)**
Materials: ruler set square, compass, protractor

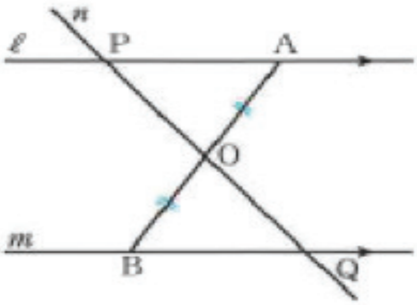
ASK-MT and Assessment	
Attitudes/Values	Enjoy and share ideas on how to make a proof using the conditions for congruence.
Skills	Make a proof using the conditions for congruence.
Knowledge	Making a proof using the conditions for congruence.
Mathematical Thinking	Think of about how to proof the segments and corresponding angles of triangles using the congruence of figures and how to write the superposition and conclusions.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Review

We know that in congruent figures, corresponding segments are equal in length and corresponding angles are equal in measure. Because of this, we often use the conditions for congruent triangles as premises in proofs demonstrating that segment or angles equal.

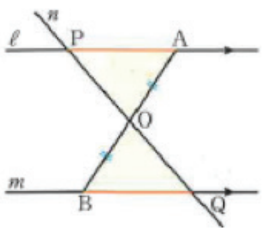
Example.1 **Make a proof using the conditions for congruence**



In the figure on the right, $l \parallel m$. Point O is the midpoint of segment AB, which connects poinrt A on l to point B onm. Line n passes through point O and intersects lines l and m at points P and Q, respectively.
How can we show that AP = BQ

The hypothesis and conclusion in the above figure are as follows.
Hypothesis $l \parallel m$, AO = BO **Conclusion** AP = BQ
Let's use the following line of reasoning to get from the hypothesis to the conclusion

(1) To deduce a conclusion AP = BQ, we need to focus on △OAP and △OBQ with sides AP and BQ
(2) Mark the figure to indicate sides of equal length and angles of equal measure in triangles △OAP and △OBQ
(3) Decide which conditions for congruent triangle we should use to show △OAP≡△OBQ

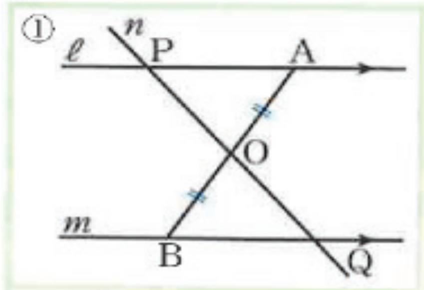


Teaching and Learning Activities

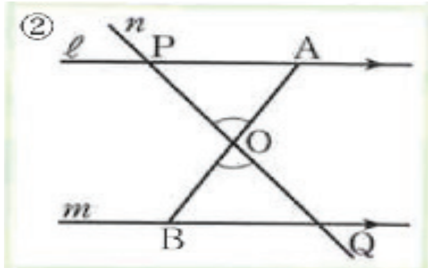
We can now write a proof using the information from example.1

Proof

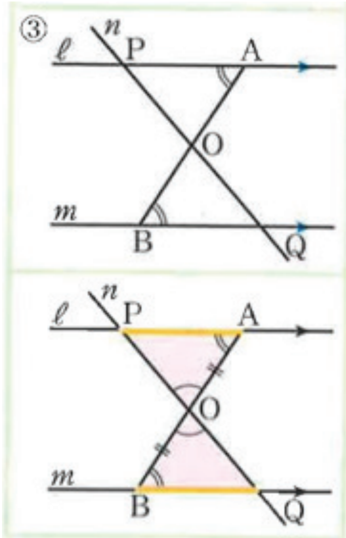
We show that △OAP and △OBQ are congruent.
First ,
OA = OB......................................①
because O is the midpoint of AB.



Since vertical angles are equal, we have
∠AOP = ∠BOQ............................ ②



Also ∠OAP = ∠OBQ..........................③
Since alternate interior angles are equal when $l//m$.



Thus ①, ② and ③ indicated that one pair of sides and the Angles on both sides are equal, so
△OAP △OBQ

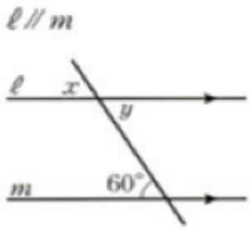
From the fact that corresponding sides of congruent figures are equal in length, it follows that
AP = BQ

Unit Checkpoint

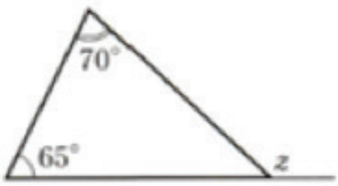
Review on congruence of triangles

1. Find the measure of ∠x, ∠y, and ∠ z in the following figures below.

(a)



(b)



2. Demonstrate that

△ABC ≡ PQR

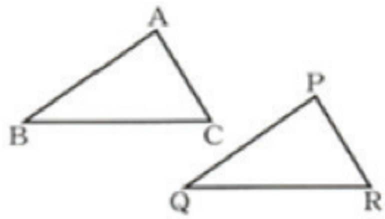
Fill in the □ with the sides that satisfy the conditions for congruence.

AB = PQ, BC = QR

□ = □

AB = PQ, ∠A = ∠P, □ =□

∠A = ∠P, ∠B = ∠Q, □=□



3. Segment AB and segment CD intersection at point O.

Prove that if

AO = BO and CO =DO

Then

AC = BD

(a) State the hypothesis and the conclusion.

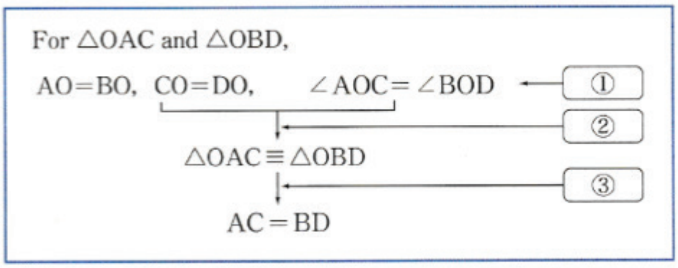
(b) The proof follows the line of reasoning in the figures below

Select the premises that satisfy ① through ③ from (A) and (C)

(A) Conditions for congruent triangles.

(B) Properties of congruent figures.

(C) Properties of vertical angles.



L38 : Properties of isosceles triangles

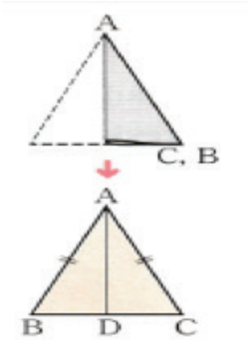
Lesson Objective: To discover the properties of isosceles triangles to develop a conclusion.**(8.2.2.3)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Share ideas on how to develop hypothesis and conclusion using isosceles triangles.
Skills	Make hypothesis and conclusion utilizing the properties of isosceles triangles.
Knowledge	Properties of isosceles triangles.
Mathematical Thinking	Think about how to conclude hypothesis of isosceles triangles coherently and logically.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

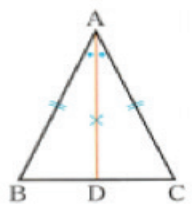
Introduction

What can we find out?
Draw a circle that intersects line l in the figure on the, left using points A as the centre. Mark the intersection point B and C. Draw △ABC. If we folded the triangle so that sides AB and AC lined up, what would we find?



Example.1 : **Hypothesis of the properties of isosceles triangles**
Knowing that AB = AC in △ABC tells us several things
For example, we might predict that ∠B =∠C in the above.
We can write this as , For △ABC,
If AB = AC, then ∠B = ∠C·······(A)
Using the conditions for congruent triangles to prove (A) above.

Proof:
Draw the bisector of ∠A intersecting BC at D.

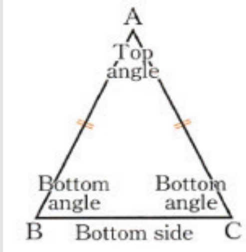


For △ABD and △ACD, Since AD is the bisector of □A,
∠BAD = ∠CAD ·····①
From the hypothesis, we know that
AB=AC···· ②
And since AD is common to both triangles,
AD=AD·····③
Since ①, ② , and ③ indicate that two pairs of sides and the angles between them are equal, so △ABD≡△ACD
Corresponding angles in congruent figures are equal, so we know that, ∠B =∠C
TN: When we think it through in a coherent and logical manner, it is necessary to clearly state the meaning of the words we use, like; An isosceles triangle is a triangle with two equal sides.

Example.2 **Bisector of the top angle of an isosceles triangle**
In example.1, we proved we proved that ∠B and ∠C from △ABD≡△ACD
What can we find?
We can also derive the following information from △ABDACD.
BD = CD······① and ∠ADC = ∠ADC..........②
From ② and ∠ADB + ∠ADC =180°, it follows that ∠ADB = 90°.
In other words AD⊥BC..............③
From now on, we will often use the properties of isosceles triangles that you have just learned as premises to the properties of other figures.

Teaching and Learning Activities

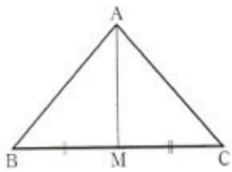
Key Ideas



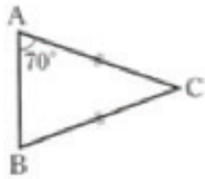
A clear statement of what a word or phrase means is called definition
In isosceles triangle ABC where AB = AC,
The angle∠A created by the equal sides is called the top angle,
the side BC across from the top angle is called the bottom side
and the angles ∠B and ∠C
Basic statements that have been proven true are called “theorems
Bisector of the top angle of an isosceles triangle
The bisector of the top angle of an isosceles triangle is perpendicular to the bottom side of the triangle and divides it into two equal parts.

1. The triangles on the right are isosceles triangles whose equal sides are marked with identical symbols. Find the measures of unknown interior angles.

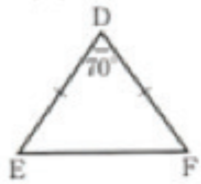
2. In isosceles triangles ABC where AB = AC and M is the midpoint of bottom side BC,
∠BAM =∠CAM and AM⊥BC
Write the hypothesis and conclusion above using the symbol
Prove above



(a)



(b)



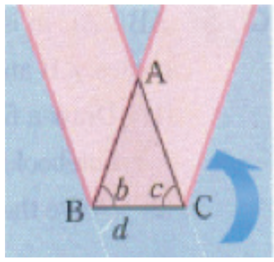
L39: Triangles with two equal angles (1)

Lesson Objective : To identify properties of isosceles triangles. **(8.2.1.3)**
Materials: set square, ruler, protractor

ASK-MT and Assessment	
Attitudes/Values	Appreciate and share ideas on how to identify properties of isosceles triangles.
Skills	Identify properties of isosceles triangles involving parallel lines.
Knowledge	Properties of isosceles triangles involving parallel lines.
Mathematical Thinking	Think about how to identify properties of isosceles triangles including parallel lines.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson

Teaching and Learning Activities

Introduction

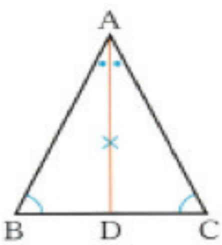


What can we find out?
Fold a ribbon along the segment BC, as shown in the figure on the right. What is the relationship between ∠B and ∠C in the overlapping part △ABC?

Example: **Properties of isosceles triangles**

Based on the properties of parallel lines, we know that ∠c and ∠d are equal from the above diagram, we also know that ∠d and ∠b are lined up because of the fold.
It therefore follows that in △ABC,∠B and ∠C are equal.
△ABC also appears to be an isosceles triangle in the above.
Now let's prove that:
If ∠B and ∠C are equal in △ABC, then AB = AC.................(B)

Proof



Fill in the □ in the proof of (B) below.
Draw the ∠A intersecting BC at D.
For △ABD and △ACD,
The hypothesis tells us that
∠BAD = ∠ □................①
∠ B = ∠ □.................. ②
Since the sum of the interior angles of a triangle is 180° and because of □ and □, it follows that
∠ADB =.......................③
Since AD is common to both triangles, we know that
AD = AD......................④
① , ③, and ④ indicate that one pair of sides and the angles on both sides are equal, so
△ABD=△ACD
Corresponding sides in congruent figures are equal, so we can conclude that
AB=AC

Teaching and Learning Activities

Key Ideas

Triangles with two equal angles

A triangle with two equal angles is an isosceles triangle

Exercises

AB = AC in isosceles triangle ABC. Bisectors of bottom angles ∠B∠C are drawn so that they intersect at P.

(a) Draw a figure matching the above description in your notebook.

(b) Prove that △PBC is an isosceles triangle.

L40: Triangles with two equal angles (2)

Lesson Objective: To identify properties of two statements of congruent figures are conversed. **(8.2.1.3)**
Materials: blackboard

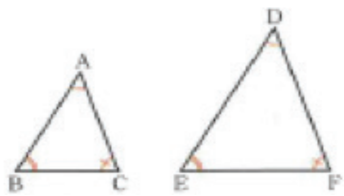
ASK-MT and Assessment	
Attitudes/Values	Appreciate the understanding of ideas on how properties of two statements of congruent figures are conversed
Skills	Explore properties of two statements of congruent figures that is converse of the other.
Knowledge	Congruent figures that is converse of the other
Mathematical Thinking	Think about how to express congruent figures that is converse of the other
Assessment	Use the ask-mt to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1 **Two statements when one is the converse of the other**
Based previous lesson statements we have proved so far,
In $\angle ABC$, if $AB = AC$, then $\angle B = \angle C$
In $\angle ABC$, if $\angle B = \angle C$, then $AB = AC$.
If we compare (A) and (B), we see that the hypothesis and the conclusion are switch.

Example.2 **When two statements are true but not congruent**

State the converse of the following statements.
(a) For $\triangle ABC$ and $\triangle DEF$,
If $\triangle ABC$ $\triangle DEF$, $AB = DE$, $BC = EF$, and $CA = FD$
(b) For $\triangle ABC$ and $\triangle DEF$,
If $\triangle ABC \equiv \triangle DEF$, then $\angle A = \angle D$, $\angle B = \angle E$, and $\angle C = \angle F$



From the properties of congruent figures, if follows that (a) and (b) in *example.2* are true.
If we look at the converse of these statements, we know that the converses of (a) in *example.2* are true. However, the triangles shown on the right are not congruent so the converse of (b) is not true.
In other words, the converse of a true statement is not necessarily true.

Key Ideas
- When two statements have this kind of relationship, we say that one is the converse of the other.
- When you need to explain why a statement is not true, show it with an example.

State the converse of the following statements. Then figure out whether the converse is true.

(a) For two integer a and b, if a and b are both odd, then $a + b$ is even.

(b) For $\triangle ABC$, if $\angle A = 90°$, then $\angle B + \angle C = 90°$

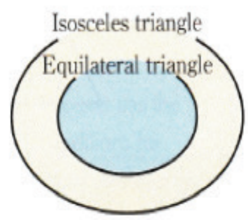
L41: Equilateral triangles

Lesson Objective : To identify and prove properties of equilateral triangles. **(8.2.2.2)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Share ideas on how to identify and prove properties of equilateral triangles.
Skills	Identify and prove properties of equilateral triangles.
Knowledge	Properties of equilateral triangles and their proofs.
Mathematical Thinking	Think about how to identify and prove the properties of equilateral triangles.
Assessment	Use the ask-mt to assess the students learning progress during the lesson.

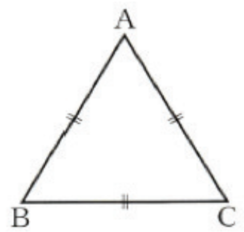
Teaching and Learning Activities

Example.1 **How to Prove properties of equilateral triangles**



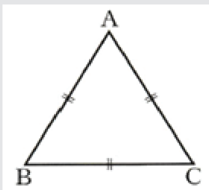
We can define equilateral triangles in the following way.
A triangle with three equal sides is called an equilateral triangle.

Using this definition, we can think of equilateral triangles as a special kind of isosceles triangles. In other words, an equilateral triangle has all the properties of an isosceles triangle as well.
For example, since the bottom angles of an isosceles triangle are equal, we know that for equilateral triangle △ABC,
Since AB = AC,
It follows that ∠B =∠C
Since BC = BA,
It follows that ∠C =∠A
This means that ∠A = ∠B = ∠C, so we can say that all three angles of an equilateral triangle are equal.



Key Ideas

A triangle with three equal sides is called an equilateral triangle



Exercises

Prove that for △ABC, from example.1
If ∠A =∠B=∠C, then AB = BC = CA

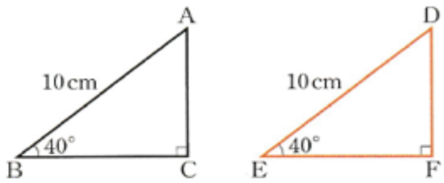
L42: Congruent right triangles

Lesson Objective: To identify properties of congruent right triangles. **(8.2.2.1)**
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Appreciate and share how to identify properties of congruent right triangles.
Skills	Identify properties of congruent right triangles.
Knowledge	Properties of congruent right triangles.
Mathematical Thinking	Think about how to use properties of congruent right triangles to prove hypothesis.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

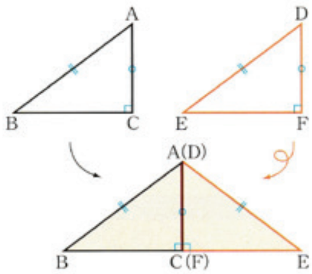
Teaching and Learning Activities

Introduction



The two right triangles in the figure on the right are congruent. How do we know?
For △ABC and △DEF in the above, ∠A = ∠D = 50°, making a pair of sides and the two angles on both sides of them equal. It therefore follows that △ABC≡△DEF.
TN: Two right triangles are congruent if their hypotenuse and one pair of acute angles are equal.

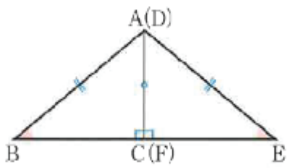
Example.1 **Condition and properties of congruent right triangles**



Let's look at the two right angle triangles △ABC and△DEF where ∠C =∠ F=90°, AB=DE, AC= DF
Use the following strategy to see whether the two triangles are congruent.

(A) Flip △DEF over and line up the equal sides AC and DF to make the figure on the right.
(B) Since ∠ACB = ∠DEF = 90°, point B, C and E are all the same line, creating △ABE.
(c) What kind of triangle is △ABE that you made in (B) above?
Which angle do you know is equal to ∠B?

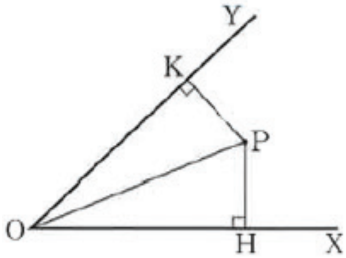
TN: Based on what we found in (c), we know that ∠ B = ∠E. Because of this and also ∠C =∠F = 90° and AB = DE,



We know that in the two right triangle △ABC and △DEF, the hypotenuse and one pair of acute angles are equal. It is therefore follow that △ABC ≡ △DEF
Two right triangles are congruent if their hypotenuse and one pair of sides are equal.

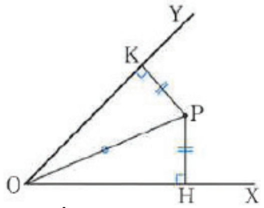
Teaching and Learning Activities

Example.1 **Proofs that use the conditions for congruent right triangles**



Point P lies inside ∠XOY. Two lines PH and PK are drawn from
P perpendicular
To sides OX and OY. Prove that OP divides ∠XOY into
two equal parts when PH and
PK are equal in length.
Approach Demonstrate that the two triangles △POH and △POK, which contain Angles △POH and ∠POK respectively, are congruent.

Proof



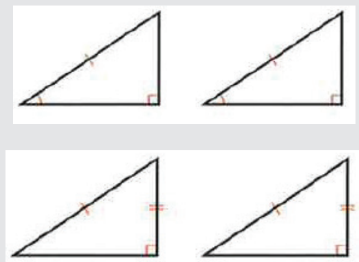
For △POH and △POK, since PH⊥OX and PK⊥OY, it follows that
∠PHO =∠PKO = 90°...............①
From the hypothesis, we know that
PH = PK..................②
And since PO is in both triangles, it follows that
PO = PO..................③
Since ①, ② and ③ indicate that the hypotenuse and one other pair of sides on the two
right triangles are equal, it follows that
△POH=△POK
Since corresponding angles in congruent figures are equal, it follows that
∠POH = ∠POK
Therefore. OP divides ∠ XOY

Key Ideas

- The side opposite the right angles in a right triangle is called the hypotenuse.

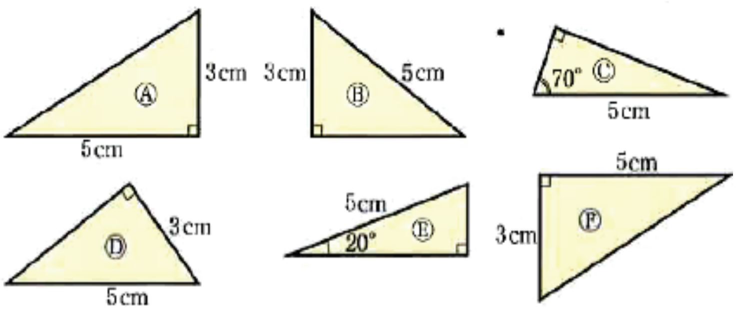
Conditions for congruent right triangle

Two right triangles are congruent in each of the following situations.
① When their hypotenuses and one pair of acute angles are equal.
② When their hypotenuses and one pair of sides are equal



Exercises

1. Sort the triangles into pairs f congruent figures. Then state the conditions for congruence that you used



L43: Properties of parallelograms and proof.

Lesson Objective : To identify and prove properties of parallelogram. **(8.2.2.3)**
Materials: Blackboard

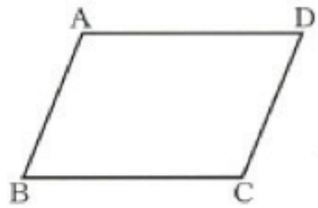
ASK-MT and Assessment	
Attitudes/Values	Appreciate and share ideas on how to identify and prove properties of parallelogram.
Skills	Identify and prove properties of parallelogram.
Knowledge	Parallelogram and their proofs.
Mathematical Thinking	Think about how to prove properties of parallelogram.
Assessment	Use the ask-mt to assess the students learning progress during the lesson.

Teaching and Learning Activities

Review

We can define a parallelogram in the following way;
A quadrilateral in which both pairs of opposite sides are parallel is called a parallelogram

Example.1 **How to Prove properties of parallelogram**



Let's learn the various properties of quadrilaterals and the conditions for special quadrilaterals.
Parallelogram property ❶ on the previously can be written this way.
For quadrilateral ABCD

Hypotenuse AB//DC and AD//BC
Conclusion AB = DC and AD = BC

Let's prove it.

Proof

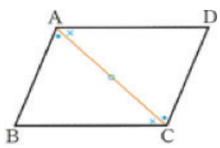


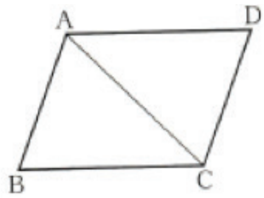
Diagram diagonal AC, for △ABC and △CDA,
Alternate interior angles of parallel lines are equal.
Therefore,
AB//DC tells us that ∠BAC = ∠ DCA.................①
AD// BC tells us that ∠BCA=∠DAC....................②
and since AC is common to both triangles, it follows that AC = CA...........③
Since ①, ②, and □ indicate that one pair of sides and the angles on both sides are equal, it follows that △ABC ≡ △CDA

Since corresponding sides in congruent figures are equal, we can conclude that
AB = CD, BC = DA
We can also use the symbol ▱ to write parallelogram ABCD as ▱ ABCD.

Teaching and Learning Activities

Parallelogram property ①

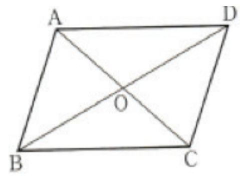
Fill in the hypothesis and conclusion for parallelogram property ②: “opposite angles in a parallelogram are equal in measure”



For parallelogram ABCD, ③

Hypothesis : ☐ **Conclusion:** ☐

TN: The proof of parallelogram property ① on the previous page demonstrates that △ABC ≡△CDA. Use this fact to prove parallelogram property ②



Parallelograms property ③**,**

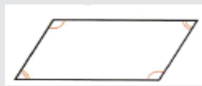
“the diagonals of a parallelogram intersect at their midpoints”, can be written this way for ◇ ABCD diagonals that intersect at O.
For quadrilateral ◇ABCD
Hypothesis: AB//DC, AD//BC
Conclusion: AO=CO, BO=DO

We can prove it if we demonstrate that △OAB and △OCD are congruent in the figure above. In doing so, we can use parallelogram property ①, which we already proved, to say that AB=AC,

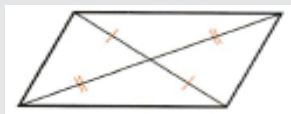
Key Ideas

- A quadrilateral in which both pairs of opposite sides are parallel is called a  n.

Properties of parallelograms

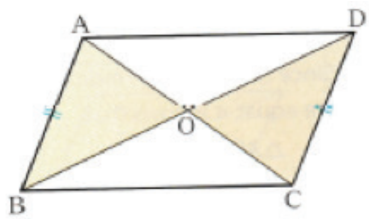


① Two pairs of opposite parallel sides in a parallelogram are equal in length.
② Two pairs of opposite angles in a parallelogram are equal in measure.
③ The diagonals of a parallelogram insert at their midpoints.



Exercises

Prove parallelogram property ③ for ◇ ABCD on the right.



L44 : Conditions of parallelogram

Lesson Objective: To utilize conditions of parallelograms to determine the conditions of the other quadrilaterals **(8.2.2.3)**

Materials: Blackboard

ASK-MT and Assessment	
Attitudes/Values	Appreciate and share ideas on how to identify conditions of parallelograms.
Skills	Identify and explain conditions of parallelograms.
Knowledge	Conditions of parallelograms.
Mathematical Thinking	Think about how to determine conditions of parallelogram.
Assessment	Use the ask-mt to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1 **How to explain conditions of parallelograms and prove**

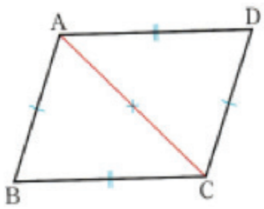
Draw various quadrilaterals ABCD according to the information below. What kind of quadrilaterals can you make?

AB = DC = 4cm and AD = BC = 6cm

There is more than one way to draw the quadrilateral in the above, but we can predict that every quadrilateral we draw will be a parallelogram. Let's try to prove it?

"For quadrilateral ABCD, If AB = DC and AD=BC, Then AB//DC and AD//BC"

Proof



Draw diagonal AC

From the hypothesis, we know that

AB=CD.....①

BC=DA.....②

And since AC is common to both triangles, it follows that

AC=CA.......③

Since ①, ②, and ③ indicate that three pairs of sides are equal, so

△ABC ≡△CDA

Since corresponding angles in congruent figures are equal, we know that ∠BAC =∠DCA and ∠ACB =∠CAD

Therefore, since alternate interior angles are equal,

AB//DC and AD//BC

Based on the proof above, we can say the following.

A quadrilateral in which both pairs of opposite sides are equal is a parallelogram.

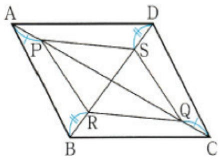
This is converse of parallelogram property ①. What about the converse of parallelogram properties ② and ③?

Teaching and Learning Activities

Example.2 **Using the properties of parallelograms diagonals to prove**

Point P and Q lie along diagonal AC of ◇ ABCD
so that AP = CQ. R and S lie along diagonal BD so that BR=DS.
What kind of quadrilateral is PRQS?

From the above, we can predict that quadrilateral PRQS is a parallelogram. Using the properties of parallelogram diagonals of PRQS? Prove it.
Proof
The diagonals of ◇ ABCD intersect at O.
We know that the diagonals of a parallelogram intersect at their midpoints, so



OA = OC......(1)
OB = OD......(2)
OP= OQ......(3)
OR = OS......(4)

(1) and AP = CQ tell us that while (2) and BR = DS tells us that
Since (3) and (4) indicate that the diagonals that intersect at their midpoints, we can conclude that quadrilateral PRQS is a parallelogram.

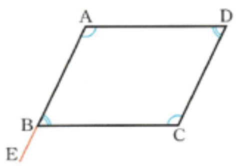
Key Ideas

- A quadrilateral in which both pairs of opposite sides are equal is called a parallelogram.

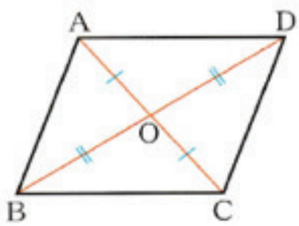
Conditions for parallelograms
We can summarize what we've learned so far like this.
A quadrilateral is a parallelogram in each of the following situations
(1) When two pairs of opposite sides are parallel (definition)
(2) When two pairs of opposite side are equal
(3) When two pairs of the opposite angles are equal
(4) When the diagonals intersect at their midpoints
(5) When one pair of opposite sides is both equal and parallel

Exercises

1. Prove the following for quadrilateral ABCD, focusing on the measure of ∠A + ∠B. If ∠A = ∠C and∠B =∠D. then quadrilateral ABCD is a parallelogram



2. Prove the following when the diagonals of quadrilateral ABCD intersect at O. If AO = CO and BO = DO, then quadrilateral ABCD is a parallelogram.



L45: Special parallelograms

Lesson Objective : To utilize conditions of parallelograms to determine the conditions of the other quadrilaterals **(8.2.2.3)**

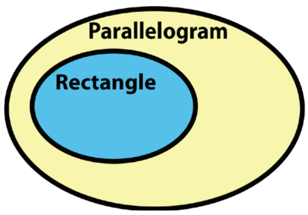
Materials: blackboard

ASK-MT and Assessment	
Attitudes/Values	Share ideas on how to identify conditions of parallelograms to determine conditions of the other quadrilaterals.
Skills	Identify conditions of parallelograms to determine conditions of the other quadrilaterals..
Knowledge	Conditions of parallelograms.
Mathematical Thinking	Think about how to determine conditions of parallelogram.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

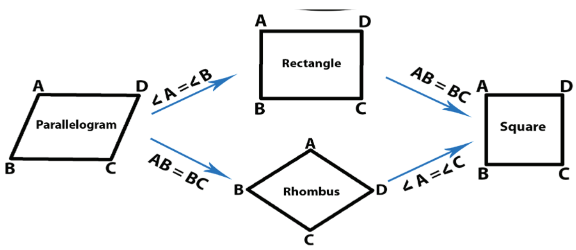
Example.1 **How to utilize conditions of parallelograms**

We can define rectangles, rhombuses, and squares in the following way.
A quadrilateral with four equal angles is a rectangle.
A quadrilateral with four equal sides is a rhombus.
A quadrilateral with four equal sides and four equal angles is a square.
Based on the above definitions, we can say that in a rectangle, opposite pairs of angles are equal.



Let's learn what conditions should be added to sides and angles of a parallelogram in order to make it a rectangle or rhombus
What will happen?

What kind of quadrilateral is □ABCD when we add each of the following conditions?
(a) $\angle A = \angle B$
(b) $AB = BC$
(c) $\angle A = \angle B$ and $AB = BC$
We can now learn the following relationship based on what we learned in the above



What relationship between diagonals AC and BD in □ABCD would make quadrilateral ABCD a rectangle of rhombus?

Teaching and Learning Activities

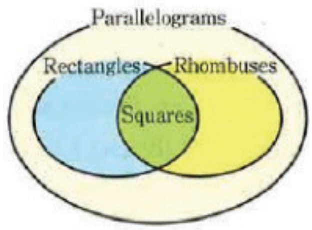
Key Ideas

- Properties of the diagonals of quadrilaterals

① The diagonals of rectangle are equal in length
② The diagonals of a rhombus are perpendicular
③ The diagonals of a square are perpendicular and equal in length.

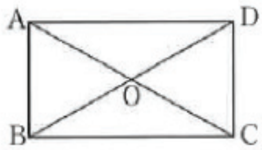
Exercises

Present it in your own words!

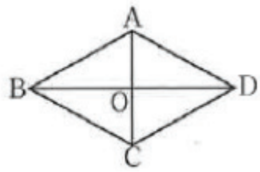


Can we say that a rhombus is a parallelogram? What about a square? Based on what we've learned so far, we know that rectangles, rhombuses, and squares are all parallelograms.

In other words, all of these quadrilaterals have the properties of parallelograms. For examples, the diagonals of a rectangle intersect at their midpoints.
We can also say the following about diagonals in rectangles or rhombuses.



(A) The diagonals of a rectangle are equal in length.



(B) The diagonals of a rhombus are perpendicular .
(1) Prove (A) and (B) above
(2) What can you say about the diagonals of a square?

L46 : Shapes of figure without changing the area

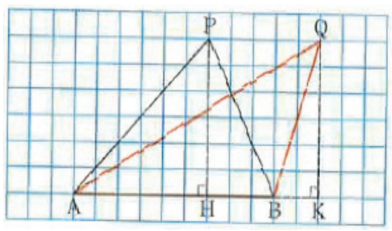
Lesson Objective: To determine area of triangles with parallel lines and properties of triangles. **(8.2.2.3)**
Materials: Coins, blackboard, record sheet

ASK-MT and Assessment	
Attitudes/Values	Appreciate and share idea on how to determine area of triangles with parallel lines and properties of triangles.
Skills	Determine area of triangles with parallel lines and properties of triangles.
Knowledge	Area of triangles with parallel lines and properties of triangles.
Mathematical Thinking	Think about how to determine area of triangles with parallel lines and properties of triangles and prove them.
Assessment	Use the ask-mt to assess the students learning progress during the lesson.

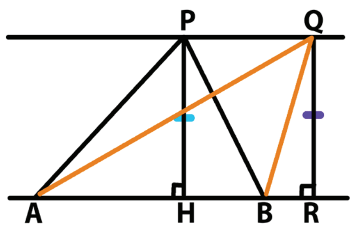
Teaching and Learning Activities

Example.1 **Changing the shape of the figure without changing the area**

What can we find out?
Compare the areas of △PAB and △QAB.



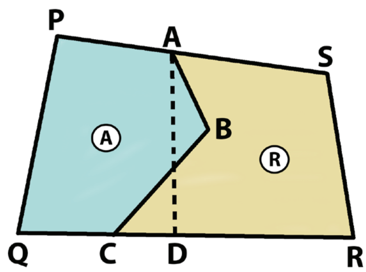
In the figure on the right, PH = QK when PQ//AB.
△PAB and △QAB also
Share bottom side AB,
Since the bottom sides and heights of the two triangles are equal, their areas are equal as well.



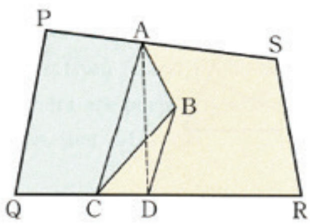
Important: The symbol △PAB may indicate the area of △PAB.
△PAB = △QAB means that the areas of the two triangles are equal

Example.2 **Figures of equal area**

The figure on the right show two plots of land, (A) and (B), whose boundary is the bent line ABC. You want to change the position of segment AD, which passes through point A, without changing the area of two plots. How should we determine the position of point D?



Approach: If we determine point D using a figure like the one below,
Quadrilateral PQDA = pentagon PQCBA
and Quadrilateral PQDA = quadrilateral PQCA +△ACD
Pentagon PQDA = quadrilateral PQCA +△ACB
Therefore, △ACD =△ACB



Explain how to find the position of point D in Approach above.

Teaching and Learning Activities

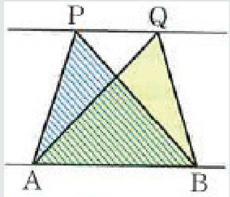
Key Ideas

Triangles with a shared bottom side

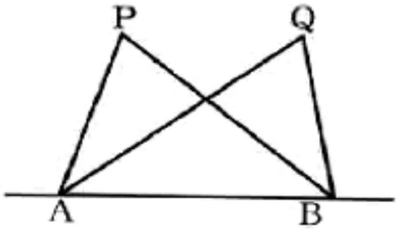
For two points A and B on a single line and two points P and Q on the same side of that line,

 If PQ// AB, then △PAB =△QAB

If △PAB = △QAB, then PQ// AB

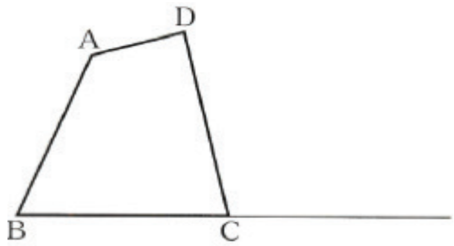


Exercises



1. If, △PAB and △QAB share side AB. Prove the following when vertices P and Q are on the same side of AB, as shown in the figure on the right.
   If △PAB = △QAB, then PQ// AB

2. You extend side BC out from C in quadrilateral ABCD.
You want to place E on that line so that △ABC has the same area as quadrilateral ABCD.
Find the position of point E and draw △ABE.

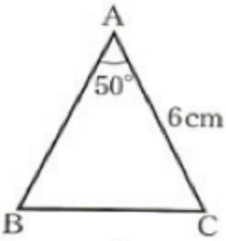


Unit Checkpoint

Review on congruence of triangles

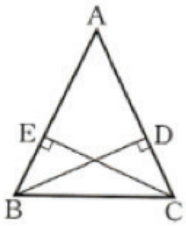
Review on Congruent triangles

1. △ABC in the figure on the right is an isosceles triangle where AB = AC.
Fill in the correct numbers in the □ AB = □cm
∠C = □°



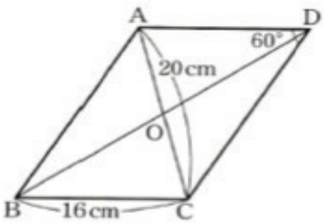
Isosceles triangles

2. ABC is an isosceles triangle where AB = AC.
Line BD is drawn from B perpendicular to AC is Drawn from C perpendicular to AB.
Prove that BE = CD



Congruent right triangles

3. Fill in the correct numbers in the ▱ for
□ ABCD in the below.
AD = □ cm
OA = □ cm
∠ABC = □ °
∠ABC = □ °



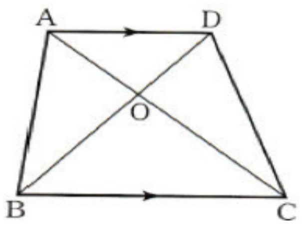
Properties of parallelograms

4. Are the quadrilateral below parallelograms?

(a) Quadrilateral ABCD, where AB// DC and ∠A = ∠C
(b) Quadrilateral ABCD, where AD// BC and AB = CD

Conditions for parallelograms

5. In the figure on the right,
AD// BC. Find all pairs of triangles whose areas are equal



Parallel lines and areas

L47: Likelihood of events happening

Meaning of probability

Lesson Objective : To understand and explain the idea of probability. **(8.4.1.2)**
Materials: Coins, blackboard, record sheet

ASK-MT and Assessment	
Attitudes/Values	Enjoy tossing coins to understand the idea of probability.
Skills	Explain the idea of probability through tossing of two coins.
Knowledge	Idea of probability.
Mathematical Thinking	Think about how to explain the idea of probability through tossing of two coins.
Assessment	Use the ask-mt to assess the students learning progress during the lesson.

Teaching and Learning Activities

Introduction – Open Discussion
Namari and Ranu want to the same piece of cake, and have to decide Who gets it. Namari thought about it and came up with the following idea.

How to decide
Toss two 10 toea coins.
If they both come up with the same side Namari gets the cake
If they come up with different sides, Ranu get the cake.

Namari's thinking
The two 10 toea coins can come up as one of the 3 ways

Of these, there are two ways A and C for the coins to come up the same, and only one way B for them to come up differently, so it is more likely that they will come up the same way.

Example.1 **Likelihood of events happening**

Do an experiment to see if Namari's thinking is correct.

(a) Toss two 10 toea coins at the same time and record whether they come in pattern A, B, or C
(A) both tails
(B) heads and tails
(C) both heads

Teaching and Learning Activities

(b) Repeat over and over as you continue to toss the coins. Record the number of times they come up (A), (B), or (C) on the table.

The table below shows the results of the experiment.

Tosses	10	20	30	40	50	60	70	80	90	100
(A)	3	6	7	13	14	18	21	26	28	29
(B)	6	10	16	18	21	24	29	33	39	46
(C)	1	4	7	9	15	18	20	21	23	25

(Expected results if it continues to 1000)

Toss	10	20	30	40	50	60	70	80	90	100
(A)	3	6	7	13	14	18	21	26	28	29
(B)	6	10	16	18	21	24	29	33	39	46
(C)	1	4	7	9	15	18	20	21	23	25

150	200	300	400	500	600	700	800	900	1000
40	56	81	101	135	150	176	200	226	251
74	94	145	198	248	302	348	395	451	497
36	50	74	101	117	148	176	205	223	252

Exercises

What did the results of the experiment tell you from example.1? Is Namari's thinking correct? Discuss it with your classmate

L48 : Definition of probability

Lesson Objective: To understand and express the meaning of probability. **(8.4.1.2)**
Materials: Coins,blackboard,record sheet

ASK-MT and Assessment	
Attitudes/Values	Show confident to express the meaning of probability using numbers.
Skills	Express the meaning of probability using numbers through simple experiments.
Knowledge	Meaning of probability.
Mathematical Thinking	Think about how to express the meaning of probability using numbers through simple experiments.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example1 **How to use numbers to express the likelihood of an event**
The table below shows the results of the experiment from the previous lesson

Tosses	10	20	30	40	50	60	70	80	90	100
(A)	3	6	7	13	14	18	21	26	28	29
(B)	6	10	16	18	21	24	29	33	39	46
(C)	1	4	7	9	15	18	20	21	23	25

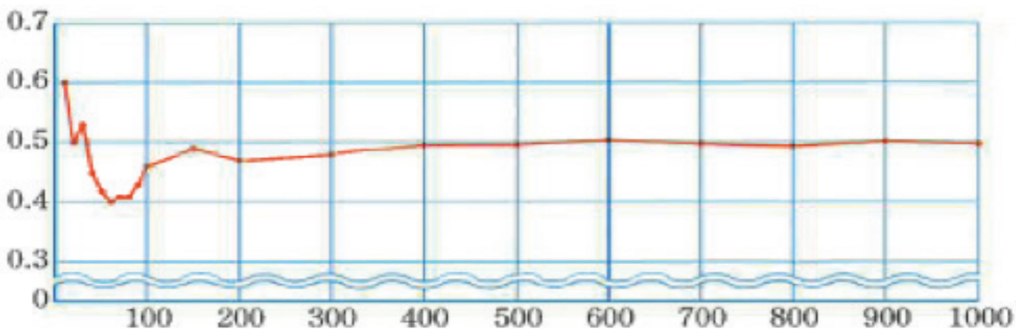
(Expected results if it continues to 1000)

Toss	10	20	30	40	50	60	70	80	90	100
(A)	3	6	7	13	14	18	21	26	28	29
(B)	6	10	16	18	21	24	29	33	39	46
(C)	1	4	7	9	15	18	20	21	23	25

150	200	300	400	500	600	700	800	900	1000
40	56	81	101	135	150	176	200	226	251
74	94	145	198	248	302	348	395	451	497
36	50	74	101	117	148	176	205	223	252

From the above table, it tells us that (B) is more likely to occur than (A) or (C).
Let's take a close look at situation (B). Use the table, find

Relative frequency of (B)=(Number of times(B)occured)/(Number of tosses)



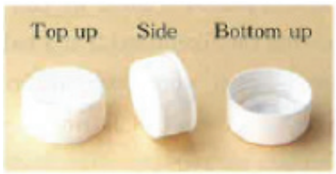
Teaching and Learning Activities

TN: When the number of tosses is low there is more variation in the relative frequency of (B) but that variation lessens as the numbers of tosses increases.
The relative frequency of (B) approaches 0.5 as the number of tosses increases.
We can think of this 0.5 as expressing the likelihood that event (B) will occur.
Using the term, we can say that when we toss two coins, the probability of one coming up heads and one coming up tails is 0.5

Key Ideas

- When the number of tosses is lo there is more variation in the relative frequency of (B) but that variation lessens as the numbers of tosses increases.
- The relative frequency of (B) approaches 0.5 as the number of tosses increases.
- A number that expresses the likelihood that a given event will occur is called the probability of that event.

Exercises



1. Use the table of expected results to state the probability of (A)

2. The table below show the results of the "Experiment Finding the likelihood of a plastic bottle cap landing in different ways".

Tosses	100	200	300	400	500	600	700	800	900	1000
Top up	28	53	82	109	138	166	187	210	236	263
Side	10	20	37	51	67	85	98	107	122	135
Bottom up	62	127	181	240	295	349	415	483	542	602

Use the term "probability" to explain the likelihood that the cap will land top up, on its side, or bottom up

L49: Usage of numbers to express probability

How to find probability

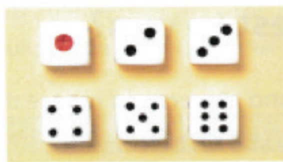
Lesson Objective : To acquire how to use the number of possible outcomes to find the probability. **(8.4.1.1)**
Materials: dices, record sheets, blackboard

ASK-MT and Assessment	
Attitudes/Values	Enjoy using the number of possible outcomes to find the probability.
Skills	Recognize how use the number of possible outcomes to find the probability.
Knowledge	Number of possible outcomes to find the probability.
Mathematical Thinking	Think about how to use the number of possible outcomes to find the probability.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1 **Using the formula P=a/n**

What is the probability of the number 1 coming up when you roll a die?
If we did an experiment in above, we would find that the probability of rolling the number 1 is close to 1/6 .
This probability can be obtained as follows.

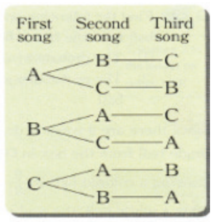


(A) There are six ways for a number to come up: 1, 2,3,4,5 and 6.
(B) The likelihood of each one is the same.
(C) There is only one way for the number 1 to come up.
Therefore, the probability of rolling the number 1 is

$$\frac{\textit{Number of ways for event } \text{(C)to happen}}{\textit{Number of ways for event (A)to happen)}} \quad \frac{1}{6}$$

This matches our experimental results.

TN: In (B) above, we say that rolling each number is equally possible
When each event is equally possible, it allows us to find the probability in terms of a ratio between the numbers of ways for each event to happen.



Example.2 **Using the tree diagram**

If we want to find the probability that a given event will occur, we have to make sure that we have not overlooked or double counted any of the possible events.
The school broadcasts three songs during lunch time A, B and C. What are the possible sequences that the three songs could be played in?

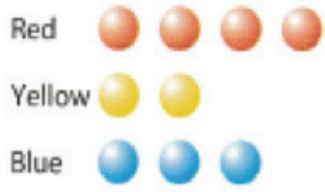
We often use diagrams like the one on the right to organize and count all possible events.

The tree diagram tells us that there are a total of 6 ways for the songs to be played in the above.

Teaching and Learning Activities

Example.3 **Probability of drawing balls**

You draw a single ball from a box containing 4 red balls, 2 yellow balls and 3 blue balls. We can find the probability of drawing a red ball using the following process.



(A) There are a total of 9 ways to draw a ball.
(B) Drawing each ball is equally possible.
(C) There are 4 ways to draw a red ball.

Therefore, the probability of drawing a red ball is .4/9
Since there are 9 ways to draw a coloured ball when drawing a single ball from the *example.2* above, the probability of drawing a coloured ball is 9/9=1 . Since there are 0 ways to draw a white ball, the probability of drawing a white ball is 0/9=0

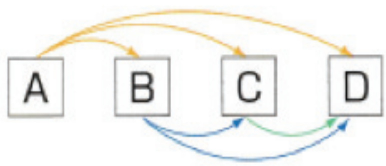
This means that we can say the following about probability.
The probability of something that will definitely happen is 1.
The probability of something that will never happen is 0.
In addition, if we let p be the probability that a particular outcome will occur, the range of the value p is $0 \leq p \leq 1$

Key Ideas

How to find the probability
Let the total number of ways to occur be n, where every event is equally possible.
Of those, if the number of ways for the event A to occur is a, we say that the probability of A happening is p=a/n

1. Four teams compete in a soccer tournament: A, B, C and D. Each team plays every other team once. How many games do the teams play in all?
   To find the answer to Ex.1, we can use a table like the one on the right or a figure like the one below.

	A	B	C	D
A		○	○	○
B			○	○
C				○
D				



2. Find the following probabilities when drawing a single ball from the box example.1

(a) The probability of drawing a blue ball
(b) The probability of drawing a blue ball or a yellow ball

3. Find the following probabilities when rolling a single die.

(a) The probability that the number 6 or lower will come up
(b) The probability that the number 7 or higher will come up

L50: Probabilities when tossing coins

Lesson Objective: To find the probability of tossing certain number of coins **(8.4.1.2)**
Materials: Coins,blackboard,record sheet

ASK-MT and Assessment	
Attitudes/Values	Enjoy finding the probability of tossing coins.
Skills	Find the probability of tossing coins.
Knowledge	Probability of tossing coins.
Mathematical Thinking	Think about how to find the probability of tossing.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example 1: **Find the probability of tossing two coins**

Tail *Heads*

Find the probability of one coin coming up heads and one coin coming up tails when you toss two coins at the same time.

Approach : When tossing one coin, we know that it is equally possible for it to come up heads or tails.

If we label two coins A and B and toss them at the same time,
there are four ways for them to come up heads or tails. These are shown in the table on the right. We can say that each of these events is also equally possible.

A \ B	Heads	Tails
Heads	(heads ,heads)	(heads ,tails)
Tails	(tails, heads)	(tails ,tails)

When we look at the two coins separately, we see that there are four ways for the coins to come up heads or tails - all of which are equally possible.
(heads, heads) (heads, tails) (tails, heads) (tails, tails)
Because there are two ways for one coin to come up heads and one coin to come up tails, the probability of this event is 2/4=1/2

Example.2 **Probabilities when tossing three coins**

Find the probability of at least two coins coming up heads when you toss three coins at the same time.

Approach
Label the three coins A, B and C. Using the diagram to think of all the ways they can come up heads or tails.

"At least two coins coming up heads" means that either all three coins come up heads or that two come up heads and one comes up tails.

Teaching and Learning Activities

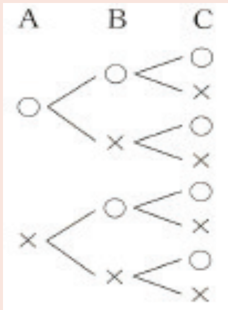
Approach

Label the three coins A, B and C. Using the diagram to think of all the ways they can come up heads or tails.

“At least two coins coming up heads” means that either all three coins come up heads or that two come up heads and one comes up tails.

Solution

Label the three coins A, B and C. Using to represent heads and × to represent tails, make a diagram showing all events. We get eight different ways, all of which are equally possible. Of these there is one way for all three coins to come heads
(O, O, O)
and three ways for two to come up heads and one to come up tails
(O, O, X) (O, X, O) (X, O, O)
This means there are total of four ways for at least two coins to come heads.
Therefore, the possibility we get is 4/8 = 1/2



Exercises

1. You toss two coins at the same time. Find the probability of both of them coming up heads
2 Find the following probability when tossing three coins at the same time.

(a) The probability that all three will come up tails.
(b) The probability that at least one will come up heads

3. You have three cards like the ones shown on the right. You shuffle them well and draw one card at a time, arranging them from left to right to make a three-digit integer. Find the probability that the integer will be even number.

L51: Probabilities when rolling dices and drawing straws

Lesson Objective : To find the probability of rolling two dices and drawing straws. **(8.4.1.3)**
Materials: Dices, straws, improvised materials, blackboard

ASK-MT and Assessment	
Attitudes/Values	Enjoy finding the probability of rolling two dices and drawing straws.
Skills	Find the probability of rolling two dices and drawing straws.
Knowledge	Probability of various events.
Mathematical Thinking	Think about how to find the probability of rolling two dices and drawing straws.
Assessment	Use the ASK-MT to assess the students learning progress during the lesson.

Teaching and Learning Activities

Example.1 **Finding the probability of rolling two dices**

Find the following probabilities when rolling two dice at the same time.
(a) The probability that they will come up the same number.
(b) The probability that they will come up different numbers.

Approach
If we label the dice A and B, the table on the right tells us that there are 6 × 6 = 36 (ways)
that the number can come up. The events were both dice come up the same are marked with □

A \ B	⚀	⚁	⚂	⚃	⚄	⚅
⚀	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
⚁	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
⚂	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
⚃	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
⚄	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
⚅	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Solution

If we label the two dice A and B, there are a total of 36 ways that the numbers can come up.
(1) Since there are six ways that the same numbers can come up, the probability of this event is 6/36 = 1/6
(2) Since there are 30 ways of different numbers coming up, the probability of this event is 30/36 = 5/6

(*Number of ways for the events where differentnumbers come up to happen*) = (*Numbers of ways that all possible events can happen*) - (*Number of ways for the event where the same number come up to happen*)

Exercises

This means that we can find the probability of different numbers coming up using the expression below.(probability of different numbers coming up) 1 — (probability of the same number coming up)
In general we let p be the probability that event A will occur.
The probability that A will not occur = 1 — p
There are five straws, and two of them are winners
Two people draw one straw each- A first, then B. find the following probabilities
(a) The probability that A will draw a winning straw.
(b) The probability that B will draw a winning straw.

Approach
Label the winning straw 1 and 2 and the losing straws 3, 4 and 5.
Make a tree diagram showing the ways that A and B can draw the draw.

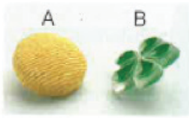
Unit Checkpoint

Review on probability

Review on Probability

Definition of probability

1. The table on the right shows the number of times that buttons A and B landed right side up or upside down being tossed. Which button is more likely to land Right side up, A or B?



How to find probability

2. The probability of rolling a 1 using a single die is 1/6. Which of (a) through (c) below explains what this Probability means?

 (a) If you roll the die six times, it will always come up 1 of those times
 (b) If you roll the die six times, It will only come up 1 once
 (c) If you roll the die 3000 times, it will come up 1 about 500 times.

3. Fill in the □.

(a) The probability of something that will definitely happen is □
(b) The probability of something that will never happen is □
(c) If we let p be the probability that event A will occur, the probability that A will not occur is □.

4. You shuffle a deck of 52 playing cards (one that does not contain a joker) and draw one. Answer the following questions.

 How many total ways are there to draw a card?
 How many ways are there to draw an ace?
 Find the probability of drawing an ace

5. Find the following probabilities.

(a) The probability that a person will draw a winner out of five straws when two of them are winning straws.
(b) The probability that an odd number will come up when rolling a single die
(c) The probability that all three coins will come up heads at the same time.

Assessment, Recording and Reporting

Assessment, recording and reporting is an integral part of the delivery of any curriculum used in the schools.

The primary purpose of assessment is to improve students' learning and teachers' teaching as both respond to fulfilling the following:

- inform and improve students' progress and achievements in learning.
- provide valuable information that enable teachers, schools and Department of Education to make decisions about how to improve the quality of teaching and learning in the education system.
- inform teachers of the progress of students learning in order to adjust teaching and planning to improve student learning.
- inform parents and guardians, about their children's progress and achievements.
- schools and systems, about teaching strategies, resource allocations and curriculum; and other educational institutions, employers and the community, about the achievements of students in general or of particular students.

Effective and meaning assessment must be maintained at all times. The content standards written in the syllabus are expected curriculum for this grade prescribed by units and sets the basis for planning and conducting on - going assessment.

Ongoing classroom assessment is done to:

- Support students learning
- Monitor students learning needs
- Diagnose students learning needs
- Evaluate teaching program and
- Inform students reporting process.

Assessment Strategies and Methods

1. Assessments Methods

Assessment is an integral part of students learning and can be done using different methods.

Teachers are encouraged to use two or more types of assessment when assessing students learning. Standards Based Curriculum specially promotes three types of assessment. These are;

- Assessment *As* Learning **(AAL)**
- Assessment *For* Learning **(AFL)**
- Assessment Of Learning **(AOL)**

2. Assessment Strategies

2.1 Answering Questions and Tasks during the Lesson

The use of questioning or an activity during the teaching of every lesson is an example of AAL that assesses student's achievement (key concepts) of the lesson. It allows student the opportunity to reflect on their own learning and identify areas of their strength and weakness during the lessons.

2.2 Exercises

All exercises given in every lesson are example of AFL. It is not used to evaluate learning but to help learners learn better. It helps the students to see the learning in relation to the goals.

2.3 Checkpoints

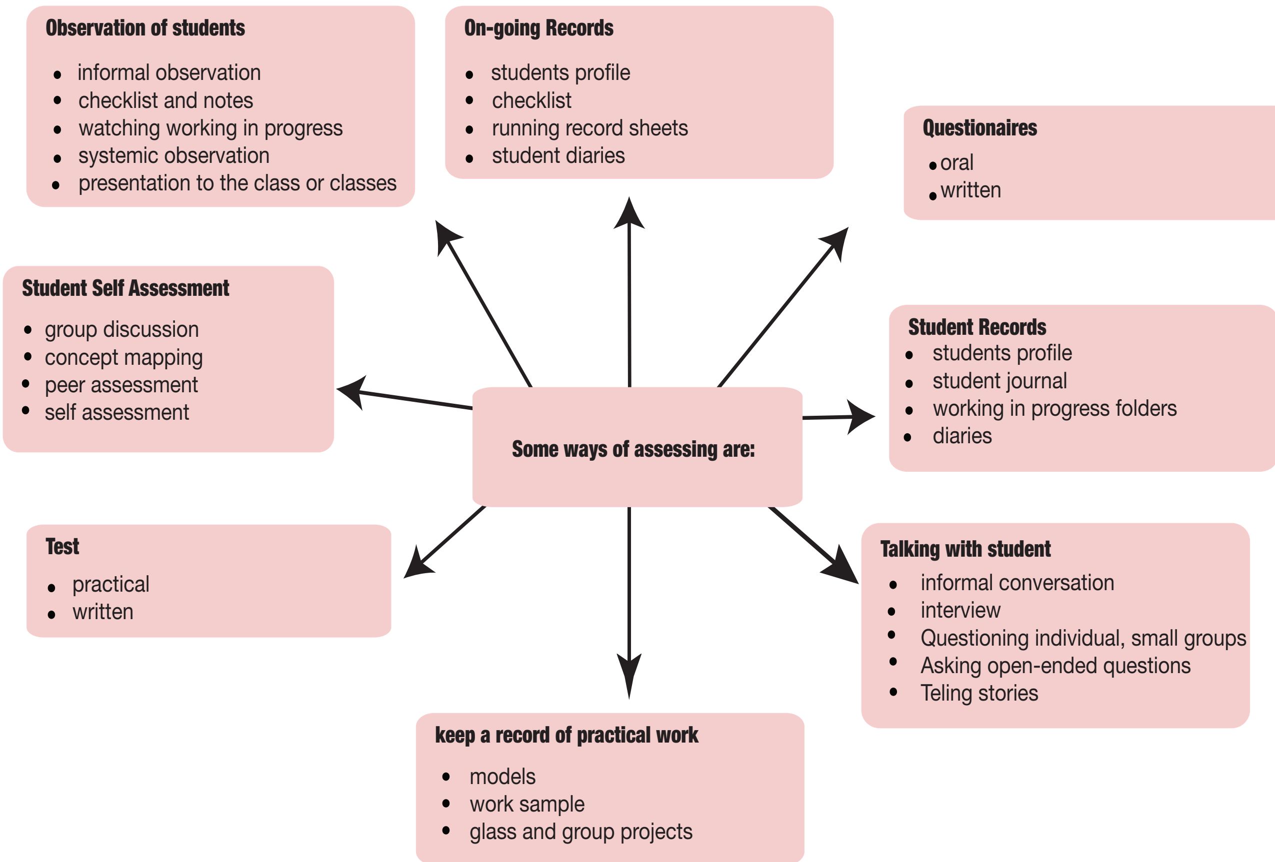
In the Teacher guide there is a check point. It is a Unit Review exercise that appears at the end of the all guided lessons of a particular unit. The exercises are based on basic key concepts to recheck the understanding of students. Teachers are encouraged to use these as classroom assessments apart from tests and assignments. This Unit Review exercises is an example of AOL that teachers can use to measure, record and report on a student's level of achievement in regards to specific learning expectations.

2.4 Written Test/Assignment/Homework

These assessment strategies are used to assess students' performance of their learning formatively or summative. Class teachers prepare these with careful considerations of;

- the knowledge and skills to assess the students
- the level of language to be used
- the construction of questions - clear and precise
- the intended content to be assess
- how much each question is worth and how to award scores for each questions.

Below are some other ways of assessing apart from what has been give above.



Assessment Plan and Tasks

It is important to plan assessment for the whole year using the content overview and the yearly or term plans. Assessment.

Assessment tasks form the basis of the assessment processes of assessing each learner in relation to the content standards. Assessment tasks are learning activities created from the benchmarks. These are written and specifically designed and planned before administering. This particular activity assessing has key concepts (ASK-MT) that must be achieved at the end of each Content Standards.

Assessment Plan

To plan assessment tasks, teachers must decide which type of assessment methods will be used to demonstrate to achieve the content standards. Standard statements are the point of reference in the process of identifying and planning assessment tasks.

In the process of writing and planning assessment tasks, the following are some points to consider;

- choose assessment methods suitable for the assessment task. (Refer to sample in Appendix.5)
- develop assessment rubric based the Key Concepts (ASK-MT). (Refer to sample in Appendix.5)
- consulting the Bloom's Taxonomy as per the student's cognitive level.

Assessment Tasks

Sample assessment tasks given are examples for teachers to use and plan their own to suit their context and the learning needs of the grade six students in the classroom. The tasks are very specific and direct the teacher to the content of learning stated in the syllabus.

Teachers should be ensured that all assessment tasks are;

- clearly stated in simple language that students can easily interpret
- link to the content standards
- balanced, comprehensive, reliable and fair
- should create assessment criteria (rubrics) to be demonstrated in each tasks and are made known to students.

Grade (8) Yearly assessment plan and task overview

According to the Grade (8) content and yearly plan, a suggested yearly assessment plan and tasks overview has been planned and placed. Teachers are encourage to utilize this according to the school overall assessment program.

Strand	Unit	Content Standard	Assessment Task
Number and Operation	Calculation of algebraic expressions	**8.1.1**	• Calculate algebra expressions of polynomials and monomials using four operations. • Organize and group algebra expressions of like terms and unlike terms, and operate to find the answer.
	Simultaneous equations	**8.1.2**	• Use simultaneous linear equations to solve various problems.
Geometrical, Measurement and Transfor-mation	Properties of Parallel lines and angles	8.2.1	• Use properties of parallel lines and angles to identify and explain various angles formed.
	Congruence of plane figures	8.2.2	• Explain congruence of triangles • Explain how to make proof using the conditions of congruence
Patterns and Algebra	Linear Functions	8.3.1	• Use algebraic equation, tables and graphs to identify functional relationships.
Statistics and Probability	Probability	8.4.1	• Find the probability for uncertain phenomena by using diagrams.

Recording and Reporting

Recording and reporting of students achievements in the classroom is very important. Teachers should use a range of tasks to ensure that commended standards statements are equally assessed and reported. This helps the teachers to reflect the effectiveness of their teachings.

Recording

Teachers must keep accurate records of students' achievement of their learning. They must report these achievements in fair and accurate ways to parents, guardians, teachers and students. Teachers should records the evidence of students' demonstrations of achievements of standards using assessment instruments that are manageable.

Examples of recording methods include;

- Anecdotal notes in a journal or dairy
- Checklist
- Portfolios of students' work
- Progressive records
- Work samples with comments written by the teacher.

Reporting

Reporting is important in assessment and should be done effectively. Teachers should report what students have done well and how they can improve further. Students' reports should be based on assessment information collected from students learning progress and other related areas such as behaviours. Schools will decide on how reports will be presented according to the needs of their communities.

Methods will include interviews and written reports. Written reports should include;

- A written record of content standards achieved by students since the previous report
- A written record of the content standards the student is now working towards
- Information about students' attitudes, values and other additional information that is specific to individual students.

Evaluation

- Evaluation is part of the process of continuously raising standards of student achievement in PNG. Assessment information used for evaluation purposes should be used in ethical and constructive ways.

- Teachers will use assessment information to evaluate the effectiveness of their teaching, learning to make improvements to their teaching practice in order to improve student learning. Evaluation tools such as written records, questionnaires, logs and diaries, submissions or records of meetings and discussion with general staff members, teaching staff, parents and other community members.

Resources

Mathematics lessons require resources both for teachers and students. The recommended list of resources in this Teacher Guide are vital for making the teaching and learning meaningful and to understand concepts more precisely and clear.

1. List of Teaching Aids /Materials

- Set squares
- Protractors (Full circle and semi of various sizes)
- Compass (various sizes)
- Wall Clock
- Rulers (1m & 30cm)
- Tape measure (various sizes)
- Multiplication table chart
- Fraction wall chart (to be Improvised by the teacher)
- Stop watches
- Base ten materials (cubes, long, flat and blocks)
- Thermometer
- Models of 2D and 3D shapes
- Scissors/blades
- Square coloured papers
- A grid square papers

2. Teaching Resources

- Melanesian School Mathematics Dictionary
- Community School Mathematics 6A & 6B
- Grade 6 Mathematics TV Students Workbook
- Grade 6 Mathematics TV Teacher resource book
- Tingting Maths Grade 6 Teacher Resources book
- Tingting Maths Grade 6 Students book
- Mathematics Essential Skills 6 Outcomes Edition for PNG

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Study with your Friends Mathematics for Elementary School, © Gakkohtosho Co. LTD, Taheshi Nara, Tosho Printing Co., LTD Japan

Gateway to the future Math 1, 2 & 3 For Junior High School, 2013, Shinko Shuppansha KEIRINKAN Co., Ltd.

Appendices

Appendix 1: STEAM or STEM

- By exposing students to STEAM and giving them opportunities to explore STEAM-related concepts, they will develop a passion for it and, hopefully, pursue a job in a STEAM field.
- Providing real life experiences and lessons, e.g., by involving students to actually solve a scientific, technological, engineering, or mathematical, or Arts problem, would probably spark their interest in a STEAM career path. This is the theory behind STEAM education.
- By integrating STEAM content and real life learning experiences at different levels of the curriculum process (e.g., Curriculum frameworks, content standards, benchmarks, syllabi, teachers' guides and students' books, curriculum design and development, annual and term school programs and lesson plans, teaching methodologies.
- Teaching methodologies – Problem and project-based learning, partnerships with external stakeholders e.g., high education institutions, private sector, research and development institutions, and volunteer and community development organizations.
- They underpin STEM education. They are the main enablers of STEM education.
- The 21st century skills movement, which broadly calls on schools to create academic programs and learning experiences that equip students with the most essential and in-demand knowledge, skills, and dispositions they will need to be successful in higher-education programs and modern workplaces.
- The term 21st century skills refers to a broad set of knowledge, skills, work habits, and character traits that are believed - by educators, school reformers, college professors, employers, and others - to be critically important to success in today's world, particularly in collegiate programs and contemporary careers and workplaces.
- Generally speaking, 21st century skills can be applied in all academic subject areas, and in all educational, career, and civic settings throughout a student's life.
- The skills students will learn will reflect the specific demands that will be placed upon them in a complex, competitive, knowledge-based, information-age, technology-driven economy and society.

Appendix 2: The 21st Century Skills, Knowledge, Attitudes and Values

The following list provides a brief illustrative overview of the knowledge, skills, work habits, and character traits commonly associated with 21st century skills:

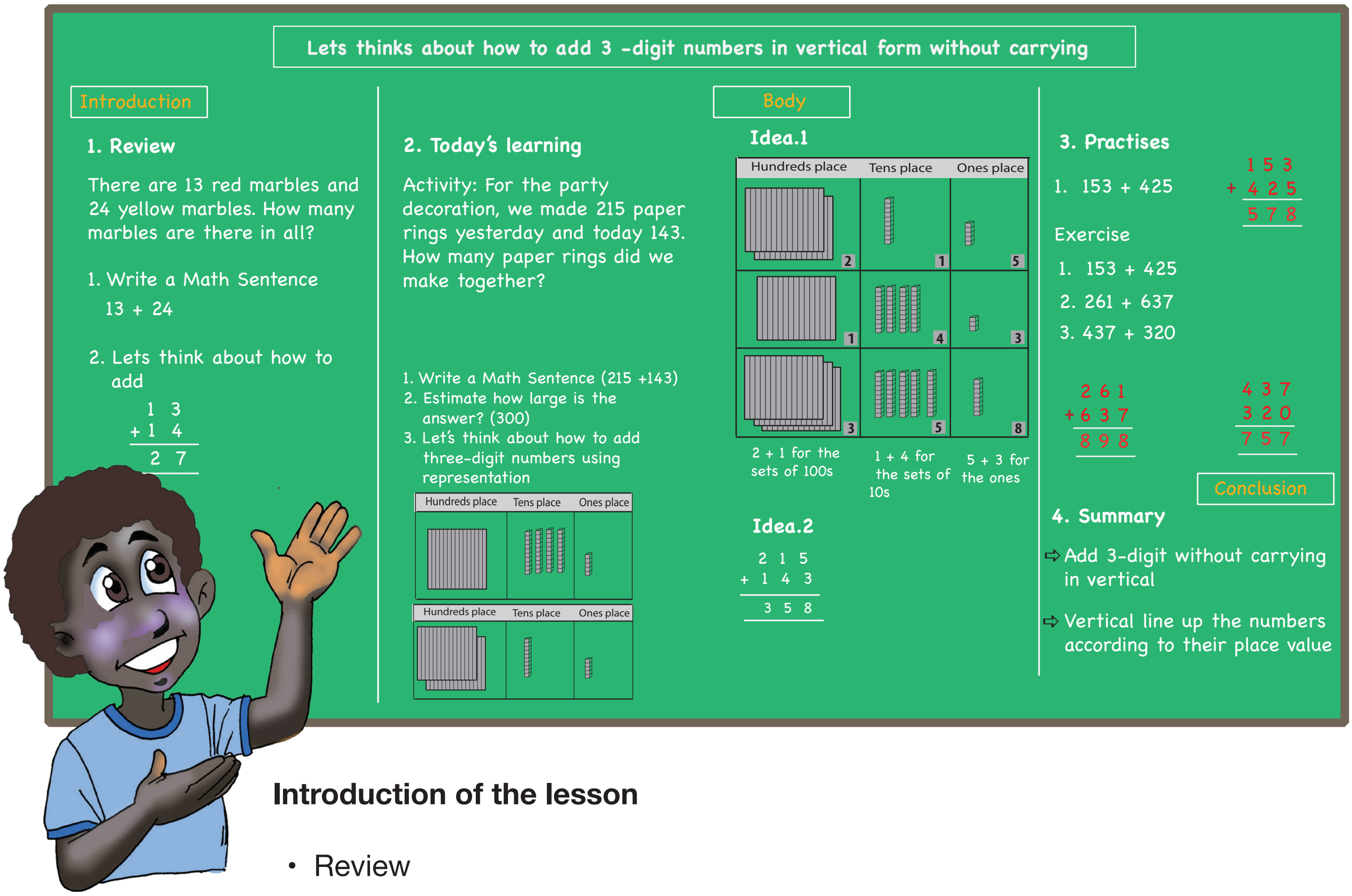
- Critical thinking, problem solving, reasoning, analysis, interpretation, synthesizing information
- Research skills and practices, interrogative questioning
- Creativity, artistry, curiosity, imagination, innovation, personal expression
- Perseverance, self-direction, planning, self-discipline, adaptability, initiative
- Oral and written communication, public speaking and presenting, listening
- Leadership, teamwork, collaboration, cooperation, facility in using virtual workspaces
- Information and communication technology (ICT) literacy, media and internet literacy, data interpretation and analysis, computer programming
- Civic, ethical, and social-justice literacy
- Economic and financial literacy, entrepreneurialism
- Global awareness, multicultural literacy, humanitarianism
- Scientific literacy and reasoning, the scientific method
- Environmental and conservation literacy, ecosystems understanding
- Health and wellness literacy, including nutrition, diet, exercise and public.

Appendix 3: The Blooms Taxonomy

BLOOM'S REVISED TAXONOMY

Remembering	Recalling information, Recognizing, listing, describing, retrieving, naming, finding. *E.g* How many ways can you travel from one place to another? List and draw all the ways you know. Describe one of the vehicles from your list, draw a diagram and label the parts. Collect "transport" pictures from magazines- make a poster with info.
Understanding	Explaining ideas or concepts, Interpreting, summarizing, paraphrasing, classifying, explaining. *E.g*. How do you get from school to home? Explain the method of travel and draw a map. Write a play about a form of modern transport. Explain how you felt the first time you rode a bicycle. Make your desk into a form of transport.
Applying	Using information in another familiar situation Implementing, carrying out, using, executing. *E.g* Explain why some vehicles are large and others small. Write a story about the uses of both. Read a story about "The Little Red Engine" and make up a play about it. Survey 10 other children to see what bikes they ride. Display on a chart or graph.
Analysing	Breaking information into parts to explore understandings and relationships Comparing, organizing, de-constructing, interrogating, finding. *E.g* Make a jigsaw puzzle of children using bikes safely. What problems are there with modern forms of transport and their uses- write a report. Use a Venn Diagram to compare boats to planes, or helicopters to bicycles.
Evaluating	Justifying a decision or course of action, Checking, hypothesizing, critiquing, experimenting, judging. *E.g* What changes would you recommend to road rules to prevent traffic accidents? Debate whether we should be able to buy fuel at a cheaper rate. Rate transport from slow to fast etc..
Creating	Generating new ideas, products, or ways of viewing things, Designing, constructing, planning, producing, inventing *E.g* Invent a vehicle. Draw or construct it after careful planning. What sort of transport will there be in twenty years time? Discuss, write about it and report to the class. Write a song about traveling in different forms of transport.

Appendix. 4 - Sample Black board Plan



Introduction of the lesson

- Review
- If the lesson happens to be first lesson of a new unit or chapter. Should have an introductory activity related to establish new ASK-MT

Body of the lesson

- Today's learning activity based on lesson objective
- Practices

Conclusion of the lesson

- Summary

Appendix 5: Sample timetable

Here is the sample timetable for you to adopt and adjust to your need,

Start		End	Sessions	Minutes
8 : 00	~	8 :25 0:25	ASSEMBLY	25
8 : 25	~	9:05 0:40	1st Class	40
9 : 05	~	9:10 0:05	break	
9 : 10	~	9:50 0:40	2nd Class	40
9 : 50	~	10:25 0:45	RECESS BREAK	30
10 :25	~	11:05 0:20	3rd Class	40
11:05	~	11:10 0:45	break	
11:10	~	11:50 0:05	4th Class	40
11:50	~	12:20 0:45	LUNCH BREAK	30
12:20	~	13:00 1:00	5th Class	40
13:00	~	13:05 0:25	break	
13:05	~	13:45 0:45	6th Class	40
13:45	~	13:50 0:05	break	
13:50	~	14:30 0:45	7th Class	40
			Daily T/L Minutes	280
			Weekly T/L Minutes	1575
			T/L Minutes without Assembly	1350

Note: 5 minutes break in the above time table sample is the preparation time for students / teachers for the next lesson.

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